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Modelling and prediction of economic growth for Nigeria under the violation of linear model assumptions: A robust principal component regression approach

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Abstract: This study was conducted to model, estimate, and predict Nigeria's economic growth (RGDP) by examining the influence of key macroeconomic drivers: internal debt (INDT), external debt (EXDT), interest rate (RINR), exchange rate (REXR), and the degree of economic openness (OPEN). Preliminary exploratory and diagnostic analyses revealed significant challenges to classical linear regression assumptions, particularly the presence of multicollinearity and outliers. To address these issues, robust principal component regression (PCR) estimation methods were employed. Principal component analysis (PCA) extracted two uncorrelated predictors (PC1 and PC2), which captured the joint variability of the original determinants while addressing collinearity. Subsequently, robust estimation techniques—namely M-estimation, S-estimation, and MM-estimation—were used to generate efficient estimated parameters. A comparative evaluation based on root mean square error (RMSE), mean absolute error (MAE), mean absolute percentage error (MAPE), and Theil's inequality coefficient established that the M-estimation method outperformed its alternatives, providing the most stable and reliable predictions of RGDP. Empirical findings revealed that both PC1 and PC2 had positive and statistically significant influences on RGDP, with contributions of 35.39% and 22.15%, respectively. These results highlight the importance of robust PCR in addressing econometric anomalies and offer valuable policy insights into how structural shocks—such as exchange rate volatility, oil price fluctuations, and COVID-19 disruptions—affected Nigeria's economic performance.
Keywords: *Debts, Economic growth, Linear model assumptions principal component, Monetary policy, Robust estimation. Trade openness.*

1. Introduction

The continued upsurge in per capita national output over an extended period is describes or terms as economic growth. It's typically a measurable expansion in the monetary value for the supplied of goods and services within an economy during a specific year. This is generally assessed through proportional variations in cumulative indicators such as gross national product or gross domestic product [1]. Thus, it is an essential macroeconomic policy that all countries across the world including Nigeria, continually strive to achieve. Regrettably, Nigeria is not doing well economically due to the low level and instability of the economic growth compared with the other nations of the World. Despite a significant amount of revenue generated mainly from oil and gas resources, since gaining independence, the global performance of the Nigeria's economy is neither encouraging nor impressive. It has been noted that stable economy is crucial for national advancement [2]. Therefore, the growth of the economy is continuously considered as a key policy by the government and its developmental

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objectives. Fundamentally, economic progress is influenced by policies designed to transform or restructure real economic sectors for better performance. According to [3], sustainable economic growth depends mainly on the country's capability to attract investment, ensure productivity and efficiency with the available resources.

However, the lack of judicious use of available resources, lack of savings, and failure to invest during an economic boom to support and sustain the viable economic sectors are the main obstacles to a country's economic development [4]. In developing countries like Nigeria, savings provide the needed capital to drive investment that can lead to an improved economy. The more the savings, the better the access to capital and the productive activities that strengthen income generation and reduce the government's reliance on borrowing. Lack of sufficient generated revenues lead to inadequate or no saving that resulting to a low level of economic growth. Thus, it can be stressed that savings are vital antidote for economic development. [2] posited that immediately after independence, the main concerns is on, how to reduce extreme poverty through a robust economic growth and development, how to ensure adequate health care facilities, how can illiteracy be tackled, how to ensure a safety democratic governance and steady political activities, how to maintain the integrity and sustainability of the normal environment, how can a violence and crime rate be reduced, become the main focus of directing investment from international capital. It can be emphasized that, long terms and broad-based economic growth are key to generate adequate and sufficient incomes to enable Nigeria reach her full potential in trade and investment partner with the other countries in the world. Thus [5] opined that Nigeria's economic growth remains quite uncertain when related to other emerging developing economies across the globe. In response to the problems, the Nigerian government and policymakers have announced a range of macroeconomic plans intended to stimulate justifiable growth. These initiatives comprise the execution of monetary and fiscal policies, the embracing of export advancement approaches, the quest for changing importation procedures, and, at times, the introduction of austerity plans. Recently, subsidy removal, floating of exchange rate, ongoing tax reform and so on are introduced.

Hence, the ultimate objectives of the policies are economy's stability, promote employment generations, enhance growth output, sustain development, ensure poverty reduction and adequate food security. These objectives are vital in achieving and promoting long terms economic growth. It must be noted that policies that are valuable for long terms growth will attract numerous divided opinions among scholars and researchers. De Long and Summers [6] stressed the importance of macroeconomic policies in driving growth in the long-run. [7] as cited in Ismaila and Imoughele [7] and Ebiwonjumi, et al. [8] opined the significant and effective role of monetary policy in advancing the economy compared to alternative policy measures. Thus, there is a need to support monetary indicators over fiscal indicators in determining economic stability. [10] posited that monetary policy indicators with effective or reliable fiscal policy indicators ascertained better productive activities. Barro [9] as cited in Ebiwonjumi, et al. [2] asserted that growth in human capital development such as, education, training and re-training of personnel, no doubt contributes to the economy.

Moreso, at the implementation of various macroeconomic policy interventions, it is evidently revealed that, Nigeria's economic performance showed substantial instabilities over the period under assessment. The national growth rate stood at 8.2% in 1990 but declined gradually to 5.4% in 2000, 4.6% in 2001, and 3.5% in 2002. An outstanding rebound happened in 2003, with growth rising to 9.6%, shadowed by another decline to 5.8% in 2005. Subsequently, uncertain increases were recorded, with growth rates of 6.4% in 2008 and 7.3% in 2011. The average economic growth rate was 7.0% from 2012 to 2014, which fell to 2.7% in 2015 and -1.6% in 2016. This bounced back to 0.8% in 2017, 1.9% in 2018, and then to 2.0% in the first half of 2019. In 2021, the economic growth rates stood at 1.1%, due to the impact of pandemic in 2020. This further increased to 3.3% in 2022 and declined to 2.9% in 2023. In addition, 3.0% growth rate was projected for 2024 (International monetary fund, 2023). Hence, for Nigeria to achieve her ultimate goal, it is imperative to have accurate and reliable economic growth rate. Thus, to put Nigeria on a sustainable economic trajectory, government and policymakers need to focus on the economic growth drivers such as debts both internal and external, monetary policy instruments and degree of economic openness that will be considered for this study.

As observed over the years, to model and predict economic growth become a difficult task due to the dynamics associated with the various macroeconomic variables such as debts, monetary policy instruments and degree of economic openness. Theoretically, to grow economy in any developing nation like Nigeria, where the internally generated revenue is insufficient to engender growth and development, the endowed human and mineral resources need to be transformed to spur growth and development through infrastructural development and industrial revolutions, there is need for borrowing either through internal or external sources. However, to make use of resources which is not belonging to the nation comes with payment determined by interest and exchange rate as well as the ratio of importation to exportation, a measured of economic openness of a given country. Thus, the complexity of modeling and predicting economic growth using the classical least square estimation method for linear model and as such the need for appropriate, reliable and flexible approaches to achieve this purpose. This serves as a great motivation behind this study for Nigeria.

2. Literature

Da'silva [10] investigated the nexus between economic openness and growth output in Nigeria (1970–2010). The key macroeconomic variables employed for the analysis includes real GDP, trade openness, monetary policy instruments, and changes in unemployed populaces. An OLS regression technique used for the analysis revealed a positive and statistically significant relationship between trade openness and growth output. Based on these results, the study emphasizes the importance of leveraging export-led revenue generation, focusing on the diversification of the economy, and formulating policy frameworks that promote exports as critical approaches for promoting sustainable growth output in Nigeria. Ugbor [11] investigated the causal relationship that exists nexus between Nigeria's economy openness and its growth, with specific emphasis on the comparative dynamics before and after the operation of the Structural Adjustment Programme (SAP). The analysis covered two different periods: the pre-SAP era (1970Q1–1985Q4) and the post-SAP era (1986–2011). Augmented Dickey–Fuller (ADF) and Phillips–Perron (PP) unit root tests, alongside Granger causality and cointegration techniques were applied to assess the basic relationships, on quarterly data on degree of economy openness, government spending, and investment. The empirical findings showed that degree of economy openness exerted a stronger positive influence on economic growth during the post-SAP period when compared to the pre-SAP era.

Ude Damian [12] assessed the relevance of trade openness within the context of Nigeria's trade policy framework. In the study, advanced econometric techniques, including Autoregressive Conditional Heteroscedasticity (ARCH), Generalized Autoregressive Conditional Heteroscedasticity (GARCH), and Granger causality tests were employed. The analysis showed a significant relationship between the degree of openness and trade policy outcomes. Furthermore, the findings indicated that both lending interest rates and the exchange rate of the naira against the US dollar play critical roles in influencing economic growth. Based on these findings, it can be emphasized that policymakers should adopt trade policies that are growth-enhancing and also conducive to sustainable macroeconomic stability. Adigwe, et al. [13] evaluated the effect of monetary policy on Nigeria's economic growth employing OLS estimation technique. Findings from the study revealed positive contributions of monetary policy to economic growth; however, its effectiveness was weakened by the opposing influenced of inflation. Based on these results, the study stressed the need for strategic use of monetary policy instruments, specifically interest rate regulations, liquidity control, and the management of exchange rate, to provide an enabling environment for investment and sustainable economic development. Okorontah and Odoemena [14] examined the impact of exchange rate fluctuations on Nigeria's economic growth over the period 1986–2012. The study using secondary time series data sourced from the Central Bank of Nigeria (CBN) Statistical Bulletin, on Real GDP was utilized as a proxy for economic growth, while exchange rate, money supply, and inflation served as the explanatory variables. In the analysis, OLS estimation method, Johansen cointegration test, and the Error Correction Mechanism (ECM) to assess both short-run and long-run dynamics among the variables. In the findings, it was revealed that exchange rate fluctuations had insignificant influence on economic growth during the period under study.

