Total degree of maximal product of two constant intuitionistic fuzzy graphs

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Abstract: This paper explains about the Total degree of Maximal product of Two constant IF graphs. Fuzzy graphs are derived from crisp graphs. Various properties of IF graphs are extended from Fuzzy graphs. Maximal product of Fuzzy graph structures with applications have been discussed in different papers and extended to IF graphs. Constant IF graphs are special type of IF graphs which have same degree for all its vertices. IF Graphs have many applications including the investigation of images by image segmentation, Brain mapping etc, Maximal product of fuzzy graphs is applied in various fields like effective logistic Management, Agricultural product mapping etc. Here in this paper, the Total degree of the vertices in maximal product of Constant IF graphs are studied in detail with definition and various examples and theorems.

Keywords: Fuzzy graph, IF graphs, Investigation of images, Maximal product, Theorems.

1. Overview

Zadeh introduced the principle of fuzzy sets in the year 1965. After his, introduction, many generalisations of this fundamental concepts have been developed.by many Mathematicians in different field related to the Fuzzy. In 1999 Atanassov introduced the notion of an IF set. In 2002 Atanassov along with Shannon further explained about generalisation of an IF Fuzzy graphs. Subsequently, in 2006 & 2009, various properties of IF graph have been discussed by Parvathy & Karunambigai on identical fields. In 2012 Karunambigai, Parvathy & Bhuvaneswari explained in details, the structure of an IF graph on its arcs and the properties of complete IF graph and Constant IF graph. In 2019, Sitara, Muhannad Akram and Muhammad Yusaf introduced maximal products of fuzzy graph structure and analysed the properties with examples. In 2021, Mala, Shanmugapriya & Santhosh Kumar explained the degrees of vertices and edges for the maximal product of an IF Ideals of $M\Gamma$ groups in Near rings.

2. Preliminaries

In this part of the article, few descriptions of an IF graphs, Constant IF graphs and maximal product of an IF graphs are presented.

Definition: 2.1 [1]

Let The Set E Be Fixed. An If Set A In E Takes The Form $A = \{\alpha, M_A(A), \Gamma_A(A)/A \in e\}$ Where The Degrees Of Membership And Non – Membership Of The Element $A \in E$ Are Indicated By The Functions $M_A: E \rightarrow [0,1]$ And $\Gamma_A: E \to [0,1]$ Where $0 \leq M_A(A) + \Gamma_A(A) \leq 1$

Definition: 2.2 [2]

The set $G = \{ < \alpha, \beta >, \mu_G(\alpha, \beta), \gamma_G(\alpha, \beta) / < \alpha, \beta > \in VxV \}$ is said to be an IF graph if these functions $\mu_G: VxV \rightarrow [0,1]$ and $\gamma_G: VxV \rightarrow [0,1]$ define the corresponding degrees of membership and non – membership of the elements $(\alpha, \beta) \in VxV$ over IFSs for all $(\alpha, \beta) \in VxV$ such that $0 \le \mu_G(\alpha, \beta) + \gamma_G(\alpha, \beta) \le 1$.

Using one of this Cartesian Product the following definition is obtained.

Definition: 2.3 [4]

 $\begin{array}{l} \text{Maximum IF graph takes the form } G = (V, E) \text{ where } V = \{v_1, v_2, \ldots, v_n\} \text{ such that } \mu : V \rightarrow [0,1] \text{ and } \\ \gamma : V \rightarrow [0,1] \text{ represent the degrees of membership and non - membership of the element } v_1 \in V \\ \text{respectively with} \qquad 0 \leq \mu(v_i) + \gamma(v_i) \leq 1 \text{ for } i = 1,2,\ldots,n. \\ \text{If } E < VxV \text{ where } \mu : VxV \rightarrow [0,1] \text{ and } \gamma : VxV \rightarrow [0,1] \\ \text{ such that } \mu(v_i, v_j) \leq \max[\mu(v_i), \mu(v_j)] \\ \text{ and } \gamma(v_i, v_j) \leq \min[\gamma(v_i), \gamma(v_j)] \\ \text{ with } 0 \leq \mu(v_i, v_i) + \gamma(v_i, v_i) \leq 1 \text{ for every } v_i, v_i \in E \text{ for } i, j = 1,2,\ldots,n. \\ \end{array}$

