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Exploring multifractal detrended cross-correlation among major southeast Asian stock markets

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Abstract: This study investigates the multifractal behavior and cross-correlations of major Southeast Asian stock markets, offering valuable insights into their interconnectedness and market dynamics. To achieve this, three methods are applied: the Multifractal Detrended Cross-Correlation Analysis (MF-DCCA), the Q-Cross-Correlation Significance Test, and the DCCA Coefficient Method. Additionally, to assess the contributions to cross-correlation multifractality, the random permutation (shuffling) and phase randomization (surrogate) techniques are employed. The results obtained from the Q-Cross-Correlation statistic reveal significant cross-correlations between all pairs of Southeast Asian indices, emphasizing a strong interconnectedness and shared market dynamics. Furthermore, the DCCA cross-correlation coefficients for the six major indices show persistent cross-correlations, with values ranging from 0 to 1. The fluctuation functions for all pairs demonstrate a nonlinear increase with time scales and scaling exponents, indicating a power-law relationship and confirming the presence of long-range cross-correlations. In addition, the Generalized Hurst Exponent shows a non-linear decrease, while the Rényi Exponent exhibits a non-linear increase as the scaling exponents increase. Meanwhile, the Singularity Spectrum functions display inverted concave parabolic shapes, which further confirm the multifractal nature of the cross-correlations. These findings are corroborated by the non-zero values of the metrics assessing the strength of multifractality, based on the Generalized Hurst Exponent and Singularity Spectrum. Among the pairs, the Indonesia-Malaysia pair demonstrates the highest degree of multifractality, reflecting complex cross-correlations driven by long-range correlations and market-specific factors, whereas the Indonesia-Singapore pair shows the lowest multifractality. Finally, the results of the shuffling and surrogate transformations indicate a significant reduction in multifractality, thereby underscoring the role of long-term temporal cross-correlations and heavy-tailed distributions in the complex behavior of these markets. The findings offer practical implications for portfolio diversification, risk management, and market regulation and policy, emphasizing the importance of multifractal analysis in capturing long-term dependencies and complex dynamics in Southeast Asian stock markets.

Keywords: *Cross-correlation, Heavy-tailed distributions, MF-DCCA, Long-range correlations, Multifractality, Southeast Asian stock markets.*

1. Introduction

The financial markets of Southeast Asia have experienced significant growth and transformation in recent decades, emerging as dynamic hubs of economic activity and investment. Historically, these markets have faced numerous challenges, such as the 1997 Asian financial crisis and various global

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economic downturns, which have significantly influenced their development and strengthened their resilience. Countries such as Malaysia, Indonesia, the Philippines, Singapore, Thailand, and Vietnam exhibit complex behaviors influenced by global economic shifts, regional integration, and the evolving dynamics of trade and financial policies. The Association of Southeast Asian Nations (ASEAN) plays a critical role in fostering regional cooperation, economic integration, and cultural exchange among its member states. These markets are characterized by substantial economic diversity, structural differences, and varying levels of financial maturity. The region's growth and the evolving integration of its markets into the global economy provide a fertile ground for studying the complexities and interdependencies of financial time series.

As Southeast Asia's financial markets become increasingly interconnected, understanding the underlying patterns and interdependencies among these stock markets is critical for both investors and policymakers. However, traditional statistical and econometric models frequently struggle to adequately capture the complex features of financial time series, including non-linearity, long memory, and inter-market interactions. One of the key limitations of these models is their reliance on specific assumptions about the nature of distributions, including Gaussian distributions, linearity of relationships, and stationarity. These limitations highlight the need for more sophisticated analytical tools that can reveal the hidden structures within financial data.

One such tool is the Multifractal Detrended Cross-Correlation Analysis (MF-DCCA), a powerful method for assessing multifractality and cross-correlations between financial time series. Multifractality, characterized by the presence of multiple scaling exponents within a time series, reveals the intricate structure of its fluctuations. The foundational concept of Detrended Fluctuation Analysis (DFA), introduced by Peng, et al. [1] was designed to detect long-range correlations in non-stationary time series. Kantelhardt, et al. [2] subsequently extended this framework with Multifractal Detrended Fluctuation Analysis (MF-DFA), which allows for the examination of multifractal properties over a range of scales. To explore cross-correlations between two time series, Podobnik and Stanley [3] introduced Detrended Cross-Correlation Analysis (DCCA), an extension of DFA. Zhou [4] further developed this concept into Multifractal Detrended Cross-Correlation Analysis (MF-DCCA), combining DCCA and MF-DFA. This integrated methodology provides a comprehensive analysis of the multifractal characteristics of cross-correlations, enhancing the understanding of the complex interactions between financial time series.

While multifractal analysis has been extensively applied to financial markets in developed economies, there is relatively little research focused on emerging markets, particularly in Southeast Asia. This research aims to fill this gap by applying advanced methods like MF-DCCA to a region that remains underrepresented in academic literature.

Specifically, our study seeks to address key research questions related to the degree of multifractality and the nature of cross-correlations among Southeast Asian stock markets. The application of MF-DCCA offers significant benefits to a variety of stakeholders, including investors, policymakers, financial institutions, and regulators. It enhances portfolio diversification by revealing hidden cross-market correlations and multifractal patterns, improving investment strategies. MF-DCCA also strengthens risk management by identifying market dependencies, helping to mitigate systemic risks during crises. For policymakers, it provides insights into the progress of financial integration in the region, aiding in the design of stability-focused policies. Regulators benefit from an improved understanding of market efficiency, enabling better regulations. Global investors gain deeper market insights, leading to more informed decisions. Finally, MF-DCCA contributes to financial technology by refining risk models and fostering innovation in investment strategies.

In this paper, we apply three methods: the Multifractal Detrended Cross-Correlation Analysis (MF-DCCA) method, the Q-Cross-Correlation Significance Test, and the DCCA Coefficient Method. Additionally, we discuss two techniques that assess the contribution of various sources to the overall cross-correlation multifractality of the bivariate series: random permutation (shuffling) and phase randomization (surrogate).

The rest of the paper is structured as follows: first, we provide a review of the relevant literature on multifractal and cross-correlation analysis, particularly its application in financial markets. Next, the materials and methods section details the data sources and analytical techniques used in the study. This is followed by the results and discussion section. To conclude, we highlight the main findings and explore their practical implications.

2. Literature Review

The following literature review will explore recent research on MF-DCCA, with a particular focus on financial markets.

El Alaoui and Benbachir [5] employed MF-DCCA to investigate the cross-correlations among the stock markets of Morocco, Tunisia, Egypt, and Jordan within the MENA region, finding significant multifractal cross-correlations among these markets. In a related study, Xinsheng, et al. [6] also utilized MF-DCCA to examine the interrelations between onshore and offshore Chinese Renminbi (RMB) markets, revealing that the highest short-term cross-correlation occurred between the Chinese Yuan and the British Pound, while the Malaysian Ringgit exhibited the strongest long-term correlation.

Continuing the exploration of multifractality, Burugupalli [7] analyzed the cross-correlations between Gold and WTI Crude Oil using the same MF-DCCA methodology. His findings indicated that short-term cross-correlations displayed a stronger multifractal nature compared to those observed over longer periods. Qingsong, et al. [8] investigated the relationship between the Hang Seng China Enterprises Index and RMB exchange markets, employing both MF-DCCA and multifractal cross-correlation analysis (MF-CCA). They identified significant cross-correlations, noting that onshore RMB markets demonstrated a higher degree of multifractality than offshore markets.

Li, et al. [9] expanded on this theme by examining the dynamic relationship between the RMB exchange index and stock market liquidity in Shanghai and Shenzhen. Their study, which applied MF-DCCA, revealed strong multifractal cross-correlations that challenge the Efficient Market Hypothesis, especially during tightening monetary policy periods. Ferreira, et al. [10] investigated the long-range correlations between major global stock markets and their respective exchange rates against the USD, uncovering varying effects across different regions, with European markets showing minimal impact, while significant effects were observed in the Indian market and positive correlations in Japan, likely attributable to its monetary policy.

In the context of market reforms, Qingsong, et al. [11] assessed the implications of China's 2015 RMB exchange rate reform on cross-correlations among the CNH, NDF, and CNY markets. Their analysis indicated a reduction in both persistence and multifractality following the reform, with shifts in the behavior of short- and long-term correlations. Yanjun and Cheng [12] analyzed cross-correlations between the Shanghai Stock Exchange Composite and the S&P 500 indices, noting an increase in significant interactions post-financial crisis, characterized by multifractal features that were particularly evident in small fluctuations.