Nwannebuike, et al. [15] carried out an empirical investigation into the relationship between foreign debt and economic growth in Nigeria. based on the analysis of collected data for the study, it was found that while external debt positively influenced economic growth in the short run, it had adverse impact in the long run. This was supported in [16] where ARDL and Bound tests was used for analyzing data that was sample from the Tanzanians between 1971-2011. The result obtained revealed external debt and economic growth were positively related during the period under consideration [17] examined the relationship between exports, imports, and economic growth in Turkey over the period 1960–2015, considering annual time series data sourced from the World Development Indicators. The analytical techniques employed were unit root tests to evaluate stationarity, the Johansen cointegration test to assess long-run relationship, and a Vector Autoregression (VAR) method to capture dynamic interrelationship among the variables. Thus, from the findings, no evidence of autocorrelation was found among the series. However, the results showed a unidirectional causal relationship running from exports to economic growth and from imports to economic growth, thereby stressing the important role of trading played in influencing Turkey's economic growth [18] investigated the relationship between international trade and economic progress across 17 Arab countries over the period 1995–2013. In the study, exports and imports were considered the key explanatory variables, while Gross Domestic Product (GDP) was used as the dependent variable. The empirical results showed that both exports and imports positively and significantly influenced economic growth within the sampled nations.

Ucan, et al. [19] assessed the nexus between exports and economic growth in Turkey by utilizing time-series data covering the period from 2006 to 2015. In the empirical model considered in the study, gross domestic product (GDP) was used as the dependent variable, while exports and imports were used as the explanatory variables. Johansen cointegration technique and Granger causality tests were applied to examine the long-run and short-run dynamics. In the results, the existence of a unidirectional causal relationship running from exports to economic growth was revealed. Thus, emphasized that an increase in exports led to a positive and significant impact on the development of the Turkish economy [20] empirically evaluated the impact of international trade on Nigeria's economic growth using OLS estimation method. The results revealed a positive relationship between the identified explanatory variables and economic growth except foreign direct investment and exchange rates, which were found to be negatively influenced economic progress. It can be stressed from these findings that, Nigeria's trade policy should not dependent only on oil exports. Rather, more effort should be placed on diversification of the export base, through the promotion of non-oil and non-primary export commodities.

Arshad and Usman [21] evaluated the relationship between exports and economic growth in Pakistan over the period 1992–2015, an annual time series data sourced from the World Development Indicators (WDI) and International Monetary Fund (IMF) databases were considered for the study. GDP was used as the dependent variable, while exports, foreign exchange reserves, and foreign direct investment (FDI) were incorporated as explanatory variables. The analysis was done using regression technique along with unit root, the Johansen cointegration approach, and Granger causality. The results showed that both exports and foreign exchange reserves had a positive and a statistical significance influenced on the economy [22] Conducted research to investigate the influence of international trade on Nigeria's economy over the period 1985–2015. The analysis employed secondary data on key macroeconomic indicators, including interest rates, balance of trade, exports, and trade openness. To ensure robustness of the study, unit root and cointegration tests, followed by estimation using a Vector Error Correction (VEC) model. Empirical results revealed that, the relationship between imports and trade openness is statistically insignificant in the long run. Also, the results further revealed, a unidirectional causality from trade openness to economic growth. Based on these findings, the study highlighted the importance of policy procedures designed to boost exports while restricting undue dependence on imported goods, thereby enhancing the contribution of international trade to sustainable economy.

Alper [23] assessed exchange rate volatility on international trade in Turkey during the period 2002–2013. Data from fifteen European countries was gathered and explored. The Generalized Autoregressive Conditional Heteroscedasticity (GARCH) (1, 0) employed for the analysis, showed that volatile exchange rate reduced inflows in the short-run, but importation was revealed to be positive and negative in the long run [24] investigated dynamics exchange rate and Nigeria's economy over the period 1970–2013. The analytical techniques employed were OLS method, unit root test and co-integration analysis, and error correction model (ECM). The dynamics of exchange rate variability during the period was captured by using standard deviation. The results indicated that exchange rate fluctuations influenced on economy was positive but statistically insignificant. The study posited that while exchange rate variations contributed to grow the economy, the influenced was inadequate, reflecting the efforts of monetary authorities to design and implement policies capable of mitigating the adverse effects of exchange rate volatility on the Nigerian economy.

Ayomitunde, et al. [25] examined the impact of monetary policy on Nigeria's economic growth over the period 1990–2017. The study employed Autoregressive Distributed Lag (ARDL) bounds testing approach. Empirical results revealed a significant positive effect of monetary policy rate on Nigeria's economy in the short run, while inflation influenced was positive in both the short-run and long-run. These results underscored the relevance of executing well-calibrated monetary policy approaches that fostering on sustainable economic transformation [26] investigated external debt and economic growth in Nigeria over the period 1999–2015, with the aim of assessing the impact of the former on the latter. In the analysis, cointegration technique along with a vector error correction model (VECM) were employed to evaluate the time-series data. Empirical results revealed that external debt and Nigeria's economic growth were significant and inversely related, suggesting that increasing debt profiles served as constraint to the progress of the economy

AL-Tamimi and Jaradat [27] study focused on Jordan for the period 2010–2017, and employed a descriptive-analytical technique to evaluate the influence of external debt on economic growth. The results showed a statistically significant negative influenced, stressing that unrestricted external borrowing adversely affected economic transformation. In contrast, [28] carried out an empirical investigation on Nigeria covering the years 1980–2016, using data from the Central Bank of Nigeria. The Generalized Method of Moments (GMM) technique employed for the study revealed a statistically significant positive relation between external debt and economy position, positing that external borrowing contributed to the expansion of the economy during the period under investigation [29] examined the impact of government debt on economic growth in Nigeria over the period 1998–2018. In the analysis, descriptive statistics, unit root testing, cointegration techniques, and a vector error correction model (VECM) were employed to examine both short-run and long-run dynamics. Findings from the study indicated that external debt had a negative influenced on growth of the economy in both the short and long run, where domestic debt effect was revealed to be positive and significant for both periods.

Fatbardha, et al. [30] examined exchange rate volatility and economic growth in Central and Eastern European (CEE) countries. The analysis was done based on data from fourteen CEE economies spanning the period 2002–2018. Exchange rate volatility measured using both the standard deviation and Z-score, a widely recognized robust indicators were used. A fixed-effects panel least squares estimation technique was employed, the empirical findings indicated that exchange rate volatility had a statistical significance and adverse effect on the economy of the region. Based on findings, the importance of exchange rate stability was emphasized, suggesting that policymakers need to adopt a unified and consistent exchange rate policy approach to boost and sustain the economy on the path of growth [31] researched on trade policy and Nigeria's economy by employing the Autoregressive Distributed Lag (ARDL) method. The findings revealed that price-related indicators and the adjusted trade ratio had a positive influence on the economy in both the short and long run. Also, the long-run dynamic analysis indicated that economy responded favorably to trade policy measured. Based on these results, the study emphasized the importance of policy interventions that encourage international trade expansion and foster innovation as strategies for sustaining economy on growth trajectory.

Ring, et al. [32] investigated external debt and economic growth, utilizing institutional quality as a moderating variable. panel GMM) least square method was employed to explore the data from 23 countries covering the period 2011–2014. The results revealed the significant of the institutional quality for the moderated impact of external debt on the economy. Explicitly, strong governance and sound institutional practices were found to enhance the positive effects of debt utilization on growth, while mitigating its adverse consequences. The findings further suggested that effective debt management, supported by reliable and sustainable policy frameworks, is important for controlling external debt burdens and fostering long-term economic development [33] examined public debt accumulation and Nigeria's economy over the period 1980–2018. Employing the Autoregressive Distributed Lag (ARDL) model, the empirical results revealed that external debt exerted a negative influence on long-term economic growth, despite exhibiting short-run growth-enhancing effects. In contrast, domestic debt showed adverse impact on economic growth in both the short and long run, thus reflecting a more persistent constraint compared to external borrowing.

Ebiwonjumi, et al. [34] applied an alternative estimation approach to assess economic growth and its key determinants in Nigeria, particularly under conditions of multicollinearity among explanatory variables. The analysis focused on a set of macroeconomic indicators, including real gross domestic product (RGDP), domestic debt, external debt, interest rate, exchange rate, and trade openness. Preliminary analysis of the data, complemented by the computation of variance inflation factors, confirmed the existence of multicollinearity within the dataset. The statistical challenged was addressed to ensure reliable parameter estimation, by employing ridge regression with appropriate ridge constant to show an efficient estimated parameter of the determinants.

However, there were numerous studies and reviews on the relationship between macroeconomic variables and economic growth in Nigeria, many have failed or overlooked potential violations of classical linear model assumptions, particularly the issues of multicollinearity and the presence of outliers. To address this gap in methodology, this study sort to model and predicts Nigeria's economic growth using internal debt, external debt, interest rate, exchange rate, and trade openness as explanatory variables, while explicitly accounting for multicollinearity and outlier effects. This is said to be achieved through the application of a robust principal component method. It can be emphasized that few or lack of empirical studies exists on modeling and forecasting Nigeria's economic growth under the identified conditions. By adopting this methodological approach, this study aims to fill an important gap in the literature and extend the body of knowledge in this domain thereby, contributes to the existing work in the identified area of research.

3. Material and Method

In this study, robust principal component regression method is used to model and predict economic growth under the violation of linear model assumptions particularly, the issue of multicollinearity and presence outliers in the dataset used. According to Draper and Smith [35] and [38] as cited in Ebiwonjumi, et al. [2], a principal component can be used to address the multicollinearity problem associated with a predictive model. Thus, it can be used to generate estimates and predictions that is more efficient and stable when compared with the estimates and predictions from the ordinary least squares. Empirically [36] studied macroeconomic variables, which many economists and researchers have been neglected for over long period of time in a preliminary investigation to ascertain their similarities and differences. In the study, a principal component analysis was carried out to assess nine macroeconomic variables and the results revealed the level of redundancy among the variables in the correlation matrix. Also, Scree plots of principal components revealed the grouping of variables into three factors or components.