Definition: 2.4 [7]

Let $G(\mu, \gamma)$ be an IF graph, the μ – degree of a vertex v_i is

$$d_{\mu}(\mathbf{v}_{i}) = \sum_{(\mathbf{v}_{i}, \mathbf{v}_{j}) \in \mathbf{E}} \mu(\mathbf{v}_{i}, \mathbf{v}_{j})$$

and the γ – degree of the vertex v_i is

$$d_{\gamma}(\mathbf{v}_{i}) = \sum_{(\mathbf{v}_{i},\mathbf{v}_{j})\in E} \gamma(\mathbf{v}_{i},\mathbf{v}_{j})$$

the degree of the vertex is

$$d(\mathbf{v}_i) = \{\sum_{(\mathbf{v}_i, \mathbf{v}_j) \in E} \mu(\mathbf{v}_i, \mathbf{v}_j), \sum_{(\mathbf{v}_i, \mathbf{v}_j) \in E} \gamma(\mathbf{v}_i, \mathbf{v}_j)\}$$

and $\mu(v_i, v_j) = \gamma(v_i, v_j) = 0$ if $(v_i, v_j) \notin E$.

Definition: 2.5. [5]

Let $G(\mu, \gamma)$ be an IF graph with $d_{\mu}(v_i) = k_i$ and $d_{\gamma}(v_j) = K_j$ for all $v_i, v_j \in V$ of the IF graph G(V, E), the graph is denoted as (k_i, k_j) - IFG (or) Constant IFG of degree (k_i, k_j) Definition: 2.6. [7]

Let G(V, E) be an IF graph with $G(\mu, \gamma)$, the total degree of a vertex $v \in V$ is defined as

$$td(u) = \sum_{(v_i, v_j) \in E} d_{\mu}(v_i, v_j) + \mu(v_i), \sum_{(v_i, v_j) \in E} d_{\gamma}(v_i, v_j) + \gamma(v_i)$$

If the total degree of each vertex in G is the same and it is denoted as (r_1, r_2) , then G is called an IF graph of total degree (r_1, r_2) or a (r_1, r_2) totally Constant IF graph.

Definition: 2.6. [6]

Let $GI_1(V_{I_2}E_{I_2}\mu_{I_2}\gamma_{I_2})$ and $G_{I_2}(V_{I_2}E_{I_2}\mu_{I_2}\gamma_{I_2})$ be two graphs of IFIMFGNR I_1 and I_2 is near ring N^{*} then $GI_1 * GI_2 = (V_IE_I\mu_I\gamma_I)$ is called maximal product structure of IFMFGNR. The set of vertices $V_I = V_{I_1} \times V_{I_2}$ exist with

 $\mu_I(r_i, s_i) = \mu_{I_1}(r_i) \vee \mu_{I_2}(s_i) \text{ and } \gamma_I(r_i, s_i) = \gamma_{I_1}(r_i) \wedge \gamma_{I_2}(s_i) \text{ for all } (r_i, s_i) \in V_I$ The set of edges

$$E_{I} = \{(r_{1}, s_{1})(r_{2}, s_{2})\} / r_{1} = r_{2} \text{ and } s_{1}s_{2} \in E_{I_{2}} \text{ (or)}$$

 $\begin{array}{rl} s_1 \ = \ s_2 \text{ and } r_1 \ r_2 \ \in E_{I_1} \text{ exist with } \mu_I(r_1, s_1) \ (r_2, s_2) \\ & = \{\mu_{I_1}(r_1) \ \forall \ \mu_{I_2}(s_1 s_2) \text{ where } r_1 \ = \ r_2 \ \& \ s_1 s_2 \ \in E_{I_2} \end{array}$