Faheem, et al. [13] explored long-range dependencies and multifractality across stock indices from nine MSCI emerging Asian economies, finding diverse levels of multifractality that supported the fractal market hypothesis. Wang, et al. [14] investigated the nonlinear and multifractal cross-correlations between P2P lending and stock markets, recommending regulatory practices from the stock market to enhance risk management in the P2P sector. Junjun, et al. [15] examined the long-range cross-correlation between Bitcoin prices and the U.S. Economic Policy Uncertainty (USEPU) index, uncovering significant multifractal interactions.

Xuemei, et al. [16] focused on the cross-correlations between SHIBOR and Chinese stock market liquidity, identifying weak persistence and multifractality that diminished following recent liberalization reforms. Similarly, Liu, et al. [17] discovered strong multifractal cross-correlations between futures and spot returns in China's soybean market. Baki [18] conducted an analysis of multifractality in EUR/TRY and USD/TRY exchange rates using MF-DCCA, highlighting the complex, scaledependent nature of their cross-correlations. Yijun, et al. [19] compared the multifractal characteristics of the CSI 300 and S&P 500 indices, documenting differences in long-term memory and complexity.

In a focused investigation, Faheem, et al. [20] explored the differential responses of Islamic and conventional stock markets to economic policy uncertainty (EPU), revealing significant multifractal cross-correlations, particularly in U.S. markets. Zeyi, et al. [21] examined the multifractal features and market efficiency within the Chinese New Energy market (NEI), concluding that significant multifractality and low market efficiency stemmed from long-range correlations. Lastly, Acikgoz, et al. [22] revealed multifractal cross-correlations between green bonds and commodities using MF-DCCA. The analysis showed long-range correlations in both markets, with volatility displaying persistence across fluctuations and returns showing persistence in small fluctuations and antipersistence in large ones.

3. Materials and Methods

3.1. Materials

The dataset for this study comprises daily closing prices from indices across six major stock markets in the Southeast Asia region: Indonesia, Malaysia, Philippines, Singapore, Thailand, and Vietnam. The indices include:

- The IDX Composite is the main stock market index of the Indonesia Stock Exchange (IDX). This index includes all stocks listed on the exchange.
- The FTSE Bursa Malaysia KLCI (Kuala Lumpur Composite Index) is the benchmark stock market index for the Bursa Malaysia, representing the 30 largest companies listed on the exchange by market capitalization.
- The PSEi Composite (Philippine Stock Exchange Index) is the main stock market index of the Philippine Stock Exchange (PSE). It represents the performance of the top 30 companies that are listed on the exchange.
- The FTSE Straits Times Index (STI) is the primary stock market index that tracks the performance of the top 30 companies listed on the Singapore Exchange (SGX).
- The SET Index is the primary stock market index that tracks the performance of all common stocks listed on the Stock Exchange of Thailand (SET).
- The VN Index is the main stock market index that tracks the performance of all listed companies on the Ho Chi Minh City Stock Exchange (HOSE) in Vietnam.

The data span from 16/10/2013 to 01/11/2024, comprising nearly 2656 observations. All data were downloaded from the website www.investing.com.

The index prices were then converted into logarithmic returns:

$$r_t = ln\left(\frac{P_t}{P_{t-1}}\right) = ln(P_t) - ln(P_{t-1}) \tag{1}$$

where P_t denotes the index daily price and ln corresponds to the natural logarithm.

4. Methods

In this section, we describe three methods: the Multifractal Detrended Cross-Correlation Analysis (MF-DCCA), the Q-Cross-Correlation Significance Test, and the DCCA Coefficient method. Additionally, we discuss two techniques that assess the contribution of various sources to the overall cross-correlation multifractality of the bivariate series: random permutation (shuffling) and phase randomization (surrogate).

4.1. Description of the MF-DCCA

Consider two time series $x = \left(x(k)\right)_{1\leq k\leq N}$ and $y = \left(y(k)\right)_{1\leq k\leq N}$, where N is the length of the series. We assume that the series have compact supports, meaning that $x(k) = 0$ and $y(k) = 0$ for only a negligible fraction of the values k.

Step 1: For x and y, we determine the profiles $X = \left(X(i)\right)_{1\leq i\leq N}$ and $Y = \left(Y(i)\right)_{1\leq i\leq N}$ defined by:

$$X(i) = \sum_{k=1}^{N}(x(k) - \bar{x}) \qquad Y(i) = \sum_{k=1}^{N}(y(k) - \bar{y}) \tag{2}$$

where $\bar{x}$ and $\bar{y}$ are the means of x and y:

$$\bar{x} = \frac{1}{N}\sum_{k=1}^{N} x(k) \qquad \bar{y} = \frac{1}{N}\sum_{k=1}^{N} y(k) \tag{3}$$

Step 2: For each time scale s, we divide the two profiles X and Y into $N_s = Int(N/s)$ non-overlapping sub-time series of the same length s, where $Int(.)$ gives the integer part of a real number. Based on the recommendations of Peng, et al. [1] $5 \leq s \leq N/4$ is traditionally selected. Since N is generally not a multiple of s, a short part at the end of the profiles may be neglected. To incorporate all the ignored parts of the series, the same procedure is repeated starting from the end of the profile. Thus, we obtain $2N_s$ intervals $I_{v,s} = \left(I_{v,s}(j)\right)_{1\leq j\leq s}$ defined by:

$$I_{v,s}(j) = (v-1)s + j \tag{4}$$

for $v = 1,2,\cdots,N_s$ and:

$$I_{v,s}(j) = (N - v - N_s)s + j \tag{5}$$

for $v = N_s + 1,2,\cdots,2N_s$.

We denote by $X_{v,s}$ and $Y_{v,s}$, the v^{th} sub-time series corresponding to X and Y, defined by:

$$X_{v,s}(j) = X\left((v-1)s + j\right) \qquad Y_{v,s}(j) = Y\left((v-1)s + j\right) \tag{6}$$

for $v = 1,2,\cdots,N_s$ and:

$$X_{v,s}(j) = X\left((N - v - N_s)s + j\right) \qquad Y_{v,s}(j) = Y\left((N - v - N_s)s + j\right) \tag{7}$$

for $v = N_s + 1,2,\cdots,2N_s$.

Step 3: For each time scale s and for each segment $v = 1,2,\cdots,2N_s$, we measure the local trends $\tilde{X}_{v,s}$ and $\tilde{Y}_{v,s}$ by performing a degree-2 polynomial least-square regressions of the sub-time series $X_{v,s}$ and $Y_{v,s}$ on the interval $I_{v,s}$:

$$\tilde{X}_{v,s}(j) = \alpha_0^{v,s} + \alpha_1^{v,s}.j + \alpha_2^{v,s}.j^2 \qquad \tilde{Y}_{v,s}(j) = \beta_0^{v,s} + \beta_1^{v,s}.j + \beta_2^{v,s}.j^2 \tag{8}$$

We then calculate the detrended covariances:

$$f_{XY}^2(v,s) = \frac{1}{s}\sum_{j=1}^{s}\left|X\left((v-1)s + j\right) - \tilde{X}_{v,s}(j)\right|.\left|Y\left((v-1)s + j\right) - \tilde{Y}_{v,s}(j)\right| \tag{9}$$

for $v = 1,2,\cdots,N_s$, and:

$$f_{XY}^2(v,s) = \frac{1}{s}\sum_{i=1}^{s}\left|X\left((N - v - N_s)s + j\right) - \tilde{X}_{v,s}(j)\right|.\left|Y\left((N - v - N_s)s + j\right) - \tilde{Y}_{v,s}(j)\right| \tag{10}$$

for $v = N_s + 1,2,\cdots,2N_s$.

Step 4:

- The q^{th} order fluctuation functions:

For each time scale s and for a given order q, the q^{th} order fluctuation function $F_q(s)$ is defined as an average of the covariances over all segments:

$$F_q(s) = \left[\frac{1}{2N_S}\sum_{v=1}^{2N_S}\left(f_{XY}^2(v,s)\right)^{\frac{q}{2}}\right]^{\frac{1}{q}} \quad (11)$$

for $q \neq 0$ and:

$$F_0(s) = exp\left[\frac{1}{4N_S}\sum_{v=1}^{2N_S} ln\left(f_{XY}^2(v,s)\right)\right] \quad (12)$$

for $q = 0$.