Alphonsus and Raji [37] applied principal component analysis (PCA) to address the issue of severe multicollinearity among sixteen morphological variables measured from fifty multiparous Bunaji cows. The analysis showed the consolidation of correlated variables into composite scores, thereby reducing the original set of morphological traits to four uncorrelated components [38] carried out a study to identify the key determinants influencing the effectiveness of economic policy in supporting the growth of small enterprises within the food, beverage, and tobacco sector in North Sumatra, Indonesia. The study employed a principal component analysis technique to examine multiple economic indicators, including investment, wages, inflation, exchange rates, population, total labor force, gross domestic regional product (GDRP), industrial GDRP, interest rates, and total credit. Findings revealed that population and credit were the two dominant factors shaping the sector's performance.

Nugrahadi, et al. [38] employed principal component analysis (PCA) and hierarchical regression to examine the influence of macroeconomic variables on Kenya's economic performance. In that investigation, 18 macroeconomic indicators spanning the period 1970–2019 were obtained from the Kenya National Bureau of Statistics and the World Bank. PCA was applied to reduce dimensionality and extract components. Subsequently, a hierarchical regression was used to fit a model based on the extracted components for the prediction of growth trajectory. However, the present study seeks to model and predict economic growth (RGDP) based on its identified key macroeconomic drivers while addressing econometric challenges such as multicollinearity and the presence of outliers. To achieve this, a robust principal component regression method is employed. Hence, this analytical technique has not previously been applied to the context of Nigeria, thereby positioning this work to contribute novel insights and bridge an existing gap in the literature on economic growth modeling.

4. Methodology

In this section, a robust principal component regression is discussed to address the violation of linear model assumption such as multicollinearity problem and outliers. Also, this study presents a comprehensive and detail examination of robust estimation methodologies, with particular focus on the M-estimator, S-estimator, and MM-estimator methods.

4.1. Test for Multicollinearity (Variance Inflation Factor (VIF))

Multicollinearity is a statistical phenomenon in which two or more predictor (explanatory) variables in a regression model show a high degree of correlation with one another. In such cases, a given independent variable says $x_1, x_2, \ldots, x_{k-1}$ may share substantial linear relationships with other predictors in the model such as $x_2, \ldots, x_k$, thereby causing confusion in the estimation and interpretation of regression parameter estimates Marquardt [39] and Belsley, et al. [40] and cited by Murray, et al. [41]. Also, challenges may emerge when including or deleting a predictor (explanatory variable), as such adjustments can lead to substantial alterations in the estimated regression coefficients. It is important to know that variations in any of the model's variables can lead to corresponding changes in the estimated parameter values. Moreso, the distribution of parameter estimates, particularly the mean and variance of a given β_i may be either underestimated or overestimated. Such inappropriate estimation can result in deflated or inflated variances, which, in turn, influence the accuracy of inference and the reliability of decisions made from the interpretation of $\beta_i,\ i = 1, 2, \ldots, k$ [40]. The normal description of $\beta_i, i = 1, 2, \ldots, k$ as an average change in the dependent variable as results of a unit or one percent changes in x_i will be incorrect or invalid, when x_1 and x_2 or x_2 and x_3 are strongly correlated, holding x_2 constant when x_1 is increasing will not make any meaningful sense. Thus, [41] said that, a formal method to be used in determining the collinear challenge is VIF, which account for the amount of inflated variance in estimated regression coefficients in comparison with the uncorrelated predictors or explanatory variables Murray, et al. [41] as cited in Ebiwonjumi, et al. [34] and Ebiwonjumi, et al. [2]. To compute the VIF, the standardized regression model given as:

$$Y_i = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_k X_n + U_i\ , \tag{1}$$

is used. This can be written in vector and matrix form as:

$$\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_k \end{bmatrix} = \begin{bmatrix} X_1'X_1 & X_1'X_2 & \dots & X_1'X_k \\ X_2'X_1 & X_2'X_2 & \dots & X_2'X_k \\ \vdots & \vdots & \ddots & \vdots \\ X_k'X_1 & X_k'X_2 & \dots & X_k'X_k \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_n \end{bmatrix} + \begin{bmatrix} U_1 \\ U_2 \\ \vdots \\ U_k \end{bmatrix}, \quad (2)$$

4.2. Variance Inflation Factor (VIF) Formula

In this study, the formula that can be used to estimate VIF within the context of a simple regression model. In addressing multicollinearity, it is vital to know that collinearity can be established through numerical predictors in a regression setting, where two or more predictors may exhibit near-linear or near-planar relationships [40, 42]. Several diagnostic approaches are available to detect such issues. Pairwise correlation coefficients are typically employed when only two predictors are involved, whereas condition indices and VIF are widely used in the presence of multiple predictors [41]. Specifically, emphasis is on the role of VIFs, as their computation offers a measure for each predictor individually, thereby enabling the identification of variables that contribute most significantly to multicollinearity. In the case of a multiple regression model with k predictors, $X_i, i = 1, \dots, k$. the VIFs correspond to the diagonal elements (r^{ii}) of the inverse of the correlation matrix R_{kxk} which is constructed from the set of predictors [40, 42]. According to [41], the VIF for the r^{th} predictor can be formally expressed as

$$\text{VIF}_i = r^{ii} = \frac{1}{1 - R_i^2}, i = 1, 2, \dots, k \ , \quad (3)$$

where VIF for a predictor X_i is defined in relation to the multiple correlation coefficient R_i^2 obtained from regressing X_i on the remaining $k-1$ predictors. As noted in [41], there is no collectively acceptable threshold that clearly differentiates between low and high VIF values. Nevertheless, several scholars have proposed heuristic cut-off points; for instance, [43] recommended that VIF values exceeding 5 or 10 may signal problematic multicollinearity. Furthermore, prior studies by [44-46] emphasized that high VIF values inflate the variance of estimated regression coefficients, thereby overstating their standard errors relative to the case where no correlation exists among the predictors. In this regard, a VIF greater than 10 is often considered as evidence for severe multicollinearity. However, it has also been proposed that the assessment of VIF should not rely solely on fixed thresholds; researchers are advised to account for additional factors, such as sample size, which may influence the stability and variability of coefficient estimates.

4.3. Outliers

The identification and assessment of extreme values, along with their possible impact, represent a critical step in modeling, interpreting, and generalizing the volatility of real GDP and the associated determinants of economic growth in this study. Consequently, it is important to conduct rigorous data exploration and diagnostic testing to detect the presence of outliers. For this purpose, the Grubbs' test is employed.

4.4. Grubbs' Test

The Grubbs' test for outlier detection has been recommended by the International Statistical Organization (ISO) as standardized test due to its ability to combine all observed values of the variable under investigation when computing the test statistic, without necessitating the removal of extreme observations. The approach involves calculating the deviation of a suspected outlier from the sample mean and standardizing this value using the sample standard deviation. An observation is considered as a possible outlier if it deviates significantly from the mean in either direction. The null hypothesis (H_0) for the Grubbs' test states that no outliers are present in the dataset. According to ISO (1994), the test statistic G_m is defined as follows

$$\widehat{G}_m = \frac{|X_*^s - \overline{X}|}{Sd}, \tag{4}$$

$$\widehat{G}_m = \frac{|\text{Suspect value} - \overline{X}|}{Sd}, \tag{5}$$

4.5. Robust Estimators

This study uses three robust estimators to obtain a robust principal component estimate.

4.6. M-estimator

Among the various approaches to robust regression, M-estimation represents the most widely employed technique [47]. This method achieves an efficiency level that is comparable to that of ordinary least squares (OLS). Unlike OLS, which minimizes the sum of squared residuals, M-estimation seeks to minimize a chosen function of the residuals. Formally, the objective function for an M-estimator can be expressed as

$$min\sum_{i=1}^{n} p\frac{re_i}{S} = \min\sum_{i=1}^{n}\rho\left(\frac{Y_i - X'\hat{\beta}_i}{S}\right), \tag{6}$$

The statistics S serves as an estimator of scale, typically derived from a linear combination of the residuals. The associated function quantifies the individual contribution of each residual to the overall objective function. For the function ρ to be considered appropriate, it is generally expected to satisfy the following properties:

$\rho(re) \geq 0,\ \rho(0) = 0, \rho(re) = \rho(-re)$), and ($\rho(re_i) \geq\ \rho(re_i^{\cdot}) \, for \, |re_i| \geq\ |re_i^{\cdot}|$.

The system of normal equations that can be used to explain the minimization problem is derived by computing the partial derivatives of the expression in (6) with respect to β and equating them to zero. This procedure leads to the following expression given by

$$\sum_{i=1}^{n}\varphi\left(\frac{Y_i - X'\hat{\beta}_i}{S}\right)X_i = 0\ , \tag{7}$$

The system of normal equations used to solve the minimization problem is derived by computing the partial derivatives of the expression in (6) with respect to β and equating them to zero. This approach leads to the following given expression

$$X'\varphi X\hat{\beta}_i = X'\varphi Y\ . \tag{8}$$

4.7. S-Estimator

According to [48], the S estimators are implicitly constructed from a scale statistic represented by $s(\theta)$, which denotes a robust M-estimator of the residual scale, given in terms of $re_1(\theta),\ re_2(\theta), \dots,$ $re_n(\theta)$. These estimators are formally defined through the minimization of the overall dispersion of the residuals. To obtain the final scale estimate, the objective is to minimize the dispersion function defined over the residuals, expressed as

$S(re_1(\theta),\ re_2(\theta), \dots,\ re_n(\hat{\theta}))$ such that the scale estimator is given by

$\hat{\sigma} =\ S(re_1(\theta),\ re_2(\theta), \dots,\ re_n(\hat{\theta}))$.