 $\begin{array}{l} \{\mu_{I_2}(s_2) \lor \mu_{I_1}(r_1r_2) \text{ where } s_1 = s_2 \& r_1r_2 \in E_{I_2} \\ \text{and } \gamma_I(r_1,s_1) (r_2,s_2) \\ = \{\gamma_{I_1}(r_1) \land \gamma_{I_2}(s_1s_2) \text{ where } r_1 = r_2 \& s_1s_2 \in E_{I_2} \end{array}$

 $\{\gamma_{I_1}(r_1) \land \gamma_{I_2}(s_1s_2) \text{ where } r_1 = r_2 \& s_1s_2 \in E_{I_2} \\ \{\gamma_{I_2}(s_2) \lor \gamma_{I_1}(r_1r_2) \text{ where } s_1 = s_2 \& r_1r_2 \in E_{I_1} \end{cases}$

Definition: 2.7. [6]

The vertex degree of maximal product of IFMFGNR $GI_1(V_{I_1}E_{I_1}\mu_{I_1}\gamma_{I_1})$ and $GI_2(V_{I_2}E_{I_2}\mu_{I_2}\gamma_{I_2})$ is given by: $D(G_1 * G_2) \mu_I(r_j, s_j) = \sum \mu_{I_1}(r_j r_k) \vee \mu_{I_2}(s_j) + \sum \mu_{I_2}(s_j s_i) \vee \mu_{I_2}(r_j)$ and $D(G_1 * G_2) \gamma_I(r_j, s_j) = \sum \gamma_{I_1}(r_j r_k) \wedge \gamma_{I_2}(s_j) + \sum \gamma_{I_2}(s_j s_i) \wedge \gamma_{I_1}(r_j)$

3. Maximal Product of Two Constant If Graph Definition 3.1.

Let $G_1(V', E', \mu', \gamma')$ and $G_2(V'', E'', \mu'', \gamma'')$ be two constant IF graphs then $G_1 * G_2 = (V^*, E^*, \mu^*, \gamma^*)$ is the maximal product structure of G_1 and G_2 with $V'XV'' = V^*$, the set of vertices exist with

$$\mu^*(u_i', u_j'') = \max \left\{ \mu'(u_i'), \mu''(u_j'') \right\}$$

$$\gamma^*(u_i', u_j'') = \min \left\{ \gamma'(u_i'), \gamma''(u_j'') \right\}, \text{ for all } (u_i', u_j'')$$

The set of edges E^* exist if either the first or the co-ordinate of the vertices are same.

$$\mu^*(u_i', u_j'')(u_j', u_i'') = \max \{\mu'(u_i'), \mu''(u_j'')\} \text{ if } u_i' = u_j' \text{ and } u_i''u_j'' \in E''$$
(or)
$$\max \{\mu'(u_i'u_j'), \mu''(u_j'')\} \text{ if } u_i'u_j' \in E' \text{ and } u_i'' = u_j''$$
and $\gamma^*(u_i', u_j'')(u_j', u_i'') = \min \{\mu'(u_i'), \mu''(u_j''u_i'')\} \text{ if } u_i' = u_j' \text{ and if } u_i''u_j'' \in E''$
(or)
$$\max \{\mu'(u_i'u_i'), \mu''(u_j'')\} \text{ if } u_i'u_j' \in E' \text{ and } u_i'' = u_j''$$

Example 3.2.

Consider $G_1(V', E', \mu', \gamma')$ and $G_2(V'', E'', \mu'', \gamma'')$ be to constant IF graphs and $G(V, E, \mu, \gamma)$ is their maximal product of Constant IF graphs



Example 3.3. Consider the following two Constant IF Graphs G_1 and G_2 .







Figure 2.

Let us find the maximal product of these two graphs as G.