The purpose of the MF-DCCA procedure is primarily to determine the behavior of the fluctuation functions $F_q^{XY}(s)$ as a function of the time scale s for various values of q. To this end, steps 2 through 4 must be repeated for different time scales s.

Step 5 *:* We analyze the multi-scale behavior of the fluctuation functions $F_q^{XY}(s)$ by estimating the slope of the log-log plots of $F_q^{XY}(s)$ versus s for different values of q. If the analyzed time series X and Y exhibits long-range cross-correlation according to a power-law, such as fractal properties, the fluctuation function $F_q^{XY}(s)$ will behave, for sufficiently large values of s, according to the following power-law scaling law:

$$F_q^{XY}(s) \sim s^{H_{XY}(q)} \quad (13)$$

or

$$log\left(F_q^{XY}(s)\right) = H_{XY}(q).log(s) + log(C) \quad (14)$$

where $H_{XY}(q)$ is called the generalized Hurst exponent, which is the power-law cross-correlation of the two series X and Y.

When $H_{XY}(q)$ depend on q, the cross-correlation of the two-time series is multifractal, otherwise it is monofractal. To estimate the values of $H_{XY}(q)$ for different values of q, we perform a semi-logarithmic regression of the time series $H_{XY}(q)$ on the time series $F_q^{XY}(s)$. When $q = 2$, $H_{XY}(2)$ is known as the standard Hurst exponent. When $H_{XY}(2) = 0.5$, there are no cross-correlations. When $H_{XY}(2) > 0.5$, the cross-correlations are long-range persistent, while $H_{XY}(2) < 0.5$, the two series have long-range anti-persistent cross-correlations. In addition, for positive q, $H_{XY}(q)$ describes the scaling behavior of intervals with large fluctuations. On the contrary, for negative q, $H_{XY}(q)$ describes the scaling behavior of segments with wavelet fluctuations.

$H_{XY}(q)$ is a decreasing function and to measure the degree of multifractality between the two series, we can use the variation ΔH_{XY} between the minimum and maximum values as defined below:

$$\Delta H_{XY} = H_{XY-Max} - H_{XY-Min} = H_{XY}(q_{min}) - H_{XY}(q_{max}) \quad (15)$$

The larger ΔH_{XY} is, the stronger the degree of multifractality will be.

For positive values of q, the average fluctuation function $F_q^{XY}(s)$ is dominated by segments v with large covariances $f_{XY}^2(v,s)$. Thus, for $q > 0$, the generalized Hurst exponents $H_{XY}(q)$ describe the scaling properties of large fluctuations. In contrast, for $q < 0$, the exponents $H_{XY}(q)$ describe the scaling properties of small fluctuations.

It is well known that the generalized Hurst exponent $H_{XY}(q)$ is directly related to the multifractal scaling exponent $\tau_{XY}(q)$, commonly known as the Rényi exponent :

$$\tau_{XY}(q) = q.H_{XY}(q) - 1 \quad (16)$$

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If the Rényi exponent $\tau_{XY}(q)$ increase nonlinearly with q, the cross-correlation of the two series is multifractal. Otherwise, if the Rényi exponent $\tau_{XY}(q)$ is a linear function of q, then the cross-correlation is monofractal.

Another interesting way to characterize the multifractality of the time series cross-correlations, is to use the Hölder spectrum or the singularity spectrum $f_{XY}(\alpha_{XY})$ of the Hölder exponent α_{XY}. It is well known that the singularity spectrum $f_{XY}(\alpha_{XY})$ is related to the Rényi exponent $\tau_{XY}(q)$ through the Legendre transform:

$$\begin{cases}\alpha_{XY} = \tau'_{XY}(q) \\ f_{XY}(\alpha_{XY}) = q.\alpha_{XY} - \tau_{XY}(q)\end{cases} \tag{17}$$

where $\tau'_{XY}(q)$ is the derivative of the function $\tau_{XY}(q)$.

The Hölder exponent α_{XY} characterizes the intensity of the singularity, and the singularity spectrum $f_{XY}(\alpha_{XY})$ represents the Hausdorff dimension of the fractal subset with exponent α_{XY}.

When the cross-correlation between the two series is multifractal, then the singularity spectrum $f_{XY}(\alpha_{XY})$ present a concave bell-shaped curve.

The richness of the multifractality can be determined by the width of the spectrum defined by:

$$\Delta\alpha_{XY} = \alpha_{XY-max} - \alpha_{XY-min} \tag{18}$$

Thus, the wider the spectrum, the richer the multifractal behavior of the cross-correlation of the analyzed time series.

We can easily deduce the relationship between the generalized Hurst exponent $h(q)$ and the singularity spectrum $f_{XY}(\alpha_{XY})$:

$$\begin{cases}\alpha_{XY} = H_{XY}(q) + q.H'_{XY}(q) \\ f_{XY}(\alpha_{XY}) = q.\left(\alpha_{XY} - H_{XY}(q)\right) + 1\end{cases} \tag{19}$$

4.2. A Q-Cross-Correlation Significance Test

As a preliminary analysis, it is useful to examine the existence of cross-correlations qualitatively. To this end, Podobnik, et al. [23] developed the Q-Cross-Correlation statistic test denoted Q_{CC}.

Suppose $(x_t)_{1\le t\le N}$ and $(y_t)_{1\le t\le N}$ are two time series of length N. Podobnik, et al. [23] have defined the cross-correlation function C_i by : for $1 \le i \le N-1$

$$C_i = \frac{\sum_{k=i+1}^{N} x_k.y_{k-1}}{\sqrt{\sum_{k=1}^{N} x_k^2 \cdot \sum_{k=1}^{N} y_k^2}} \tag{20}$$

The cross-correlation statistic Q_{CC} is defined by : for $1 \le s \le N-1$

$$Q_{CC}(s) = N^2.\sum_{i=1}^{s}\frac{C_i^2}{N-s} \tag{21}$$

Podobnik, et al. [23] demonstrated that $Q_{CC}(s)$ is approximately $\chi^2(s)$ distributed with s degrees of freedom. The test can be used to test the null hypothesis that none of the first s cross-correlation coefficients is different from zero. The authors proposed to use the statistic by plotting the test statistic $Q_{CC}(s)$ versus the critical values $\chi^2(s)$ for a broad range of the degree of freedom s. If for a broad range of s the test statistic $Q_{CC}(s)$ exceeds the critical values at a 95% level of confidence, we can claim that cross-correlations are not only significant, but there are long-range cross-correlations. However, this test statistic, as a correlation coefficient, is a measure of linear cross-correlations, and as pointed by Podobnik, et al. [23] this cross-correlations test should be used to test the presence of cross-correlations only qualitatively.

4.3. DCCA Cross-correlation Coefficient

Based on the Detrended Cross Correlation Analysis (DCCA) and the Detrended Fluctuation

Analysis (DFA) [1, 24] proposed a DCCA Cross-Correlation Coefficient to quantify the cross-correlation between two non-stationary series. This method divide the profiles of the two series into overlapping sub-time series, contrary to the MF-DCCA which is based on the non-overlapping segments. We present below another version of the DCCA Cross-correlation Coefficient based on MF-DCCA and using non-overlapping segments.
The detrended covariance and variance functions are given by:

$$f_{XY}^2(v,s) = \frac{1}{s}\sum_{i=1}^{s}\left(X(j) - \tilde{X}_{v,s}(j)\right)\left(Y(j) - \tilde{Y}_{v,s}(j)\right) \tag{22}$$

$$f_X^2(v,s) = \frac{1}{s}\sum_{i=1}^{s}\left(X(j) - \tilde{X}_{v,s}(j)\right)^2 \qquad f_Y^2(v,s) = \frac{1}{s}\sum_{i=1}^{s}\left(Y(j) - \tilde{Y}_{v,s}(j)\right)^2 \tag{23}$$

We obtain the covariance and variance fluctuation functions by taking $q = 2$:

$$F_{XY}^2(v,s) = \frac{1}{2N_S}\sum_{v=1}^{2N_S} f_{XY}^2(v,s) \tag{24}$$

$$F_X^2(s) = \frac{1}{2N_S}\sum_{v=1}^{2N_S} f_X^2(v,s) \qquad F_Y^2(s) = \frac{1}{2N_S}\sum_{v=1}^{2N_S} f_Y^2(v,s) \tag{25}$$

The DCCA Cross-correlation Coefficient is defined by:

$$\rho_{DCCA}(s) = \frac{F_{XY}^2(s)}{\sqrt{F_X^2(s)} \times \sqrt{F_Y^2(s)}} \tag{26}$$

4.4. Sources of Cross-Correlation Multifractality

It is widely recognized that the two primary sources of multifractality in the cross-correlation of bivariate time series are long-term temporal cross-correlations and heavy-tailed distributions [2, 4]. To assess the contribution of each source to the overall cross-correlation multifractality, we apply two transformations to the original return series: random permutation and phase randomization.