Here, the residual dispersion $re_1(\theta),\ re_2(\theta), \dots,\ re_n(\hat{\theta})$ is formally defined as the solution of expression given by:

$$\frac{1}{n}\sum_{i=1}^{n} P\left(\frac{re_i}{S}\right) = K\ , \tag{9}$$

According to the approach proposed in Rousseeuw and Yohai [48], the Tukey's bi-weight function is recommended for robust estimation. In extension of this, Rousseeuw and Leroy [49] defined the function by selecting the tuning constant $c = 1.5476$ and setting $K = 0.1995$, which ensures a breakdown point of 50%. The residual function is therefore expressed in terms of $P\left(\frac{e_i}{s}\right)$, where K serves as a scaling constant.

$$p(x) = \begin{cases} \frac{x^2}{2} - \frac{x^4}{2c^2} + \frac{x^6}{6c^4} & for\ |x| \le c \\ \frac{c^2}{6} & for\ |x| > c \end{cases} . \tag{10}$$

4.8. MM-Estimator

The MM-estimator represents a refinement within the broader class of M-estimators, which combined two desirable statistical properties: the high asymptotic efficiency characteristic of M-estimators and the strong robustness, in terms of breakdown point, typically associated with S-estimators [50]. The MM-estimator is a robust estimator to have these two properties simultaneously. It is based on this concept that, this estimation relies on the principle of iteratively applying the M-estimation approach in multiple stages to obtain a stable and reliable estimate. The construction of the MM-estimator can be outlined in three sequential phases, as detailed below.

Stage 1: An estimator with a high breakdown point is employed to obtain the initial parameter estimate, denoted as $\tilde{\beta}$. This initial estimate is required to exhibit sufficient efficiency. Based on this preliminary estimate, the corresponding residuals are then computed.

$$r_i(\beta) = y_i - x_i^T \tilde{\beta} , \tag{11}$$

Stage 2: Based on the residuals obtained from the robust fitting process, the objective is given as:

$$\frac{1}{n}\sum_{i=1}^{n} \rho\left(\frac{r_i}{s}\right) = K , \tag{12}$$

where K is a constant, and ρ represent an M-estimator of scale with a breakdown point of 50%, which can be explicitly computed. The scale estimate based on the residuals $s(r_1(\tilde{\beta}), r_2(\tilde{\beta}), \ldots r_n(\tilde{\beta})$ is denoted by s_n. The corresponding objective function is defined in terms of this estimator and is denoted by ρ_0.

Stage 3 introduces the MM-estimator, which is formulated as an M-estimator of the parameter vector β. At this stage, the estimation is carried out using a re-descending score function, expressed as:

$$\emptyset_1(u) = \frac{\partial \rho_1(u)}{\partial u} , \tag{13}$$

In this stage, the scale estimate s_n obtained in stage 2 is used to derived MM-estimator $\hat{\beta}$, which is defined by the expression given as:

$$\sum_{i=1}^{n} x_{ij}\, \emptyset_1\left(\frac{y_i - x_i^T \tilde{\beta}}{s_n}\right) = 0; \ j = 1, 2 \ldots, p . \tag{14}$$

4.9. Principal Component Regression Technique

This technique offers a systematic process for addressing the violation of a linear model assumption such as multicollinearity. Unlike conventional regression methods, its implementation involves additional computational steps that are typically not part of standard regression analysis [2]. The application of principal component analysis arises from the property that any linear regression model can be reformulated using a set of orthogonal independent variable(s) [51]. The new variable(s) are obtained through the formulation and transformation of the original independent variable(s), known as the principal component(s) ([2, 51]). Thus, consider Y to be an $n \times 1$ response vector, X represent an $n \times p$ matrix of predictors, β as a $p \times 1$ vector of unknown parameters and ε as an $n \times 1$ vector of random error terms. Under this approach, there exists a transformation matrix such that the principal components can be expressed as:

$$\Psi'(X'X)\Psi = \Lambda \text{ and } \Psi'\Psi = \Psi\Psi' = 1\,, \quad (15)$$

where Λ denote a diagonal matrix, whose diagonal entries consist of the ordered characteristic roots of $X'X$ denoted by $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_p$. The matrix Ψ can then be utilized to generate a transformed set of explanatory variables as given and expressed as:

$$H = \left(H_{(1)}, H_{(2)}, \dots, H_{(p)}\right)\,, \quad (16)$$

$$X\Psi = \left(X_{(1)}, X_{(2)}, \dots, X_{(p)}\right), \quad (17)$$

The linear combinations derived from the original independent variables are denoted as the principal components. Consequently, the regression model may be reformulated and expressed in terms of these principal components as given by the expression:

$$Y = H\alpha + \varepsilon\,, \quad (18)$$

$$H'H = \Psi'X'X\Psi = \Psi'\Psi\Lambda\Psi'\Psi, \quad (19)$$

where $H = X\Psi$ and $\alpha = \Psi\beta$. The OLS method for the parameter α and its variance-covariance matrix are given as:

$$\widehat{\alpha} = (H'H)^{-1} H'Y = \Lambda^{-1} H'Y\,, \quad (20)$$

$$Var(\widehat{\alpha}) = \sigma^2 (H'H)^{-1} = \sigma^2\Lambda^{-1}\,. \quad (21)$$

Therefore, a relatively small eigenvalue of the matrix $X'X$ is an indication that the associated regression coefficient is estimated with high variance. From the expression given in (15) and (19), this can be put as:

$$H'H = \Psi'X'X\Psi = \Psi'\Psi\Lambda\Psi'\Psi = \Lambda\,, \quad (22)$$

The eigenvalues, denoted as λ_j, which represent the variance explained by the j^{th} principal component. When all eigenvalues are equal to one, the original regressors are mutually orthogonal. In contrary, if any eigenvalue is exactly zero, this implies the presence of a perfect linear dependency among the regressors. Principal Component Regression (PCR) addresses the issue of multicollinearity by fitting the regression model using a reduced subset of principal components rather than the full set. To derive the principal components estimator, the regressors are first ordered according to their associated eigenvalues, such that $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_p > 0$. In PCR, components corresponding to eigenvalues that are close to zero are excluded from the model, and the least squares estimation is employed to the retained components. The resulting estimator is given in (18) and can be expressed as:

$$Y = X\Psi\Psi'\beta + \varepsilon\,, \quad (23)$$

$$Y = H\alpha + \varepsilon\,, \quad (24)$$

where $H = X\Psi, \alpha = \Psi'\beta$ and $\Psi = \left(\psi_1, \psi_2, \psi, \dots, \psi_p\right) = (\Psi_r, \Psi_{p-r})$ is a $p \times p$ orthogonal matrix with the expression given by:

$$\left(\Psi_r, \Psi_{p-r}\right)' X'X\left(\Psi_r, \Psi_{p-r}\right) = A = \begin{pmatrix} W_r & 0 \\ 0 & W_{p-r} \end{pmatrix}, \quad (25)$$

where $0 < r \leq p$, $\mathrm{W} = \left(\psi_1, \psi_2, \psi, \dots, \psi_p\right)$, $W_r = diag(\psi_1, \psi_2, \psi_3, \dots, \psi_r)$, $W_{p-r} = \left(\psi_{r+1}, \psi_{r+2}, \psi_{r+3}, \dots, \psi_p\right)$ and $\psi_1 \geq \psi_2 \geq \psi_3 \geq \dots \geq \psi_p > 0$. The diagonal elements of this decomposition correspond to the ordered eigenvalues of $X'X$. By construction, expression obtained from (23) can be put as:

$$H = X\Psi = \left(H_r, H_{p-r}\right). \quad (26)$$

H represents $n \times p$ matrix of principal components, In particular the i^{th} principal component. Also, consider H_{p-r} that contains principal components with corresponding eigenvalues that is closed to zero, thus, emphasizing that H can be partitioning into H_r and H_{p-r}, such that H_{p-r} are eliminated. Thus, expression given in (24) can be re-written and expressed as:

$$Y = H_r\alpha_r + H_{p-r}\alpha_{p-r}\alpha + \varepsilon\,, \tag{27}$$

The estimated parameter (α) is given as:

$$\widehat{\alpha} = (W)^{-1}H'Y\,, \tag{28}$$

From expression in (8) and (28), the M-estimator of the α can be given as:

$$\widehat{\alpha}_M = \left(W_\varphi\right)^{-1}H'\widehat{\varphi}Y \tag{29}$$

where $W_\varphi = H'\widehat{\varphi}Y, \widehat{\varphi}$

is the derivative of p. Based on (27) and (28), the principal component estimator of the α can be expressed in the form given by:

$$\widehat{\alpha}_{PC} = (W_r)^{-1}{H_r}'Y\ . \tag{30}$$

To address multicollinearity among the explanatory variables under consideration, principal components with highest cumulative proportion of the variance explained are selected to replace the original variables. Follows from this, are the empirical results and discussion of various analyses, including a robust principal component regression model, its associated diagnostic tests, and the evaluations conducted, are presented in the next section.

5. Empirical Results

In this section, the empirical results of fitted models discussed under research methodology are presented. Thus, in order to assess the degree of independence among of the identified determinants of economic growth that include internal debt (INDT), external debt (EXDT), interest rate (RINR), exchange rate (REXR), and trade openness (OPEN). These are used as covariates to determine real GDP growth (RGDP). Prior to their inclusion in the model, the pairwise associations among these growth determinants are evaluated using Pearson's correlation coefficients, and the results are presented in Table 1.

Table 1.
Correlation matrix (*p*-values) of the variables.

	INDT	EXDT	RINR	REXR	OPEN
INDT	1.0000	0.6074	0.1276	-0.0924	0.8201
p-value		(0.0000)	(0.1320)	(0.2760)	(0.0000)
EXDT		1.0000	0.3876	-0.2898	0.2534
p-value			(0.0000)	(0.0000)	(0.0020)
RINR			1.0000	-0.4594	-0.0090
p-value				(0.0000)	(0.9160)
REXR				1.0000	0.2416
p-value					(0.0040)
OPEN					1.0000
p-value					

The correlation coefficients results presented in Table 1 provide primary understandings into the relationships among the major drivers of economic growth in Nigeria, namely internal debt (INDT), external (EXDT), interest rate (RINR), exchange rate (REXR), and degree of economy openness (OPEN). Correlation analysis serves as an important diagnostic tool in empirical research, as it not only illustrates the strength and direction of association between variables but also signals potential issues of multicollinearity that may bias subsequent regression estimates [52]. The results reveal that internal debt (INDT) is positively associated with external debt (r = 0.6074, $p < 0.01$), trade openness (r = 0.8201, $p < 0.01$), and, to a lesser extent, the real interest rate (r = 0.1276, which is statistically insignificant). The strong and statistically significant correlation between INDT and OPEN suggests

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that proper management of internal borrowing in Nigeria's has tendency to enhance participation in international trade.