Consider the above graph, the degree of the maximal product of the CONSTANT IF graphs are calculated using the above definition as given below:

$$\begin{split} \mathsf{D}(\mathsf{G}_{1}*\mathsf{G}_{2})_{\mu}(\mathsf{v}_{1}\mathsf{x}\mathsf{v}_{1}') &= \mu_{1}(\mathsf{v}_{1}\mathsf{v}_{2})\mathsf{V}\mu_{2}(\mathsf{v}_{1}') + \mu_{1}(\mathsf{v}_{1}\mathsf{v}_{4})\mathsf{V}\mu_{2}(\mathsf{v}_{1}') + \mu_{2}(\mathsf{v}_{1}'\mathsf{v}_{2}')\mathsf{V}\mu_{1}(\mathsf{v}_{1}) + \\ & \mu_{2}(\mathsf{v}_{1}'\mathsf{v}_{3}')\mathsf{V}\mu_{1}(\mathsf{v}_{1}') = 1.7 \\ \mathsf{D}(\mathsf{G}_{1}*\mathsf{G}_{2})_{\gamma}(\mathsf{v}_{1}\mathsf{x}\mathsf{v}_{1}') &= \gamma_{1}(\mathsf{v}_{1}\mathsf{v}_{2})\mathsf{A}\gamma_{2}(\mathsf{v}_{1}') + \gamma_{1}(\mathsf{v}_{1}\mathsf{v}_{4})\mathsf{A}\gamma_{2}(\mathsf{v}_{1}') + \gamma_{2}(\mathsf{v}_{1}'\mathsf{v}_{2}')\mathsf{A}\gamma_{1}(\mathsf{v}_{1}) + \\ & \gamma_{2}(\mathsf{v}_{1}'\mathsf{v}_{3}')\mathsf{A}\gamma_{1}(\mathsf{v}_{1}') = 0.7 \\ (i.e) \quad \mathsf{D}(\mathsf{G}_{1}*\mathsf{G}_{2})(\mathsf{v}_{1}\mathsf{x}\mathsf{v}_{1}') &= (1.7,0.7) \\ \mathsf{D}(\mathsf{G}_{1}*\mathsf{G}_{2})_{\mu}(\mathsf{v}_{1}\mathsf{x}\mathsf{v}_{2}') &= \mu_{1}(\mathsf{v}_{1}\mathsf{v}_{2})\mathsf{V}\mu_{2}(\mathsf{v}_{2}') + \mu_{1}(\mathsf{v}_{1}\mathsf{v}_{4})\mathsf{V}\mu_{2}(\mathsf{v}_{2}') + \mu_{2}(\mathsf{v}_{2}'\mathsf{v}_{1}')\mathsf{V}\mu_{1}(\mathsf{v}_{1}) + \\ & \mu_{2}(\mathsf{v}_{2}'\mathsf{v}_{3}')\mathsf{V}\mu_{1}(\mathsf{v}_{1}') = 1.8 \\ \mathsf{D}(\mathsf{G}_{1}*\mathsf{G}_{2})_{\gamma}(\mathsf{v}_{1}\mathsf{x}\mathsf{v}_{2}') &= \gamma_{1}(\mathsf{v}_{1}\mathsf{v}_{2})\mathsf{A}\gamma_{2}(\mathsf{v}_{2}') + \gamma_{1}(\mathsf{v}_{1}\mathsf{v}_{4})\mathsf{A}\gamma_{2}(\mathsf{v}_{2}') + \gamma_{2}(\mathsf{v}_{2}'\mathsf{v}_{1}')\mathsf{A}\gamma_{1}(\mathsf{v}_{1}) + \\ & \gamma_{2}(\mathsf{v}_{2}'\mathsf{v}_{3}')\mathsf{A}\gamma_{1}(\mathsf{v}_{1}') = 0.7 \\ \mathsf{D}(\mathsf{G}_{1}*\mathsf{G}_{2})(\mathsf{v}_{1}\mathsf{x}\mathsf{v}_{2}') &= (1.8,0.7) \end{split}$$

$$D(G_{1} * G_{2})(v_{1}xv_{3}') = \mu_{1}(v_{1}v_{2}) \forall \mu_{2}(v_{3}') + \mu_{1}(v_{1}v_{4}) \forall \mu_{2}(v_{3}') + \mu_{2}(v_{3}'v_{1}') \forall \mu_{1}(v_{1}) + \mu_{2}(v_{3}'v_{2}') \forall \mu_{1}(v_{1}') = 1.6$$