Random permutation (shuffling) maintains the distribution of the data's moments but removes any long-term correlations. After permutation, the data retain their statistical distribution but lack temporal correlations or memory.

Phase randomization (surrogate), on the other hand, isolates the effect of long-term correlations on multifractality. This method involves randomly altering the temporal phases of the data, disrupting long-term correlations while preserving the overall fluctuation behavior.
Several techniques for phase randomization are discussed in the literature:

- Inverse Fast Fourier Transform (IFFT) [25].
- Iterated Algorithm (iAAFT) [26].
- Statically Transformed Autoregressive Process (STAP) [27].

In this study, we employed two shuffling techniques using functions “randperm” and “randi”. For phase randomization, we utilized the Inverse Fast Fourier Transform (IFFT) method.

5. Results and Discussion

5.1. Results of Q-Cross-Correlation Significance Test

In this section, we have checked qualitatively the presence of the cross-correlations between the six Southeast Asia indices, using the Q_{CC} statistic.

For all pairs return of indices, we have plotted the decimal logarithm of the test statistic $Q_{CC}(s)$ versus the decimal logarithm of the critical values $\chi_{0,95}^2(s)$ at 95% confidence level for a broad range of the degree of freedom s, $1 \leq s \leq 2000$. The results are given in the figure below.

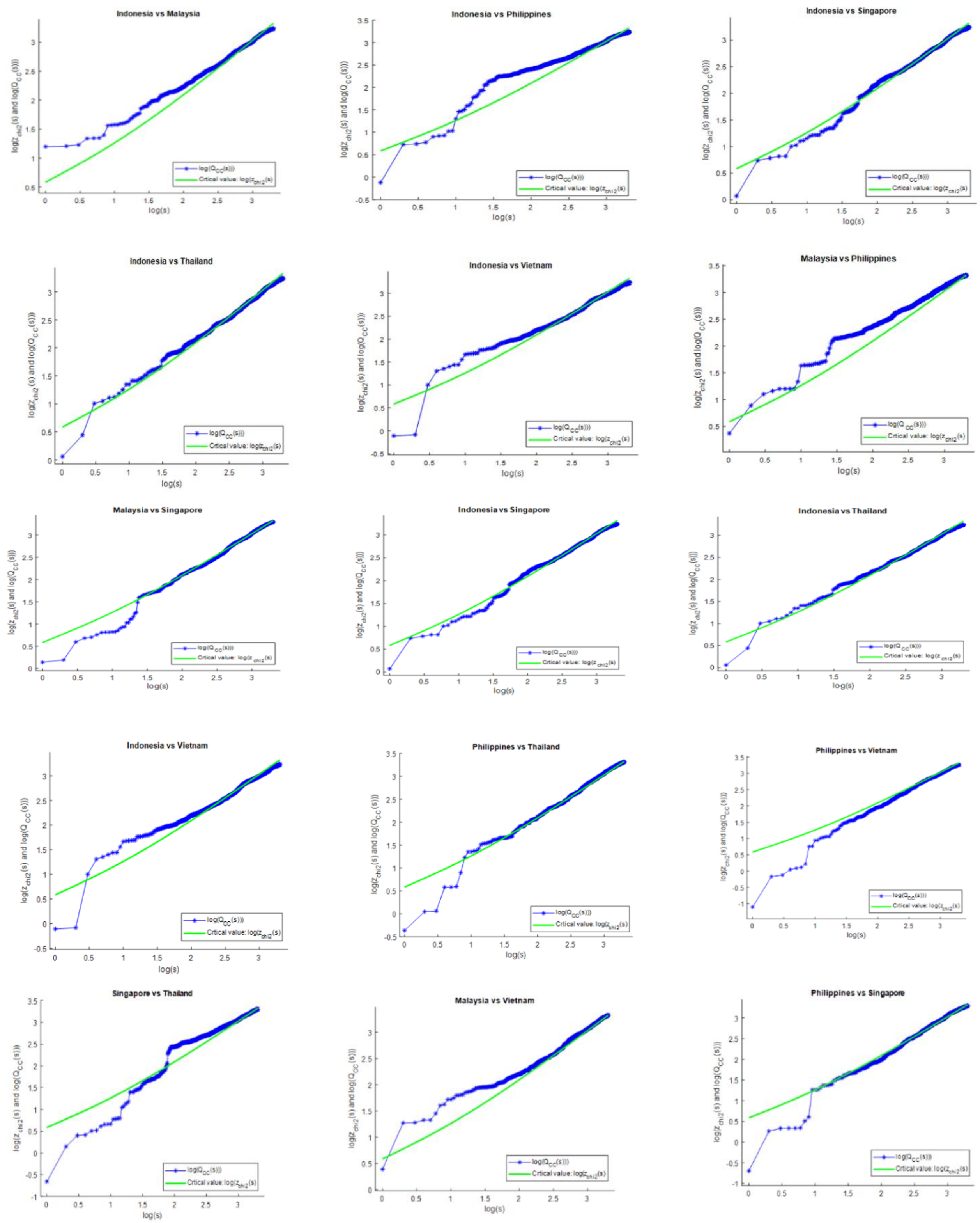

Figure 1.
$Log(Q_{CC}(s))$ and $Log(\chi^2_{0,95}(s))$ vs. $Log(s)$ for all pairs of indices.

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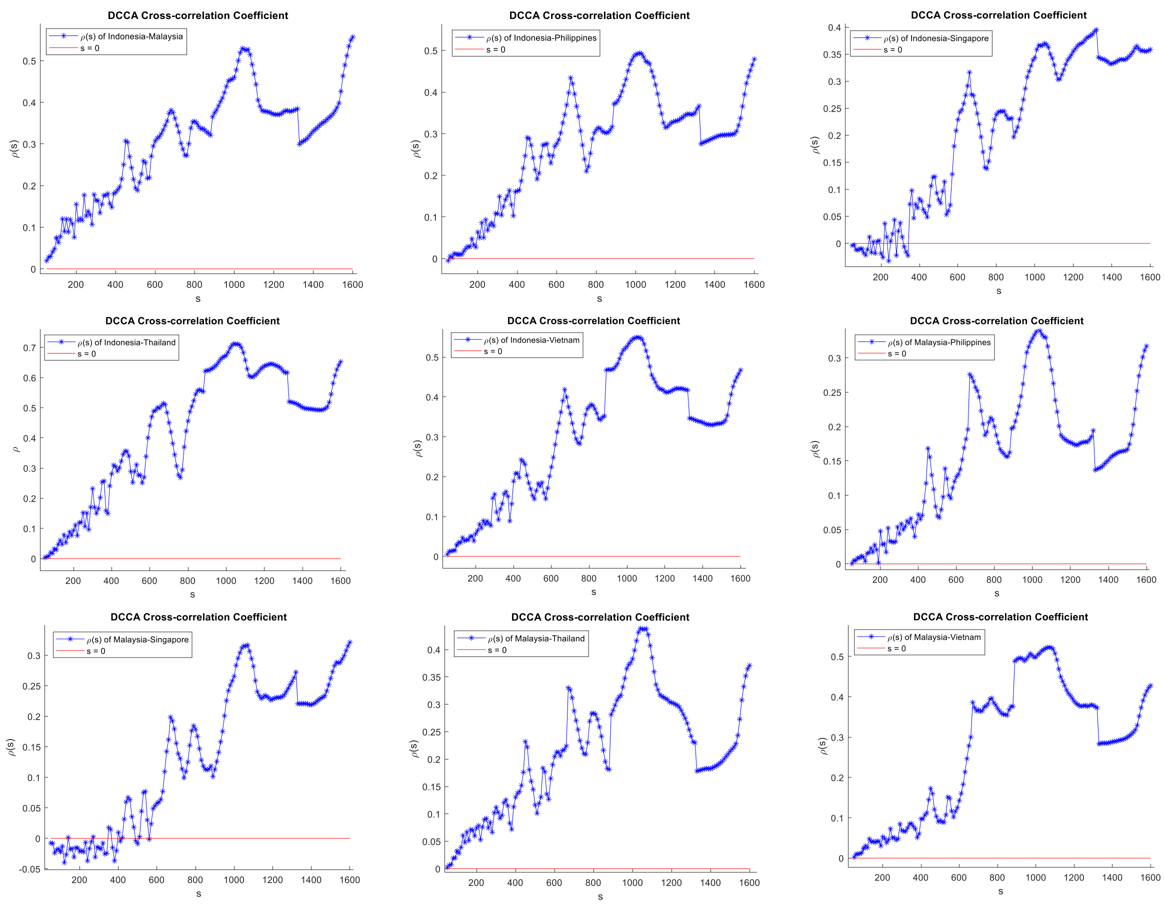
The results from the $Q_{CC}(s)$ statistics indicate that for all pairs of stock market indices in the dataset, the values are close to or deviate from the critical threshold of $\chi^2_{0,95}(s)$. This suggests that significant cross-correlations exist between these pairs, pointing to interconnectedness and shared dynamics among the major Southeast Asian stock markets. However, it is important to note that this test statistic provides a predominantly linear and qualitative assessment of cross-correlations, which limits its ability to fully capture the complex, nonlinear dependencies that may exist in financial time series.