Similarly, external debt (EXDT) shows a positive and significant correlation with the real interest rate (r = 0.3876, $p < 0.01$) and trade openness (r = 0.2534, $p < 0.01$). This implies that, in Nigeria, when the demand for external borrowing is high, it is usually accompanied with tighter monetary conditions. Furthermore, the exchange rate (REXR) exhibits a positive association with trade openness (r = 0.2416, $p < 0.01$), reflecting the conventional view that currency movements influence competition in the international market and trade flows. However, the relationships between INDT and REXR (r = −0.0924, $p > 0.10$) and between RINR and OPEN (r = −0.0090, $p > 0.10$) are weak and statistically insignificant, indicating that exchange rate fluctuations and monetary policy alone may not directly drive industrial development through internal borrowing or economy openness in the Nigerian context.

An important take away from Table 1 is the very strong positive correlation between INDT and OPEN (r = 0.8201). While this finding is economically meaningful, suggesting a close link between domestic borrowing for industrial growth and international trade integration, it also raises an important econometric concern. As noted by [37, 46, 51], the presence of a high correlation coefficient between independent variables in a regression model is an indication of possible multicollinearity, which can inflate the standard errors of the estimated parameters and weaken the reliability of inference about the parameters. In line with best econometric practice, it is therefore necessary to formally test for multicollinearity before fitting of the model. Thus, this study employs the Variance Inflation Factor (VIF) diagnostic technique, which is widely used to detect multicollinearity in linear models. The VIF test assesses the degree to which each explanatory variable can be linearly predicted from the others, with values above the threshold (10) suggesting severe collinearity ([44, 45]). The results of this test are presented in Table 2, which provides a more robust evaluation of checking the observed correlations for any possible threats to the validity of subsequent estimations.

Table 2.
Variance Inflation Factor (VIF) Results.

Variables	VIF for OLS Regression (%)
INDT	14.27
EXDT	3.53
RINR	1.44
REXR	1.76
OPEN	9.56

Table 2 presents the VIF statistics for all the identified determinants of economic growth under consideration. The results reveal that while most variables fall within the acceptable range, internal debt (INDT) with a VIF of 14.27, which is considerably higher than majorly acceptable level of 10. This emphasizes the presence of severe multicollinearity associated with this variable. Following the guidelines established in the econometric literature ([44, 45] as cited in [34, 46]), this finding affirms that INDT introduces a high degree of linear dependency with other predictors considered in fitting the linear model. The implication is that the inclusion of internal debt in the growth model has a potential to misrepresent estimated parameters, abnormally increases standard errors, and reveals ambiguous effect of other predictors. The emergence of such multicollinearity is not unexpected. Internal debt is frequently linked with other macroeconomic indicators such as external debt, openness of the economy, and changes in exchange rate. These interdependencies, while theoretically meaningful, can lead to overlapping explanatory power in econometric specifications. Consequently, the presence of high VIF values demands cautious model modification to avoid spurious results and to ensure the validity of the fitted regression model and outcomes. Having ascertain the presence of multicollinearity, the next step involves conducting **outlier** diagnostics to ensure whether extreme values in the dataset may also be contributing to the observed statistical alterations. This corresponding step is relevant because both multicollinearity and outlier effects can compromise the reliability of estimated parameters if left unaddressed.

5.1. Outlier Test

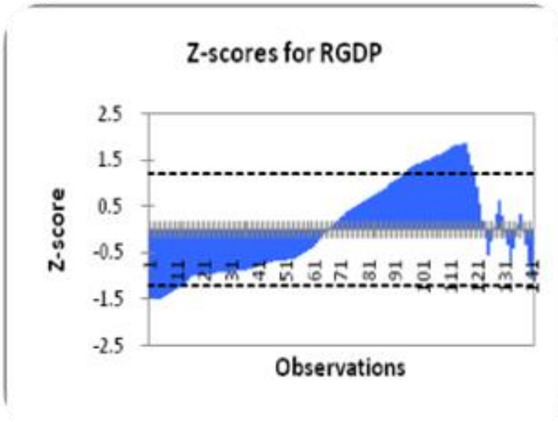
The evaluation of normality is a crucial step in ensuring the reliability of statistical inferences, particularly when parametric methods are applied. The preliminary descriptive statistics, specifically the skewness and kurtosis measures, revealed deviations from the threshold values typically associated with a normal distribution [2, 34]. Such departures suggest that the dataset may not follow a perfectly symmetric distributional pattern, raising concerns regarding the potential influence of extreme values. In empirical research, non-normality is often linked to the presence of outliers, which can unduly affect mean estimates, correlation coefficients, and regression results, eventually leading to biased or misleading inferences. Therefore, in this study, Grubb's test for outliers was employed to identify whether individual observations significantly deviated from the rest of the data series. Grubb's test is particularly suitable in this context, as it is designed to detect single extreme values in normally distributed datasets, and it offers a proper hypothesis testing procedure to examine whether the most extreme value in the data can be categorized as outlier at a given level of significance. Unlike graphical check methods or descriptive benchmarks, this test offers a more robust and statistically grounded approach to identifying anomalies in the dataset. Therefore, to ensure reliable empirical results, it is imperative to examine the existence of outliers in the dataset. Outliers, if concealed, may lead to bias of the estimated parameters, reduce statistical influence, and theoretically lead to misleading inferences concerning the dynamics of economic growth and its determinants. The results that provide critical understandings about the presence and possible impact of outliers within the variables under consideration using Grubb's test are presented in Table 3.

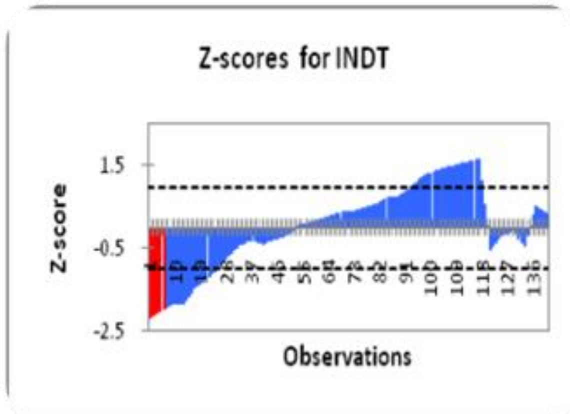
Table 3.
Grubb's Test Results.

Variable	G-statistics	G-critical value
RGDP	1.8600	0.8210
INDT	2.1890	0.8210
EXDT	2.7520	0.8210
RINR	3.2150	0.8210
REXR	2.8800	0.8210
OPEN	2.2540	0.8210

Table 3, Grubb's test (also known as the maximum normalized residual test) was employed as a formal procedure for outlier detection. The null hypothesis of Grubb's test states that no outliers exist in the dataset, whereas the alternative hypothesis indicates the presence of at least one outlier. The decision rule is based on the comparison of the computed Grubb's test statistic (G-statistic) with the corresponding critical value (G-critical). When the G-statistic exceeds the G-critical value, the null hypothesis is rejected, implying the existence of an outlier in the variable under consideration. In Table 3, the G-statistics revealed that real gross domestic product (RGDP), internal debt (INDT), external debt (EXDT), interest rate (RINR), exchange rate (REXR), and degree of economy openness (OPEN) were 1.8600, 2.189, 2.752, 3.215, 2.880, and 2.254, respectively. Each of these values was significantly higher than the G-critical value of 0.8210. This result shows robust statistical evidence for the rejection the null hypothesis across all variables, thereby establishing the existence of outliers in the dataset considered for this study.

Also, the presence of outliers was further validated through the graphical analysis displayed in Figure 1, which visually highlights extreme values within the distribution of the variables under investigation. The collective statistical and graphical evidence emphasizes the relevance of addressing these violations in succeeding analysis. Outliers may reflect structural breaks, data recording errors, or sincere economic shocks; thus, their treatment is vital for robust and valid econometric results. In empirical research of this nature, identifying outliers through Grubb's test not only validates the integrity of the dataset but also gives information about appropriate potential research methods and techniques to be adopted in the modeling. By correcting or accounting for extreme values, it alleviates biases and ensure accurate econometric estimates that reflect the true fundamental economic relationships under investigation. The results of this test are intended to provide critical understandings into the existence and possible influence of outliers within the variables under investigation. To identify and address such observations, it is not just a sound methodology but also enhances the robustness of succeeding econometric model, particularly in situations where slight deviations can lead to substantial policy or theoretical implications.