$$D(G_{1} * G_{2})_{\gamma}(v_{1}xv_{3}') = \gamma_{1}(v_{1}v_{2}) \land \gamma_{2}(v_{3}') + \gamma_{1}(v_{1}v_{4}) \land \gamma_{2}(v_{3}') + \gamma_{2}(v_{3}'v_{1}') \land \gamma_{1}(v_{1}) + \gamma_{2}(v_{3}'v_{2}') \land \gamma_{1}(v_{1}') = 0.6$$

$$D(G_{1} * G_{2})(v_{1}xv_{3}') = (1.6, 0.6)$$

Likewise apply the same technique to determine the degree of each vertex in the maximal product. $D(G_1 * G_2)(v_2xv_1') = (1.9,0.7)$,

 $D(G_1 * G_2)(v_2xv_2') = (2.0,0.7),$ $D(G_1 * G_2)(v_2xv_3') = (1.8,0.6),$ $D(G_1 * G_2)(v_3xv_1') = (1.7,0.5),$ $D(G_1 * G_2)(v_3xv_2') = (1.8,0.5),$ $D(G_1 * G_2)(v_3xv_3') = (1.6,0.4),$ $D(G_1 * G_2)(v_4xv_1') = (1.9,0.7),$ $D(G_1 * G_2)(v_4xv_2') = (2.0,0.7),$ $D(G_1 * G_2)(v_4xv_3') = (1.8,0.6).$

The above example implies the maximal product of two constant IF graphs need not be a constant IF graphs.

Definition 3.4.

If $G(V^*, E^*, \mu^*, \gamma^*)$ is the maximal product of two constant IF graphs $G_1(V', E', \mu', \gamma')$ and $G_2(V'', E'', \mu'', \gamma'')$ then the total degree of the vertices of $G(u_i', u_j'') \in V^*$ is defined as,

$$totaldeg_{\mu^{*}}(u_{i}', u_{j}'') = \sum \max \left\{ \mu'(u_{i}'u_{k}'), \mu''(u_{j}'') \right\} + \sum \max \left\{ \mu''(u_{j}''u_{k}''), \mu'(u_{i}') \right\} + \mu^{*}(u_{i}', u_{j}'')$$

Where $(u_{i}'u_{k}') \in E'$ and $(u_{j}''u_{k}'') \in E''$ and
$$totaldeg_{\gamma^{*}}(u_{i}', u_{j}'') = \sum \min \left\{ \gamma'(u_{i}'u_{k}'), \gamma''(u_{j}'') \right\} + \sum \min \{ \gamma''(u_{j}''u_{k}''), \gamma'(u_{i}') \} + \gamma^{*}(u_{i}', u_{j}''),$$

wher $i, j, k = 1, 2, 3 \dots n$

If each vertex of G has the unique total degree (k',k'') then G is said to a maximal product of IF graphs of total degree (k',k'') or (k',k'')- totally constant maximal product IF graphs.

Example 3.5

Consider the maximal product of two constant IF graphs which is obtained in Example 3.2. The total degree of all 9 vertices of **G** has calculated as follows:

 $\begin{aligned} & totaldeg_{\mu^*}(u_1', u_1'') = (0.4 + 0.4 + 0.3 + 0.3) + 0.3 = 1.7 \\ & totaldeg_{\gamma^*}(u_1', u_1'') = (0.2 + 0.2 + 0.1 + 0.1) + 0.1 = 0.7 \\ & totaldeg_{\mu^*}(u_1', u_2'') = (0.4 + 0.4 + 0.4 + 0.4) + 0.4 = 2.0 \\ & totaldeg_{\gamma^*}(u_1', u_2'') = (0.1 + 0.1 + 0.2 + 0.2) + 0.1 = 0.7 \\ & totaldeg_{\mu^*}(u_1', u_3'') = (0.4 + 0.4 + 0.5 + 0.5) + 0.5 = 2.3 \\ & totaldeg_{\gamma^*}(u_1', u_3'') = (0.2 + 0.2 + 0.1 + 0.1) + 0.2 = 0.8 \\ & totaldeg_{\mu^*}(u_2', u_1'') = 2.1 \\ & totaldeg_{\mu^*}(u_2', u_1'') = 0.9 \\ & totaldeg_{\mu^*}(u_2', u_2'') = 2.3 \\ & totaldeg_{\mu^*}(u_2', u_3'') = 2.5 \end{aligned}$