5.2. Results of DCCA Cross-Correlation Coefficient

In this section, we applied the DCCA cross-correlation coefficient to quantify the cross-correlation between the six Southeast Asia indices. The figure below shows the plots of the DCCA cross-correlation coefficient $\rho_{DCCA}(s)$ as a function of the variable s for the 15 pairs.



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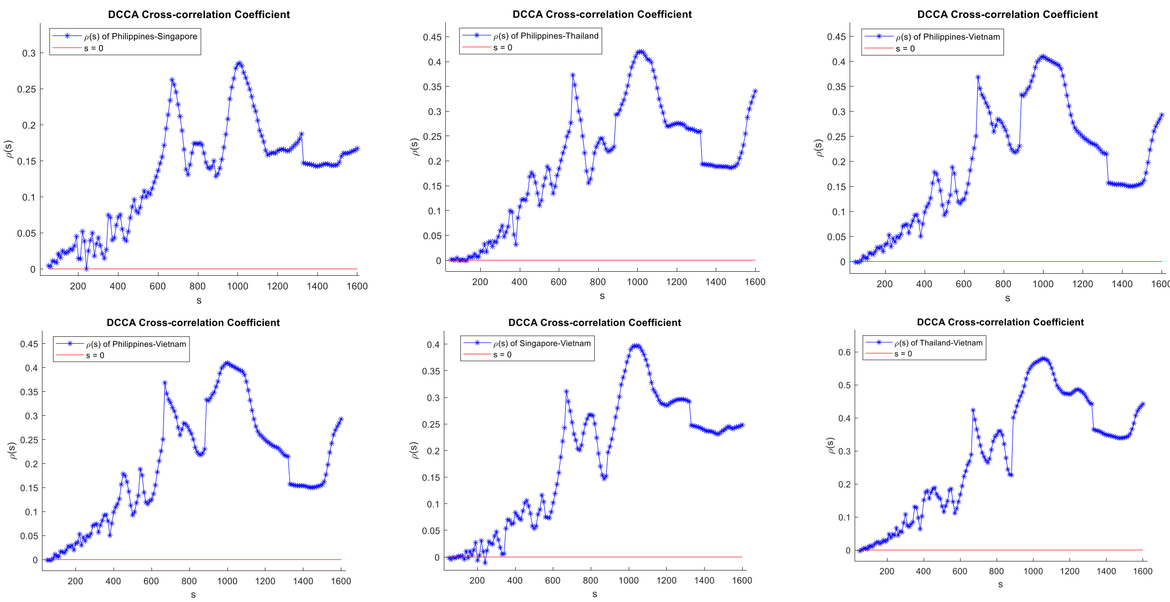

Figure 2.
DCCA cross-correlation coefficient $\rho_{DCCA}(s)$ vs. s for all pairs of six indices.

The analysis of the DCCA cross-correlation coefficients for the six major Southeast Asian indices reveals persistent cross-correlations across all market pairs, with values between 0 and 1. This indicates interconnected behavior among these markets, where an increase in one market is likely to drive increases in others.

The persistence of these correlations suggests long memory effects, implying that past market behaviors impact future dynamics, reflecting regional market interdependence. This finding highlights the challenges for portfolio diversification, as these markets are not entirely independent. Additionally, the varying strength of correlations between different market pairs, during the time scale interval, suggests that the level of integration across Southeast Asian markets is not uniform. The persistence of these cross-correlations may also heighten systemic risk, as financial shocks in one market could propagate to others. Future sections will delve deeper into the multifractal characteristics driving these persistent cross-correlations to better understand the complexity and evolution of these relationships over time.

5.3. Results from the Application of MF-DCCA

In this section, we applied the MF-DCCA technique to analyze the multifractal cross-correlation of the six southeast Asia indices.

5.3.1. Multi-Scale Behavior of the Cross Correlation Fluctuation Functions

We analyzed the multi-scale behavior of the cross-correlation fluctuation functions $F_q^{XY}(s)$ with respect to the time scales s within the interval [20:10:100, 200:100:1000] for values of q in the interval [-45:5:-5,-3.1:0.1:-0.1,0.1:0.1:3.1,5:5:45]. By regressing $Log(s)$ on $Log\left(F_q^{XY}(s)\right)$, we obtain an estimation of $H_{XY}(q)$:

$$Log\left(F_q^{XY}(s)\right) \approx H_{XY}(q).Log(s) \tag{27}$$

The figure below shows the log-log plots of $Log\left(F_q^{XY}(s)\right)$ versus $Log(s)$ for 9 values of q chosen from $\{-10, -5, -3, -0.7, 0, 0.7, 3, 5, 10\}$, for the 15 pairs of indices.