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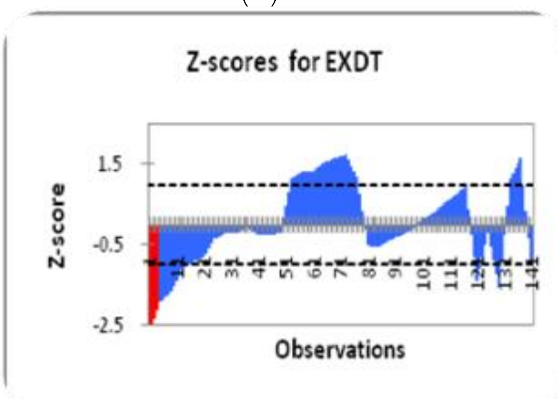

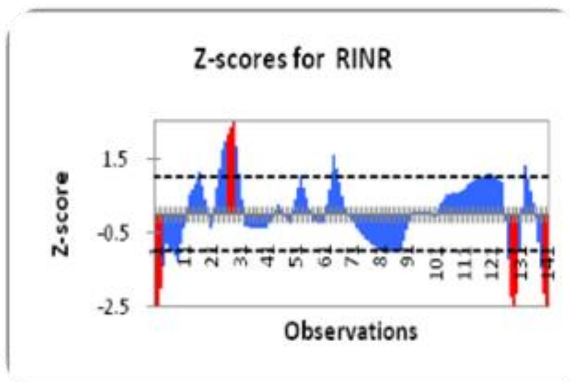

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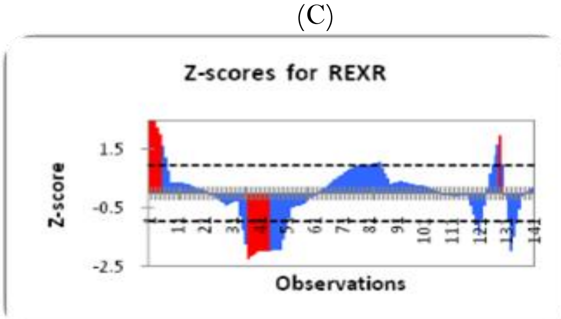

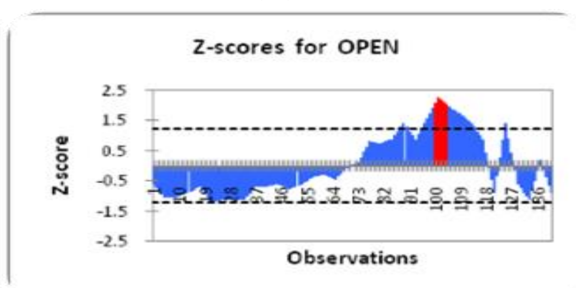

(E)

(F)

Figure 1.
The Plots showing Outliers in RGDP and Identified Drivers.

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Figure 1 provides the graphical representations of the study variables, namely real gross domestic product (RGDP), internal debt (INDT), external debt (EXDT), interest rate (RINR), exchange rate (REXR), and degree of economy openness (OPEN). Subfigure A illustrates the trend of RGDP, while subfigures B through F correspond to INDT, EXDT, RINR, REXR, and OPEN, respectively. These plots serve as an initial diagnostic tool to evaluate the statistical properties of the series and to detect distributional anomalies in their behavior. A preliminary examination of the plots reveals that several outliers exist across the variables, reflected in data points that deviate substantially from the mean as measured by standardized Z-scores. Such extreme values are particularly obvious in the macroeconomic indicators, where instabilities often reflect occurrences of policy shifts, external shocks, or structural imbalances in the Nigerian economy. The existence of these outliers climaxes the need for thoughtful modeling approaches, if not properly addressed, may misrepresent estimated parameters or bias inferential results.

Apart from the issue of outliers, the study also identifies possible multicollinearity among the predictors. For example, internal debt, external debt, and degree of economy openness show strong interdependence, consistent with theoretical expectations that internal borrowing enhances international trade incorporation. Similarly, the relationship between interest rate movements and exchange rate regulations suggests that economic variables are not completely independent. High degrees of multicollinearity, can bloat standard errors and weaken the statistical significance of distinct predictors if not corrected, thus, its complicate all efforts to draw reliable policy inferences. To address the identified problems associated with the methodology and to ensure robust estimates for the parameters, this study employs Principal Component Analysis (PCA) as a dimension reduction method. PCA is particularly appropriate in this context because it transforms correlated predictors into a set of orthogonal components that are statistically independent of each other. Doing this, eases the problem of multicollinearity while retaining the maximum potential variation in the dataset. Furthermore, PCA enhances model stability and reliability of the predictive model and its performance by focusing on information from the reduced number of composite predictors that can serve as reliable proxies for the original economic variables.

Table 4.
Principal Components Analysis.

Principal Components Analysis					
Eigenvalues: (Sum = 5, Average = 1)					
Number	**Value**	**Difference**	**Variance Proportion**	**Cumulative Variance Explained**	
PC1	2.2456	0.6149	0.4491	0.4491	
PC2	1.6307	1.0742	0.3261	0.7753	
PC3	0.5564	0.0544	0.1113	0.8866	
PC4	0.5020	0.4369	0.1004	0.9870	
PC5	0.0650	---	0.0130	1.0000	
Ordinary correlations:					
	INDT	EXDT	RINR	REXR	OPEN
INDT	1.0000				
EXDT	0.6073	1.0000			
RINR	0.1275	0.3875	1.0000		
REXR	-0.0923	-0.2898	-0.4594	1.0000	
OPEN	0.8200	0.2534	-0.0089	0.2416	1.0000

The extracted components are subsequently used as predictors in the empirical growth model, ensuring that the estimates are both efficient and unbiased in the presence of correlated regressors. Table 4 presents the results of the PCA, including the eigenvalues, percentage of variance explained, and component loadings for each variable. These results form the foundation for the subsequent econometric analysis, which evaluates the independent contribution of the derived components to Nigeria's economic growth trajectory.

Table 4 presents the outcomes of the Principal Component Analysis (PCA), which was employed to address potential multicollinearity among the predictors and to reduce the dimension of the dataset. Multicollinearity is a common issue in empirical economic research, as predictors (explanatory variables) such as internal debt (INDT), external debt (EXDT), interest rate (RINR), exchange rate (REXR), and degree of economy openness (OPEN) often show strong interdependencies. This procedure extracts a series of linear combinations of the original variables, known as principal components (PCs), which are ordered according to the proportion of total variance explained. The first component (PC1) captures the largest share of common variance among the variables, while subsequent components account for progressively smaller proportions. By construction, each principal component is uncorrelated with the others, allowing for a parsimonious representation of the underlying data structure. As shown in Table 4, the principal components derived from the five explanatory variables-INDT, EXDT, RINR, REXR, and OPEN, which summarize the systematic variation in the dataset. These components serve a dual purpose. First, they allow for dimension reduction by retaining only the most informative components, thereby simplifying the empirical model without significant loss of explanatory influence. Second, they help isolate the independent contribution of economic fundamentals to real GDP growth (RGDP), free from the biases introduced by collinearity as given in equation 31 to 35.

The retained components were selected on the basis of eigenvalues greater than unity and cumulative variance explained, consistent with the Kaiser criterion and standard PCA practice in applied economics. In particular, the first few components capture the majority of the variability in the dataset, suggesting that the dynamics of industrialization, export diversification, openness, and financial variables can be effectively summarized using a smaller subset of orthogonal dimensions. This reinforces the usefulness of PCA as both a diagnostic and corrective tool in regression analysis. From a substantive perspective, the resulting components may be interpreted as latent constructs that combine structural and policy-related aspects of economic performance. For example, a component heavily loaded on INDT and OPEN might reflect the structural transformation associated with industrial growth and globalization, while another weighted strongly on RINR and REXR could capture monetary and exchange rate conditions. These latent factors provide a clearer picture of how interrelated macroeconomic indicators shape growth outcomes when incorporated into subsequent regression analysis. Overall, the PCA results highlight that the chosen explanatory variables are not independent in their raw form but can be meaningfully restructured into uncorrelated components. This methodological step ensures that subsequent estimations linking the explanatory variables to RGDP are both statistically valid and theoretically grounded.

$$PC1 = 0.6134INDT + 0.5292EXDT + 0.2921RINR - 0.1844REXR + 0.4736OPEN, \quad (31)$$

$$PC2 = 0.2382INDT - 0.1930EXDT - 0.5331RINR + 0.6258REXR + 0.4796OPEN, \quad (32)$$

$$PC3 = -0.1676INDT - 0.2802EXDT + 0.7863RINR + 0.4716REXR + 0.2290OPEN, \quad (33)$$

$$PC4 = -0.1638INDT + 0.7211EXDT - 0.0072RINR + 0.5622REXR - 0.3701OPEN, \quad (34)$$

$$PC5 = 0.7155INDT - 0.2899EXDT + 0.1097RINR + 0.1889REXR - 0.5968OPEN. \quad (35)$$

Also, from the Table 4 and based on the Kaiser's rule of thumb that says the principal component with the associated eigenvalues greater than one is considered to fit an appropriate model. Thus, it is found from the results that the two components which are the PC1 and PC2 account for the 44.9% and 32.6% of the variance proportion. This indicates that a large portion of the variability in the original variables can be effectively summarized by these two components, thereby justifying their selection for further analysis. In contrast, the remaining components (PC3, PC4, and PC5) all possess eigenvalues less than one, with a combined contribution of only 22.5% of the total variance. According to Kaiser's criterion, these components are deemed insignificant, as they do not provide additional explanatory power beyond what could be captured by individual variables. The scree plot presents in Figure 2 also establishes the significance of the selected PC1 and PC2 as the two components to fit a robust principal component model to predict a stable and a reliable economic growth (RGDP). Also, the principal component scree plot and the cumulative proportion plot for the selection of the components is shown in Figure 2