 $\begin{array}{l} totaldeg_{\gamma^*}(u_2',u_3'')=1.0\\ totaldeg_{\mu^*}(u_3',u_1'')=1.7\\ totaldeg_{\gamma^*}(u_3',u_1'')=0.5\\ totaldeg_{\mu^*}(u_3',u_2'')=2.0\\ totaldeg_{\gamma^*}(u_3',u_2'')=0.5\\ totaldeg_{\mu^*}(u_3',u_3'')=2.3\\ totaldeg_{\gamma^*}(u_3',u_3'')=0.5 \end{array}$

Example 3.6

The following example explains the maximal product of two totally constant IF graph need not be totally constant maximal product IF graphs.





Theorem 3.7

If $G_1(V', E', \mu', \gamma')$ and $G_2(V'', E'', \mu'', \gamma'')$ are two constant IF graphs such that $\mu'(u_i') \leq 1$ $\mu''(u_i''u_k''), \mu'(u_i'u_i') \le \mu''(u_k'') \text{ and } \gamma'(u_i') \ge \gamma''(u_i''u_k''), \gamma'(u_i'u_i') \ge \gamma''(u_k'') \text{ then the total}$ degree of their maximal product $G^*(V^*, E^*, \mu^*, \gamma^*) = G_1 * G_2$ is given by

 $totaldeg_{\mu^*}(u_i', u_j'') = n_0(u_i') \mu''(u_j'') + totaldeg_{\mu''}(u_j'')$ and

 $totaldeg_{\gamma^*}(u_i', u_j'') = n_0(u_i') \gamma''(u_j'') + totaldeg_{\gamma''}(u_j'')$ for $i, j, k = 1, 2, 3 \dots n$

Proof:

If $G_1(V', E', \mu', \gamma')$ and $G_2(V'', E'', \mu'', \gamma'')$ are two constant IF graphs such that $\mu'(u_i') \leq 1$ $\mu''(u_i''u_k'')$, $\mu'(u_i'u_i') \leq \mu''(u_k'')$ then the total degree of the vertices in their maximal product is defined for $i, j, k = 1, 2, 3 \dots n$ as,

$$totaldeg_{\mu^{*}}(u_{i}', u_{j}'') = \sum \max \left\{ \mu'(u_{i}'u_{k}'), \mu''(u_{j}'') \right\} + \sum \max \left\{ \mu''(u_{j}''u_{k}''), \mu'(u_{i}') \right\} + \mu^{*}(u_{i}', u_{j}'')$$

$$Where \quad (u_{i}'u_{k}') \in E', \quad (u_{j}''u_{k}'') \in E'' \quad \text{and} \quad u_{i}' = u_{j}', u_{i}'' = u_{j}''$$

$$= \sum \mu''(u_{j}'') + \sum \mu''(u_{j}''u_{k}'') + \mu^{*}(u_{i}', u_{j}'')$$