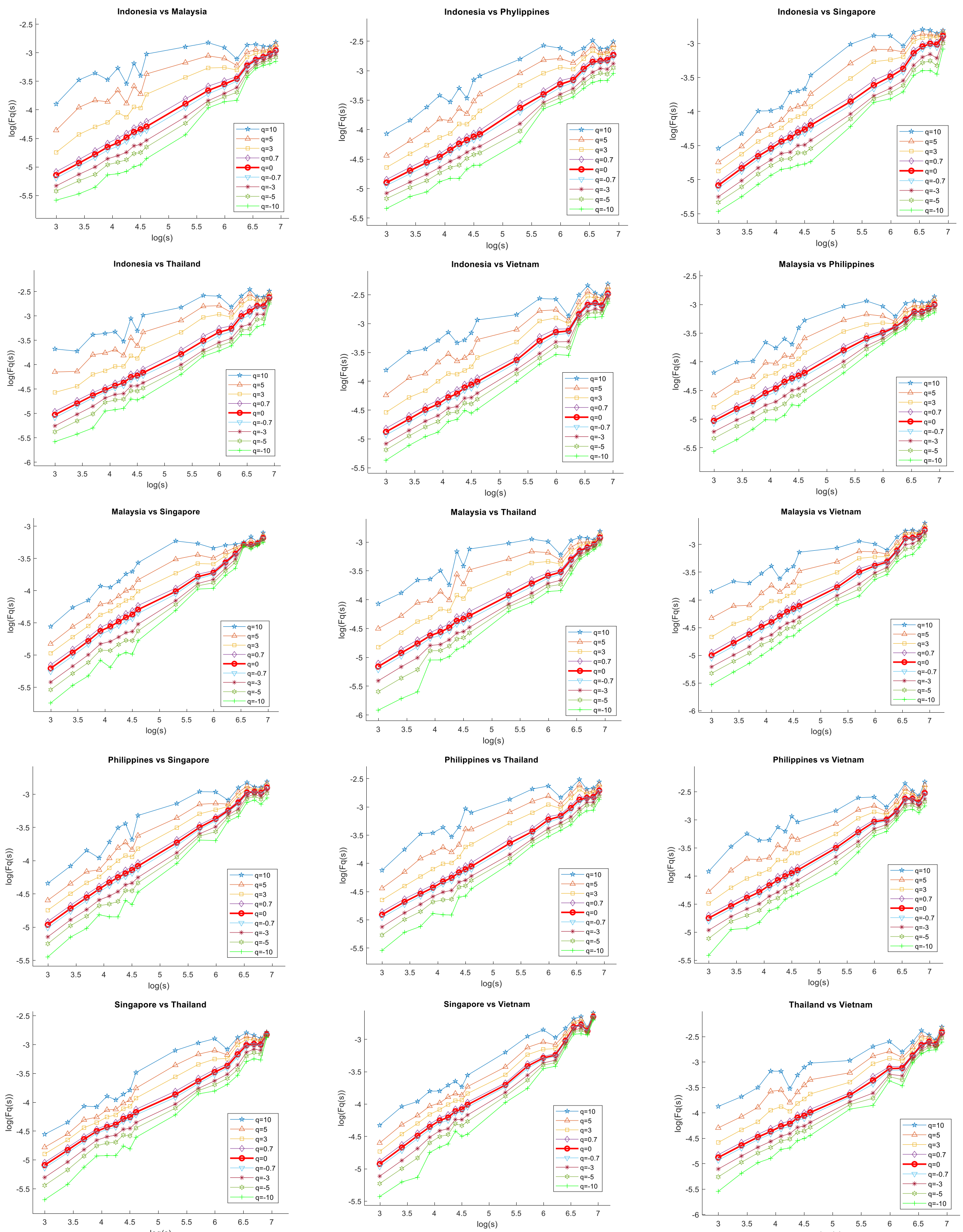

Figure 3.
$Log\left(F_q^{XY}(s)\right)$ vs. $Log(s)$ for $q \in \{-15, -5, -3, -0.7, 0, 0.7, 3, 5, 20\}$

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As illustrated in the previous figure, the fluctuation functions $F_q^{XY}(s)$ for all index pairs demonstrate a nonlinear increase with the scale s and exhibit growth with changes in q, which is indicative of a power-law relationship across all pairs. This nonlinear increase confirms the existence of significant long-range cross-correlations between the indices, as the scaling behavior of $F_q^{XY}(s)$ suggests that the correlations are not merely short-term or random but extend over long time horizons. The power-law relationship also points to the multifractal nature of these cross-correlations, as the variation in q highlights the fact that different moments of the fluctuation function exhibit different scaling properties. This is a key characteristic of multifractality, where the system's complexity cannot be captured by a single scaling exponent but instead requires a spectrum of exponents. The persistence of these cross-correlations suggests that shocks or movements in one market are likely to propagate and have a sustained impact on others. This interdependence has significant implications for risk management and portfolio diversification, as it highlights the potential for shared vulnerabilities or opportunities across the region's stock markets. The growth of $F_q^{XY}(s)$ with q further indicates that the strength of cross-correlation may differ for small and large fluctuations, with larger movements possibly exhibiting stronger multifractality.

5.3.2. Multifractality and Persistence of the Cross-Correlations

The figure below shows the plots of the generalized Hurst exponent $H_{XY}(q)$, the Rényi exponent $\tau_{XY}(q)$ and the Singularity spectra $f_{XY}(\alpha)$ for all the pairs of indices.

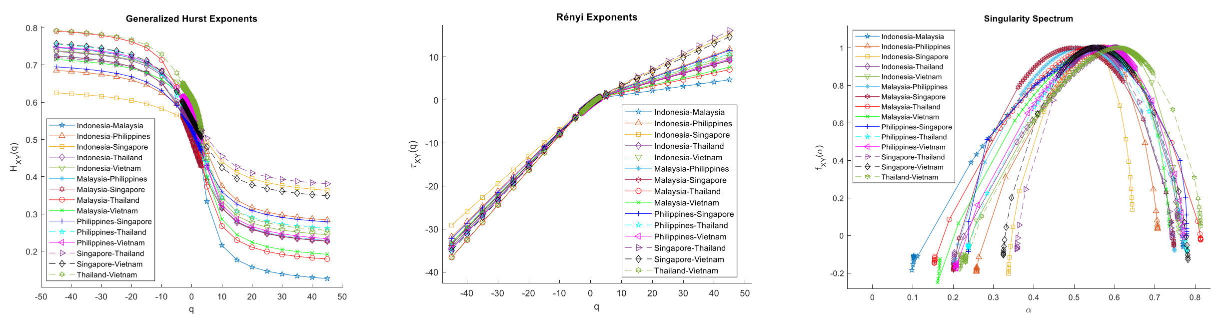

Figure 4.
Plots of $H_{XY}(q)$, $\tau_{XY}(q)$ and $f_{XY}(\alpha)$ for all pairs of southeast Asia indices.

The results from the figure illustrate important aspects of the multifractal behavior in the cross-correlations between the pairs of Southeast Asian stock market indices. As the parameter q varies from -45 to 45, the Generalized Hurst exponent $H_{XY}(q)$ shows a non-linear decrease. This behavior suggests that the correlations between smaller and larger fluctuations differ significantly, indicating a heterogeneous structure in the cross-correlation scaling. A decreasing $H_{XY}(q)$ points to stronger persistent correlations for smaller fluctuations (negative q) and weaker persistence for larger ones (positive q), a hallmark of multifractality.

Similarly, the non-linear increase in the Rényi exponent $\tau_{XY}(q)$ across all pairs further confirms the presence of multifractal scaling. This indicates that the fluctuations in cross-correlations are governed by a range of different scaling exponents rather than a single uniform behavior, a key feature of multifractal systems.

Furthermore, the Singularity spectrum functions $f_{XY}(\alpha)$, which exhibit inverted concave parabolic shapes, provide strong evidence for multifractality. The bell-shaped parabolas indicate that the pairs of indices have a wide range of singularities, reinforcing the finding that these cross-correlations are not monofractal, but rather exhibit a rich multifractal nature.

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These results suggest that the relationships between the Southeast Asian stock markets are highly complex and governed by multifractal dynamics. The existence of multifractality implies that the markets exhibit long-range dependencies and scale-invariant behavior, which could be driven by underlying factors such as market liquidity, investor sentiment, or external economic shocks. This complexity may affect portfolio diversification and risk management strategies, as it points to a non-linear interdependence between the markets that traditional methods may not fully capture.

The strength of multifractality of the cross-correlations could be measured by the difference between the smallest and largest values of $H_{XY}(q)$:

$$\Delta H_{XY} = H_{XY-Max} - H_{XY-Min} = H_{XY}(q_{min}) - H_{XY}(q_{max}) \quad (28)$$

or by the width of the Spectrum, given by:

$$\Delta\alpha_{XY} = \alpha_{XY-max} - \alpha_{XY-min} \quad (29)$$

The table below present the degrees of multifractality for the 15 pairs of indices based on ΔH_{XY} and $\Delta\alpha_{XY}$.

Table 1.
Degrees of multifractality of the 15 pairs cross-correlations based on ΔH_{XY} and $\Delta\alpha_{XY}$.

Pairs of indices	ΔH_{XY}	$\Delta\alpha_{XY}$	**Pairs of indices**	ΔH_{XY}	$\Delta\alpha_{XY}$
Indonesia vs Malaysia	0.620	0.668	Malaysia vs Vietnam	0.523	0.569
Indonesia vs Philippines	0.401	0.449	Philippines vs Singapore	0.416	0.462
Indonesia vs Singapore	0.262	0.308	Philippines vs Thailand	0.494	0.541
Indonesia vs Thailand	0.509	0.553	Philippines vs Vietnam	0.514	0.562
Indonesia vs Vietnam	0.493	0.540	Singapore vs Thailand	0.339	0.385
Malaysia vs Philippines	0.495	0.545	Singapore vs Vietnam	0.408	0.458
Malaysia vs Singapore	0.497	0.546	Thailand vs Vietnam	0.536	0.582
Malaysia vs Thailand	0.612	0.659			

The analysis of cross-correlations between each pair of Southeast Asian stock indices reveals clear evidence of multifractal behavior, as demonstrated by the values of both ΔH_{XY} and $\Delta\alpha_{XY}$.

Firstly, the fact that ΔH_{XY} is non-zero for all pairs suggests that the cross-correlations are characterized by multifractality rather than monofractality. A $\Delta H_{XY} = 0$ would imply uniform scaling behavior across different fluctuations, indicative of monofractal time series. Since ΔH_{XY} is positive for all pairs, this means that the different pairs of indices exhibit varying levels of persistence across different time scales, confirming multifractal behavior. This non-uniform scaling suggests that small and large fluctuations are governed by different dynamics, which is a hallmark of multifractal systems.

Similarly, the non-zero values of $\Delta\alpha_{XY}$ further emphasize the multifractality of the cross-correlations. The width of the Singularity spectrum reflects the range of singularities or scaling exponents present in the data. A broader spectrum, as observed here, indicates that the cross-correlations are influenced by a wide variety of scaling behaviors, typical of multifractal systems. In contrast, a $\Delta\alpha_{XY} = 0$ would denote a monofractal structure.