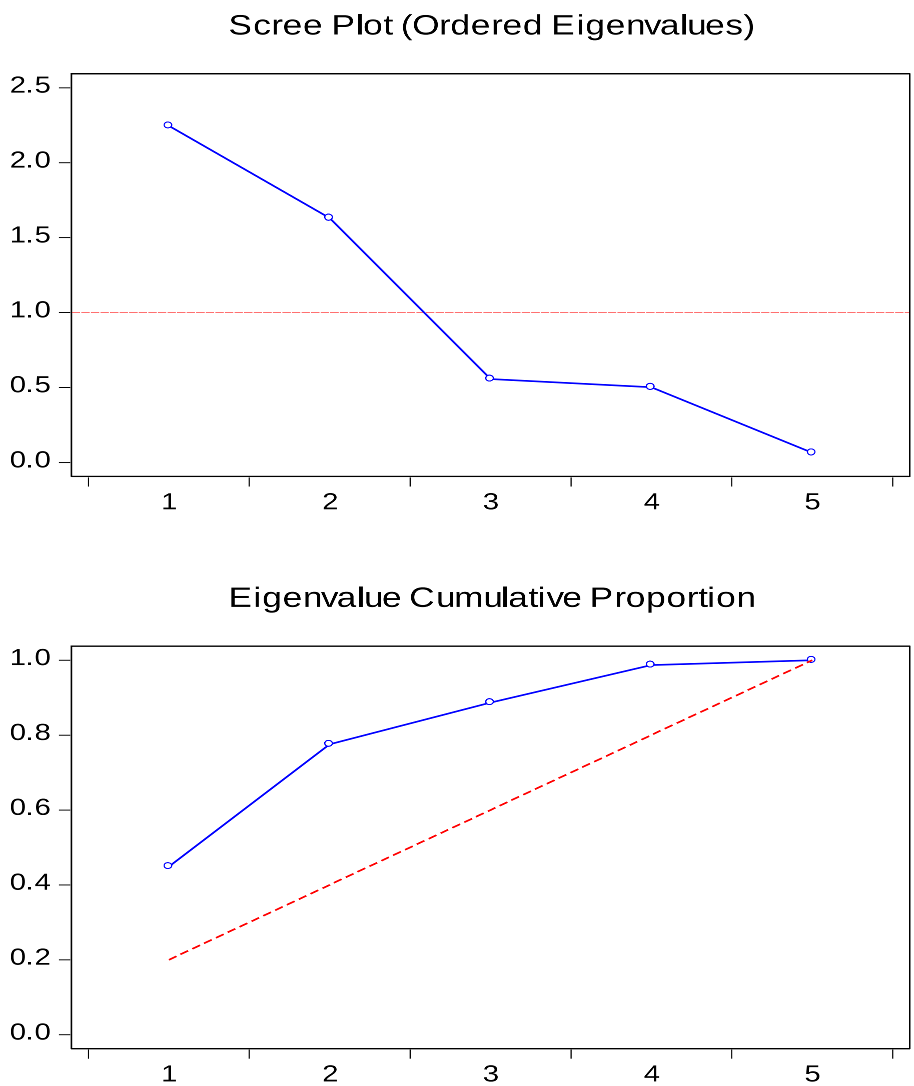

Figure 2.
The Scree Plot and Cumulative Proportion Plot Showing the Number of Component of Economic Growth Drivers to be Selected.

In Figure 2, we present the scree plot and the cumulative proportion of the eigenvalue of the components for the identified economic growth drivers. The results show that the PC1 with associated eigenvalue of 2.2456 and PC2 with the associated eigenvalue of 1.6307 have the highest variance proportion estimates of 44.9% and 32.6%. The cumulative variance proportion that accounts for the 77.5% of the extracted components for the identified economic growth drivers. Also, when compare with the others extracted components such as PC3, PC4, and PC5 that have an eigenvalue that is less than one and their total variance cumulative proportion of 22.5%. The plot confirms that the inclusion of PC1 and PC2 alone captures more than 75% of the variance, which is generally considered adequate for a parsimonious yet robust model in applied economic research.

From a methodological perspective, the extraction of PC1 and PC2 provides a stable foundation for constructing a principal component model to predict real gross domestic product (RGDP). These components effectively condense the multidimensional dataset into two orthogonal factors, thereby reducing multicollinearity while retaining the essential information. This dimension reduction enhances the reliability and stability of subsequent econometric modeling, ensuring that the predictors of economic growth are both interpretable and statistically sound. Accordingly, the PCA model for RGDP can be expressed as a linear combination of the selected principal components (PC1 and PC2). This formulation not only improves the predictive efficiency of the model but also aligns with best practices in empirical investigation, where parsimony and interpretability are key considerations. Thus, based on the eigenvalue of the extracted principal component presented in the Table 4 and the scree plot shown in the Figure 2, the formal representation of the PCA-based growth model is thus specified as:

$$\mathrm{RGDP} = \propto_0 + \propto_1 \mathrm{PC1} + \propto_2 \mathrm{PC2} + \in_i, \tag{36}$$

where $\propto_0$ denotes the intercept, $\propto_1$ and $\propto_2$ are the estimated coefficients associated with the principal components, and $\in_i$ is the error term. Hence, to address the outliers in the model, the results of the fitted robust principal components to efficiently predict stable and reliable values for RGDP are presented in Table 5

Table 5.
Robust Principal Component Regression (PCR) Results.

	M-estimator		**S-estimator**		**MM-estimator**	
Variable	**Estimate**	***p*-value**	**Estimate**	***p*-value**	**Estimate**	***p*-value**
C	10.2851	0.0000	10.3244	0.0000	10.2847	0.0000
PC1	0.3539	0.0000	0.3715	0.0000	0.3546	0.0000
PC2	0.2215	0.0000	0.2122	0.0000	0.2224	0.0000
R-squared	0.6617		0.7125		0.7031	
Adjusted R-squared	0.6568		0.7083		0.6988	
Rw-squared	0.9248				0.9172	
Adjust Rw-squared	0.9248				0.9172	
Rn-squared statistic	1048.933	0.0000	1316.421	0.0000	1076.843	0.0000

Table 5 shows the robust principal component regression analysis, which was employed to estimate the parameters of the model explaining RGDP. The analysis draws upon two extracted components-principal component one (PC1) and principal component two (PC2) identified as the most appropriate linear combinations of the predictors for capturing the basic dynamics of economic growth. The selection of these components was guided by their eigenvalue magnitudes and cumulative variance contribution, ensuring that the crucial information rooted in the data was preserved while justifying issues of redundancy and noise. It is important to stress that the dataset used in this study consists of quarterly time series macroeconomic indicators, which are prone to econometric complications such as multicollinearity among predictors and the presence of outliers that may misrepresent classical regression results. To address these challenges, we implemented robust regression techniques within the principal component process. Specifically, three robust estimators were used:

Based on this, a robust M-principal component regression (M-estimator) was applied with a bi-square weighting function, a tuning constant of 4.685, and median-centered scaling. This configuration is particularly effective in down-weighting extreme observations, thereby reducing the influence of outliers on the estimated coefficients while retaining efficiency for normally distributed errors. To ensure robustness in the presence of high-leverage points and heteroscedastic errors, a robust S-principal component regression (S-estimator) was employed with a tuning constant of 1.5476 and a breakdown point of 0.5, applied over 200 trials. This estimator prioritizes resistance against atypical observations by focusing on minimizing the scale of residuals, thereby providing stable coefficient estimates under contamination. and a robust MM-principal component regression (MM-estimator) was fitted, combining the high breakdown point of the S-estimator with the efficiency of the M-estimator. The procedure was designed with an S-estimator starting point (tuning constant = 1.5476, breakdown = 0.5, trials = 200), followed by refinement using an M-estimator with a bi-square weight function (tuning constant = 4.685). This hybrid approach is particularly advantageous, as it achieves robustness to outliers while maintaining asymptotic efficiency under standard conditions. These were fitted to address the two identified linear model assumption violations through the incorporation of robust estimators within the principal component regression approach namely: robust M-principal component regression (M-estimator), robust S-principal component regression (S-estimator), robust MM-principal component regression (MM-estimator) methods. This is to ensures that the parameter estimates for PC1 and PC2 are more reliably capture the systematic relationship between the predictors and RGDP, independent of data anomalies, thus, provide a thorough statistical and robust methodology underpinning for interpreting the impact of PC1 and PC2 in explaining economic growth dynamics.

In Table 5, the results of a M-estimation method shows that the PC1 and PC2 have positive impact on the RGDP, thus, it increases the RGDP by 35.39% and 22.15%, respectively. These findings underscore the importance of both components in driving economic performance, with PC1 emerging as the more dominant factor. Also, a robust S-estimation method shows that the PC1 and PC2 positively influence the RGDP, and as such increases the RGDP by 37.15% and 21.22% respectively. This demonstrate and validate the robustness of this relationship by the S-estimation method, which yields slightly stronger effects of PC1 in enhancing RGDP. These estimates also confirm that the explanatory power of the components remains stable across alternative estimation strategies. Similarly, the results of a robust MM-estimation method reveals that the PC1 and PC2 have positive influence on the RGDP, thus, it leads to a respective 35.46% and 22.24% increases in the RGDP during the period under investigation. These results is closely aligned with that of the M-estimation technique, emphasizing strong robustness in the detected relationships. The estimated p-values for the PC1 and PC2 are less than 0.05, and thus, indicate the statistical significance of the PC1 and PC2 in examining and predicting the RGDP in Nigeria. This finding confirms that both components play a crucial role in explaining variations in RGDP and can serve as reliable predictors of macroeconomic performance. From a policy perspective, these results suggest that the underlying variables embedded within PC1 and PC2, which may represent clusters of industrial, financial, or trade-related indicators, are critical levers for stimulating sustainable economic growth.

Also, the adjusted R-square values obtained as 0.6568, 0.7083 and 0.6988 for the M-estimation, S-estimation, and MM-estimation methods respectively. These results reveal that approximately 65.68%, 70.83%, and 69.88% proportional changes in the RGDP can be explained by by the first two principal components (PC1 and PC2) under the different robust estimation methods. Thus, it can be emphasized that robust principal component estimation methods are statistical significance estimation methods that can be used in addressing model specification issues identified in this study. The robust Rn-squared statistic values obtained as 1048.933, 1316.421 and 1076.843 with the associated p-value < 0.05 are used to examine the overall significance of the M-estimation, S-estimation, and MM-estimation methods for the fitted models. Hence, the results indicate that the fitted robust principal component regression estimation methods are statistically significant in examining and predicting the RGDP using PC1 and PC2. This can be further stressed that the inclusion of the robust principal component estimation methods greatly improves the model's predictive capacity, even in the presence of multicollinearity and outlier misrepresentations. The test to confirm that the multicollinearity problem has been addressed is presented in Table 6.

Table 6.
Test for the Absence of Multicollinearity.