$$= n_{0}(u_{i}') \mu''(u_{j}'') + totaldeg_{\mu''}(u_{j}'')$$

$$totaldeg_{\gamma^{*}}(u_{i}', u_{j}'') = \sum \min\{\gamma'(u_{i}'u_{k}'), \gamma''(u_{j}'')\} + \sum \min\{\gamma''(u_{j}''u_{k}''), \gamma'(u_{i}')\} + \gamma^{*}(u_{i}', u_{j}'') \\ \text{Where} \quad (u_{i}'u_{k}') \in E', \quad (u_{j}''u_{k}'') \in E'' \text{ and } u_{i}' = u_{j}', u_{i}'' = u_{j}'' \\ = \sum \gamma''(u_{j}'') + \sum \gamma''(u_{j}''u_{k}'') + \gamma^{*}(u_{i}', u_{j}'') \\ = n_{0}(u_{i}') \gamma''(u_{j}'') + totaldeg_{\gamma''}(u_{j}'') \\ = \sum (u_{i}') \sum (u_{i}'') + u_{i}'' = u_{i}'' \\ = \sum (u_{i}') \sum (u_{i}'') + u_{i}'' = u_{i}'' \\ = \sum (u_{i}') \sum (u_{i}'') + u_{i}'' = u_{i}'' \\ = u_{i}'' + u_{i}'' = u_{i}'' \\ = u_{i}'' + u_{i}'' + u_{i}'' + u_{i}'' + u_{i}'' \\ = u_{i}'' + u_{i}'' + u_{i}'' + u_{i}'' \\ = u_{i}'' + u_{i}'' + u_{i}'' + u_{i}'' \\ = u_{i}'' + u_{i}'' + u_{i}'' + u_{i}'' \\ = u_{i}'' + u_{i}'' + u_{i}'' + u_{i}'' \\ = u_{i}'' + u_{i}'' + u_{i}'' + u_{i}'' \\ = u_{i}'' + u_{i}'' + u_{i}'' + u_{i}'' \\ = u_{i}'' + u_{i}'' + u_{i}'' + u_{i}'' \\ = u_{i}'' + u_{i}'' + u_{i}'' \\ = u_{i}'' + u_{i}'' + u_{i}'' \\ = u_{i}'' + u_{i}'' + u_{i}'' + u_{i}'' \\ = u_{i}'' + u_{i}'' + u_{i}'' + u_{i}'' \\ = u_{i}'' + u_{i}'' + u_{i}'' + u_{i}'' \\ = u_{i}'' + u_{i}'' + u_{i}'' + u_{i}'' \\ = u_{i}'' + u_{i}'' + u_{i}'' + u_{i}'' \\ = u_{i}'' + u_{i}'' + u_{i}'' + u_{i}'' + u_{i}'' \\ = u_{i}'' + u_{i}'' + u_{i}'' + u_{i}'' + u_{i}'' + u_{i}'' + u_{i}'' \\ = u_{i}'' + u_{i}''' \\ = u_{i}'' + u_{i}'' + u_{i}'' + u_{i}'' + u_{i}'' + u_{i}'' + u_{i}''' + u_{i}''' + u_{i}'' + u_{i}'' + u_{i}''' + u_{i}''$$

Here $n_0(u_i')$ is the number of edges incident at u_i' in $G_1(V', E', \mu', \gamma')$

Theorem 3.8

If $G_1(V', E', \mu', \gamma')$ and $G_2(V'', E'', \mu'', \gamma'')$ are two constant IF graphs such that $\mu'(u_i') \leq \mu''(u_j''u_k'')$, $\mu'(u_i'u_j') \leq \mu''(u_k'')$ and $\gamma'(u_i') \geq \gamma''(u_j''u_k'')$, $\gamma'(u_i'u_j') \geq \gamma''(u_k'')$ with second constant IF graph is (C_1^{**}, C_2^{**}) then the total degree of the vertices of their maximal product is

 $totaldeg_{\mu^{*}}(u_{i}', u_{j}'') = totaldeg_{\mu''}(u_{j}'') + n_{0}(u_{i}')C_{1}^{**} \text{ and}$ $totaldeg_{\gamma^{*}}(u_{i}', u_{j}'') = totaldeg_{\gamma''}(u_{j}'') + n_{0}(u_{i}')C_{2}^{**}$