The multifractal behavior observed based on both ΔH_{XY} and $\Delta\alpha_{XY}$ points to the complexity of the relationships between these stock markets. It suggests that different time scales or magnitudes of market movements affect cross-correlations in distinct ways, potentially influenced by factors like market liquidity, external shocks, or investor behavior. This multifractality adds a layer of intricacy to understanding the interconnections between markets, implying that their relationships cannot be fully captured by simple linear models or assumptions of uniform behavior.

The figure below illustrates the degrees of multifractality based on on ΔH_{XY} and $\Delta\alpha_{XY}$.

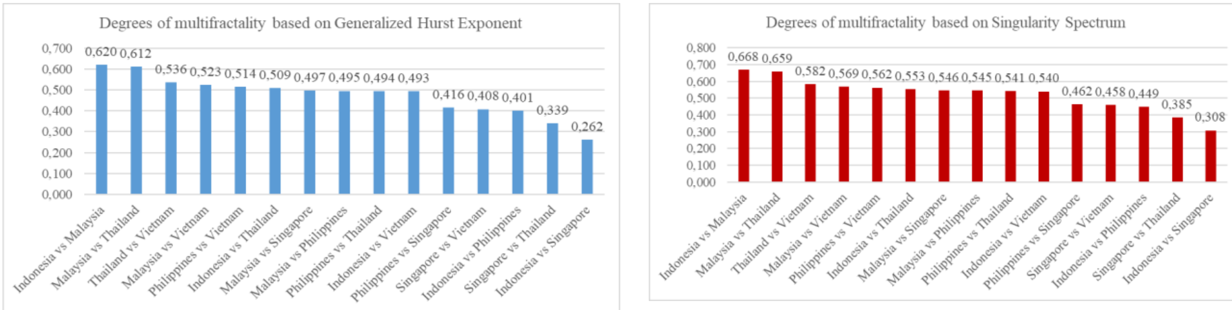


Figure 5.
Ranking of the Southeast Asian indices based on the degrees of multifractality.

As demonstrated in the figure, the rankings derived from both ΔH_{XY} and $\Delta\alpha_{XY}$ metrics reveal consistent patterns of multifractality across the pairs of indices. The pair Indonesia-Malaysia exhibits the highest degree of multifractality, indicating more complex cross-correlations and significant variations in scaling behavior. This suggests that the interdependence between these two markets is characterized by a higher level of intricacy, likely driven by a combination of long-range correlations and market-specific factors. On the other hand, the Indonesia-Singapore pair shows the lowest multifractality, suggesting that their cross-correlations are less complex. This comparative analysis highlights the varying levels of market interaction and cross-correlation complexity across the region, which could be influenced by structural differences in market dynamics, economic linkages, and investor behavior.

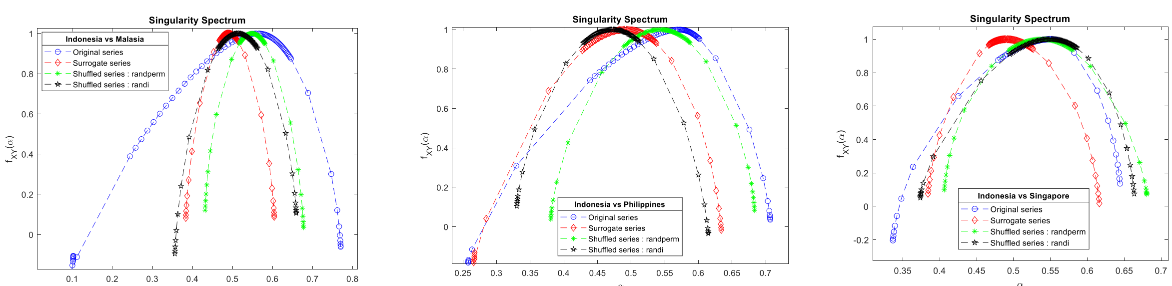
5.3.3. Source of Multifractality for the Cross-Correlations

As previously mentioned, multifractality can arise from two distinct sources: long-term temporal cross-correlations and heavy-tailed distributions. To assess the contributions of each source to the overall multifractality of cross-correlations, we employ two transformations on the original geometric return series:

(a) Shuffling
(b) Surrogate

In this study, we implemented two shuffling techniques, referred to as "randperm" and "randi." For the phase randomization, we utilized the Inverse Fast Fourier Transform (IFFT) method as described by Proakis and Manolakis [25].

The figures presented below compare the Singularity pectrum curves $f_{XY}(\alpha)$ for the 15 original pairs of index return series against those derived from the surrogate and shuffled series.



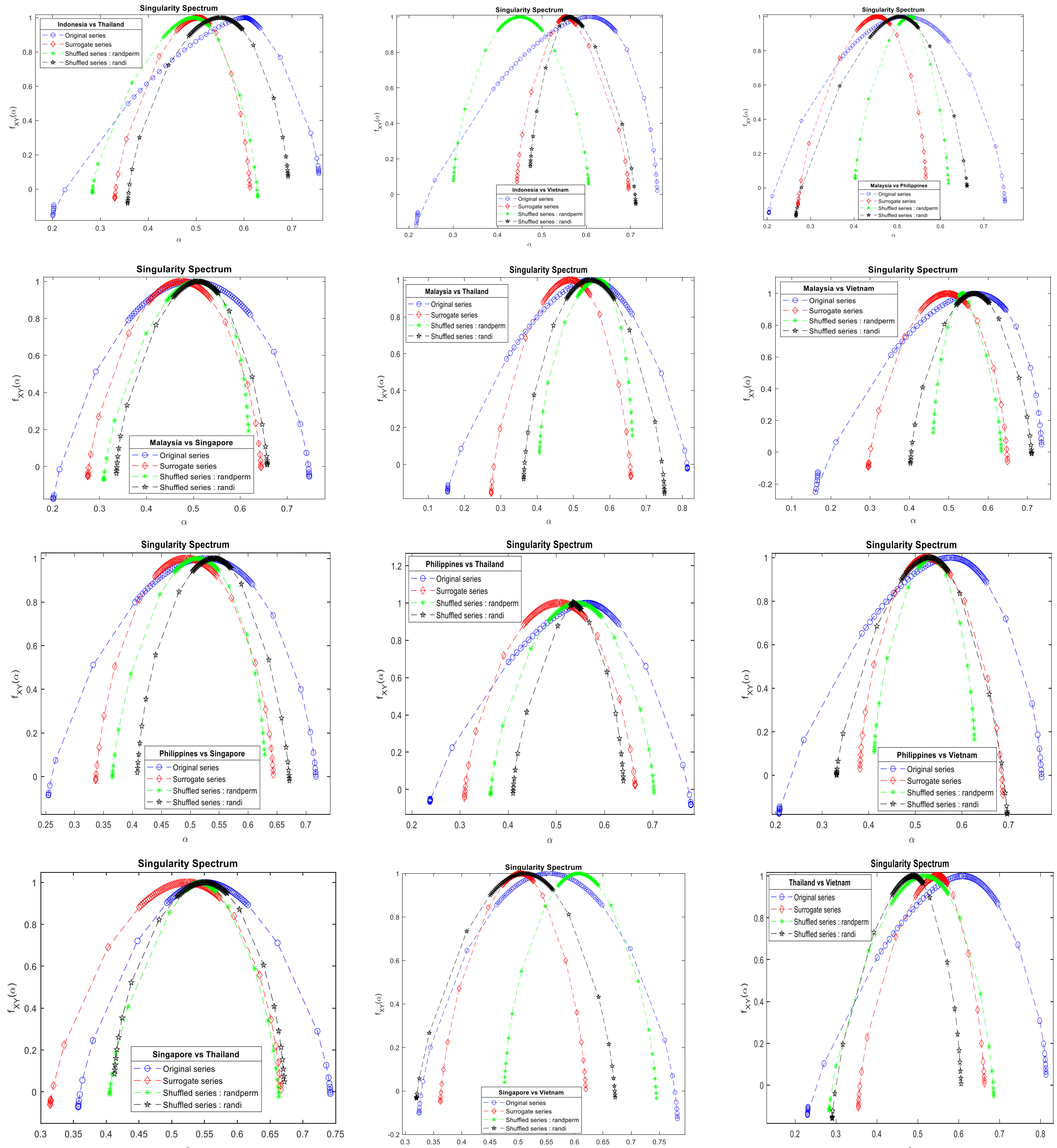


Figure 6.
Singularity spectrum $f_{XY}(\alpha)$ vs. α for original, surrogate and shuffled pairs.