Variable	Coefficient Variance	VIF
PC1	0.00026	1.0000
PC2	0.00026	1.0000

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Table 6 presents the results of the variance inflation factor (VIF) analysis, which was conducted to assess the possible problem of multicollinearity among the predictors after the extraction of the first two principal components (PC1 and PC2), which were retained as the optimal principal components based on their eigenvalues, which were the largest among the extracted factors. The selection of these components is consistent with the Kaiser criterion and principal component analysis (PCA) theory, which suggests that components with the highest eigenvalues capture the greatest proportion of variance in the dataset. These components were subsequently employed as predictors of RGDP. However, to assess the inclusion of PC1 and PC2 in the fitted models, it is necessary to ensure that multicollinearity problem has been addressed to engender the robustness of the econometric results. Hence, Table 6 reports the results of VIF test applied for this purpose, as it provides a straightforward diagnostic tool for detecting linear dependencies among explanatory variables, which shows that PC1 and PC2 are 1.0000 and 1.0000 respectively and as such indicating no correlation between the two selected principal components. Thus, according to Allison [44] and Freund and Littell [45], and as cited in Khalaf and Iguernane [46], it can be posited that the VIF < 10.00 evidently indicates the absence of the multicollinearity problem in this study. Based on this, it can be stressed that the multicollinearity problem has been addressed from the fitted robust principal component models, and as such, the estimated parameters of models are efficient, optimal, stable and reliable to predict the RGDP in Nigeria. Also, to determine the most efficient robust principal component regression method among the fitted M-estimation, S-estimation, and MM-estimation methods, the predictive power or performance evaluation metrics of the aforementioned fitted robust principal component regression methods are examined and results are presented in Table 7.

Table 7.
Performance Evaluation for Robust Estimation Methods.

Forecast RGDP	M-estimation	S-estimation	MM-estimation
Root Mean Square Error	0.1927	0.1929	0.1928
Mean Absolute Error	0.1319	0.1321	0.1320
Mean Abs Percent Error	1.2672	1.2675	1.2686
Theil Inequality Coefficient	0.0093	0.0093	0.0093
Bias Proportion	0.0103	0.0104	0.0107
Variance Proportion	0.0284	0.0132	0.0265
Covariance Proportion	0.9612	0.9763	0.9627

Table 7 presents the results to assess predictive power or performance evaluation of the fitted robust principal component regression models, which are used simultaneously to address the violation of a linear model assumptions (the presence of the multicollinearity and outliers). Thus, the results reveal a consistent pattern across multiple forecast evaluation metrics as shown in Table 7. Specifically, the root mean square error (RMSE) of M-estimation technique with a value given as 0.1927 has the smallest value, when it is compared with RMSE value of S-estimation (0.1929), and MM-estimation (0.1928) respectively. The mean absolute error (MAE), where the M-estimation method achieves has smallest value obtained as 0.1319, compared with 0.1321 for S-estimation and 0.1320 for MM-estimation. Also, the mean absolute percentage error (MAPE) of the M-estimation method records value of 1.2672, which is slightly better than S-estimation (1.2675) and the MM-estimation (1.2686).

The same results are obtained using bias proportion that shows minimum value under M-estimation (0.0103), relative to S-estimation (0.0104) and MM-estimation (0.0107). In addition, the robustness of the three estimation methods is also evaluated using Theil inequality coefficient and the variance proportion, which also reveal M-estimation outperform the other two estimation techniques, and as such, it can be asserted that the M-estimation method is the most efficient, optimal, stable and reliable robust principal component estimation method that can be used in modelling and predicting RGDP using PC1 and PC2 that is extracted from predictors considered. Thus, M-estimation method is used to

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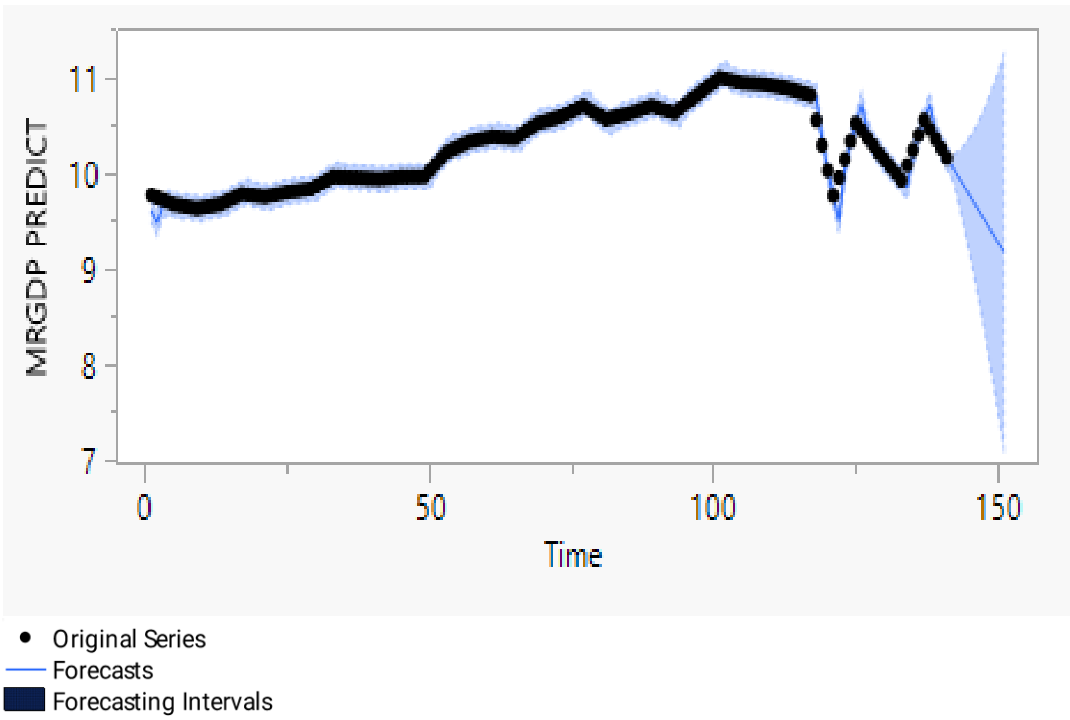
generate the predicted values of the RGDP in Nigeria for the next ten quarters as shown in the Figure 3.

5.2. Forecast for the RGDP using Robust Principal Component Regression Method

Figure 3 shows the predictive efficiency of the fitted M-estimation method that is used to generate a stable and a reliable RGDP, considered in this study for the next ten quarters. The plot in Figure 3 represents this forecast.



Author's computation (2025)

Figure 2.
Forecast Plot using Robust Principal Component Regression Method.

6. Findings, Discussion, and Policy Implications

This study investigated the modelling, estimation, and prediction of Nigeria's economic growth, with particular emphasis on the identified determinants: internal debt (INDT), external debt (EXDT), interest rate (RINR), exchange rate (REXR), and degree of economy openness (OPEN). The preliminary diagnostic tests revealed violations of the classical linear model assumptions due to the presence of multicollinearity among predictors and influential outliers within the dataset. Specifically, the variance inflation factor (VIF) confirmed a severe multicollinearity, while Grubb's test established the presence of outliers. These issues necessitated the adoption of a robust estimation approach, which has efficient capability to produce stable, consistent, and reliable results. However, in order to address these challenges emanated from the methodology, a robust principal component regression (PCR) method was employed. Principal component analysis extracted two components (PC1 and PC2), which served as orthogonal predictors of economic growth (RGDP). This transformation effectively addressed multicollinearity problem. Subsequently, three robust estimation methods that include: M-estimation, S-estimation, and MM-estimation were applied to obtain efficient, stable, and reliable parameter estimates. Among these, M-estimation method appeared to be the most efficient and optimal technique. The out performance of the M-estimator over S-estimation and MM-estimation was established through comparative evaluation and predictive metrics. It recorded the lowest values of root mean square error (RMSE = 0.1927), mean absolute error (MAE = 0.1319), and mean absolute percentage error (MAPE = 1.2672), alongside the smallest bias proportion, variance proportion, and Theil's inequality coefficient. These results emphasize the robustness, stability, and predictive reliability of the M-estimation model with all the anomalies available in the dataset.

The estimation results further revealed that both PC1 and PC2 had a positive and statistical significance impact on economic growth in Nigeria. Quantitatively, PC1 contributed approximately 35.39% while PC2 contributed 22.15% to variations in RGDP over the period under investigation. The statistical significance of these components (p-value < 0.05) emphasizes their relevance as growth-enhancing predictors. The overall performance of the model fit, confirmed by the R-squared statistic (p-value < 0.05), further validates the appropriateness of the M-estimation method in modeling economic growth under distorted and intricate data conditions. The empirical findings align with Nigeria's recent macroeconomic situations and certainties. Incidents such as the global crash in crude oil prices, the COVID-19 pandemic, persistent insecurity and terrorism, exchange rate fluctuations, depreciation and currency devaluation, changes in economic and financial policies such as the floating of the naira, and the removal of fuel subsidies have all influenced significant shocks on the identified economic growth drivers. These shocks, in turn, were transmitted to the broader economy, as reflected in fluctuations in RGDP during the period under consideration.

7. Conclusion and Recommendations

This study concludes that the M-estimation-based robust principal component regression model offers the most reliable approach for modeling, analyzing, and predicting economic growth in Nigeria, particularly when multicollinearity and outliers misrepresent classical linear model results. The model not only ensures efficient, stable, and reliable parameter estimates but also provides deeper understandings of the contribution of various key economic drivers used as predictors. Based on the results, some policy recommendations are made: since internal debt and external debt are robust growth determinants, policies should focus on enhancing the management borrowed fund to drive investment and bring value addition to the economy; the significant influence of exchange rates and interest rates suggests that stabilization policies intended at reducing volatility in these variables will foster continuous growth. Exchange rate reforms must be carefully sequenced to avoid eroding competitiveness gains in the international trade market; the adverse effect of COVID-19 and global oil price crises highlights the need for a diversified revenue base and deliberate buffers to protect the economy from external shocks; and the proved efficiency of robust regression methods underlines the requirement for policymakers and researchers to adopt advanced statistical techniques when designing, evaluating, and monitoring growth-related policies, especially in data environments prone to multicollinearity and outliers.

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Data Availability Statements:

The data used for this study were extracted from the central bank of Nigeria statistical bulletin. https://www.cbn.gov.ng

Transparency:

The authors confirm that the manuscript is an honest, accurate, and transparent account of the study; that no vital features of the study have been omitted; and that any discrepancies from the study as planned have been explained. This study followed all ethical practices during writing.

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