Theorem 3.9

If $G_1(V', E', \mu', \gamma')$ and $G_2(V'', E'', \mu'', \gamma'')$ are two constant IF graphs such that $\mu''(u_i'') \leq \mu'(u_j'u_k')$, $\mu''(u_i''u_j'') \leq \mu'(u_k')$ and $\gamma''(u_i'') \geq \gamma'(u_j'u_k')$, $\gamma''(u_i''u_j'') \geq \gamma'(u_k')$ then the total degree of the vertices in their maximal product $G(V^*, E^*, \mu^*, \gamma^*) = G_1 * G_2$ is given by

 $totaldeg_{\mu^{*}}(u_{i}', u_{j}'') = n_{0}(u_{j}'') \mu'(u_{i}') + totaldeg_{\mu'}(u_{i}') \text{ and}$ $totaldeg_{\gamma^{*}}(u_{i}', u_{j}'') = n_{0}(u_{j}'') \gamma'(u_{i}') + totaldeg_{\gamma'}(u_{i}') \text{ for } i, j, k = 1,2,3 \dots n$

Proof:

Let $G_1(V', E', \mu', \gamma')$ and $G_2(V'', E'', \mu'', \gamma'')$ are two constant IF graphs such that $\mu''(u_i'') \leq \mu'(u_i''u_k')$, $\mu''(u_i''u_j'') \geq \mu'(u_k)$ and $\gamma''(u_i'') \geq \gamma'(u_j'u_k)$, $\gamma''(u_i''u_j'') \geq \gamma'(u_k)$ then by definition of total degree, their maximal product $G_1 * G_2 = G(V^*, E^*, \mu^*, \gamma^*)$ is obtained as totaldeg $*(u_i''u_i'')$

Similarly,

$$totaldeg_{\gamma^{*}}(u_{i}', u_{j}'') = \sum \min \left\{ \gamma'(u_{i}'u_{k}'), \gamma''(u_{j}'') \right\} + \sum \min \left\{ \gamma''(u_{j}''u_{k}''), \gamma'(u_{i}') \right\} + \gamma^{*}(u_{i}', u_{j}''),$$

Where $(u_{i}'u_{k}') \in E', \quad (u_{j}''u_{k}'') \in E''$ and $u_{i}' = u_{j}', u_{i}'' = u_{j}''$
$$= \sum \gamma'(u_{i}'u_{k}') + \sum \gamma'(u_{i}') + \gamma^{*}(u_{i}', u_{j}'')$$

$$= \sum_{k} \gamma'(u_{i}'u_{k}') + \sum_{k} \gamma'(u_{i}') + \min\{\mu'(u_{i}'), \mu''(u_{j}'')\} \\= n_{0}(u_{j}'') \gamma'(u_{i}') + totaldeg_{\gamma'}(u_{i}')$$

Hence $n_0(u_i'')$ is the number of edges incident at u_i'' in $G_2(V'', E'', \mu'', \gamma'')$ constant IF graphs.

Theorem 3.10

If $G_1(V', E', \mu', \gamma')$ and $G_2(V'', E'', \mu'', \gamma'')$ are two constant IF graphs such that $\mu''(u_i'') \leq 1$ $\mu'(u_j'u_k'), \mu''(u_i''u_j'') \le \mu'(u_k') \text{ and } \gamma''(u_i'') \ge \gamma'(u_j'u_k'), \gamma''(u_i''u_j'') \ge \gamma'(u_k') \text{ with second}$ constant IF graph is $(\mathcal{C}_1^*, \mathcal{C}_2^*)$ then their maximal product has the total degree as

$$totaldeg_{\mu^{*}}(u_{i}', u_{j}'') = totaldeg_{\mu'}(u_{i}') + n_{0}(u_{j}'')C_{1}^{*} \text{ and} totaldeg_{\nu^{*}}(u_{i}', u_{j}'') = totaldeg_{\nu'}(u_{i}') + n_{0}(u_{j}'')C_{2}^{*}$$

4. Conclusion

The total degree of the vertices of the maximal product of two constant IF graphs has been explained with definitions, examples and theorems. Also, theorems on different conditions related to two different constant IF graphs on membership and non - membership values are proved explicitly.

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