As shown in the figure above, the application of both the shuffling and surrogate transformations results in a reduction of the multifractality degrees of the original series. To quantify the extent of this reduction, we calculated the values of $\Delta\alpha_{XY}$ for all 15 pairs of indices.

We ran the MF-DCCA program 100 times for each pair, and each run produced different results for the surrogate series and the two shuffled series. In contrast, the results for the original series remained consistent across all simulations. This variability in the results for the surrogate and shuffled series is

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attributed to the random permutations used by the algorithms when generating these series. Despite the variability in the surrogate and shuffled series, the $\Delta\alpha_{XY}$ values for the original series are consistently greater than those for the surrogate series and both shuffled series in all 100 simulations.

The table below presents the results from one of the 100 simulations, further illustrating this trend. This confirms that the multifractal nature of the original series is significantly reduced when subjected to the shuffling and surrogate transformations, reinforcing the fact that the multifractality in the original series is predominantly driven by long-term temporal cross-correlations and heavy-tailed distributions.

Table 2.
Degrees of multifractality of original, surrogate and shuffled series based on $\Delta\alpha_{XY}$

	$\Delta\alpha_{XY}$			
Pairs	**Original**	**Surrogate**	**Shuffled-randperm**	**Shuffled-randi**
Indonesia vs Malaysia	0.668	0.221	0.246	0.303
Indonesia vs Philippines	0.449	0.367	0.303	0.285
Indonesia vs Singapore	0.308	0.232	0.274	0.290
Indonesia vs Thailand	0.553	0.282	0.345	0.335
Indonesia vs Vietnam	0.540	0.252	0.305	0.238
Malaysia vs Philippines	0.545	0.295	0.216	0.394
Malaysia vs Singapore	0.546	0.369	0.309	0.322
Malaysia vs Thailand	0.659	0.385	0.256	0.388
Malaysia vs Vietnam	0.569	0.353	0.173	0.308
Philippines vs Singapore	0.462	0.307	0.263	0.263
Philippines vs Thailand	0.541	0.354	0.339	0.229
Philippines vs Vietnam	0.562	0.307	0.215	0.366
Singapore vs Thailand	0.385	0.351	0.258	0.259
Singapore vs Vietnam	0.458	0.258	0.269	0.352
Thailand vs Vietnam	0.582	0.308	0.402	0.315

The results indicate that $\Delta\alpha_{XY-Originate}$ is greater than both $\Delta\alpha_{XY-Surrogate}$ and $\Delta\alpha_{XY-Shuffled}$ for all 15 pairs of index returns. This indicates that the multifractality of the cross-correlations has been reduced by both the surrogate and shuffled transformations. We conclude that both long-term cross-correlations and heavy-tailed distributions contribute to the cross-correlations multifractal behavior of the 15 pairs of index returns.

This outcome underscores the significance of both long-range dependencies and the presence of heavy tails in driving the multifractal behavior observed in the 15 pairs of index returns. By demonstrating that both sources play a key role in the multifractal structure, the results reinforce the conclusion that the complex interactions between these markets are not only driven by temporal dependencies but also by the fat-tailed nature of the returns distributions.

6. Conclusion

Based on the comprehensive findings from this research, we can draw several key conclusions regarding the cross-correlations and multifractal behavior of the major Southeast Asian stock markets.

Initially, the significant cross-correlations identified through the $Q_{CC}(s)$ statistics indicate a robust interconnectedness among the stock market indices. This interconnectedness suggests that market dynamics are not isolated, but rather influenced by a complex web of relationships across the region. However, the limitations of the Q_{CC} statistic highlight the necessity for more sophisticated methods to fully capture the intricate nonlinear dependencies present in these financial time series.

Furtehermore, the DCCA cross-correlation coefficients affirm the existence of long memory effects, indicating that past market behaviors significantly impact future dynamics. This persistence in correlations not only reflects regional market interdependence but also poses challenges for portfolio diversification, as these markets are not entirely independent. The variability in correlation strengths across different pairs and time scales further emphasizes the heterogeneous nature of market integration, suggesting that systemic risk may increase during times of financial stress.

Moreover, the analysis of fluctuation functions demonstrates a power-law relationship, reinforcing the presence of long-range cross-correlations. This finding indicates that market responses are likely to propagate over extended periods, complicating risk management strategies. The evidence of multifractal characteristics in the scaling behaviors—such as the non-linear changes in the Generalized Hurst exponent and the Rényi exponent—illustrates that the cross-correlations are governed by diverse dynamics across varying fluctuations.

Finally, the results of the shuffling and surrogate transformations reveal a notable reduction in multifractality, further confirming that both long-term temporal cross-correlations and heavy-tailed distributions contribute significantly to the complex behavior of these markets. The findings suggest that these two factors are integral to understanding the multifractal nature of cross-correlations, indicating that financial shocks in one market can have pronounced effects on others due to these underlying dependencies.

Building upon the comprehensive findings and conclusions, several practical implications can be drawn for investors, portfolio managers, and policymakers.

6.1. Portfolio Management and Diversification Strategies

The demonstrated interconnectedness among Southeast Asian stock markets suggests that diversification strategies based solely on geographical separation may be less effective than previously thought. Investors should consider the multifractal nature of cross-correlations when constructing portfolios, as the persistent dependencies between markets can lead to increased systemic risk. Instead of focusing on traditional diversification across markets, investors may benefit from strategies that incorporate assets with lower correlation coefficients, even if they are from the same region. Additionally, employing risk management tools that account for long-term dependencies and multifractal characteristics will enhance portfolio resilience against market shocks.

6.2. Risk Assessment and Management

The presence of long memory effects and heavy-tailed distributions in the cross-correlations highlights the need for advanced risk assessment models that incorporate multifractality. Traditional risk metrics such as Value at Risk (VaR) may underestimate potential losses during periods of market stress due to their linear assumptions. Implementing multifractal models and stress testing portfolios under various market scenarios will provide a more nuanced understanding of potential risks. Financial institutions should consider developing and utilizing models that account for the tail dependencies and nonlinear relationships revealed in this research. Such models can enhance the accuracy of risk forecasts, allowing firms to better prepare for extreme market movements and improve their overall risk management frameworks.

6.3. Market Regulation and Policy Implications

The interconnectedness of stock markets calls for closer regulatory oversight to mitigate systemic risk. Policymakers should monitor cross-market dynamics to identify potential sources of contagion that could lead to broader financial instability. Regulatory frameworks may need to be updated to reflect the complex interdependencies between markets, ensuring that financial institutions maintain adequate capital reserves to withstand shocks in interconnected systems. Moreover, collaboration among regulatory bodies in Southeast Asia can facilitate a more coordinated approach to market surveillance and crisis management. Establishing regional frameworks for information sharing and joint responses to financial distress can enhance overall market stability.

6.4. Investor Education and Awareness

Investors should be educated about the implications of multifractal behavior in stock markets. Understanding that market correlations can change over time and are influenced by underlying factors such as liquidity and investor sentiment is crucial. Investor awareness programs could focus on the importance of adaptive strategies that account for market complexity rather than relying on historical performance or simplistic models. Encouraging investors to adopt a long-term perspective and consider the broader economic and political factors that can influence market dynamics will lead to more informed investment decisions.

6.5. Further Research and Development

The findings of this research indicate that there is a need for ongoing exploration of the cross-correlation multifractal nature of financial markets. Researchers should continue to investigate how varying market conditions, technological advancements, and geopolitical events impact the multifractal behavior of stock markets. Developing new methodologies that better capture the complexities of market interactions will be essential for advancing our understanding of financial systems.

In conclusion, this research provides significant added value by offering a deeper understanding of the multifractal dynamics driving the interdependence of Southeast Asian stock markets. By employing advanced analytical tools and providing a comprehensive, region-specific study, this work expands the theoretical and practical knowledge of financial market interconnections. It not only advances cross-correllation multifractal analysis techniques but also lays the groundwork for improved investment, risk management, and regulatory strategies in the context of increasingly interconnected global financial markets.

Transparency:

The authors confirm that the manuscript is an honest, accurate, and transparent account of the study; that no vital features of the study have been omitted; and that any discrepancies from the study as planned have been explained. This study followed all ethical practices during writing.

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