

Optimizing of the specific power consumption of an ore grinding mill, taking into account the stability of its electric drive system

Marinka Baghdasaryan^{1*}, Eduard Hakobyan¹, Gor Vardanyan¹

¹Institute of Energetics and Electrical Engineering, National Polytechnic University of Armenia, Teryan St.105, 0009 Yerevan, Armenia; m.baghdasaryan@seua.am (M.B.)

Abstract: Rising energy costs and the alarming state of the environment are creating new problems for energy-intensive industrial enterprises. This is why energy saving is currently becoming an increasingly relevant issue in all areas of economic and social development. Of particular interest are energy-intensive technological processes, which are interconnected and operate under the influence of random factors. One example is the electric drive system of a drum mill, which powers the ore crushing process. This paper presents a new approach to optimizing the specific energy consumption of electric drive systems for ore crushing mills. The essence of this approach is that the formulated energy-saving problem considers system stability. This approach eliminates unnecessary energy costs for systems operating under nonstandard conditions while ensuring the production of high-quality products. Depending on the specific gravity of the crushed ore, a comparative analysis shows that energy savings of up to 34.4% can be achieved. The stabilization condition can be used to optimize the electromechanical mill system's specific energy consumption, thereby improving operational efficiency.

Keywords: *Specific energy consumption, optimization criteria, stability condition, drum mill.*

1. Introduction

As with many industrial processes, various economic criteria are employed to evaluate the efficiency of ore crushing [1-4].

For long-term planning, operating costs, the cost of obtaining one ton of material and total costs are more appropriate efficiency criteria. The latter include costs for ore supply, electricity, consumption of various materials, production and organizational costs, as well as other production and non-production costs. The percentage distribution of these costs is shown in Figure 1. Economic indicators are largely determined by the type and quantity of equipment used, as well as its operating modes.

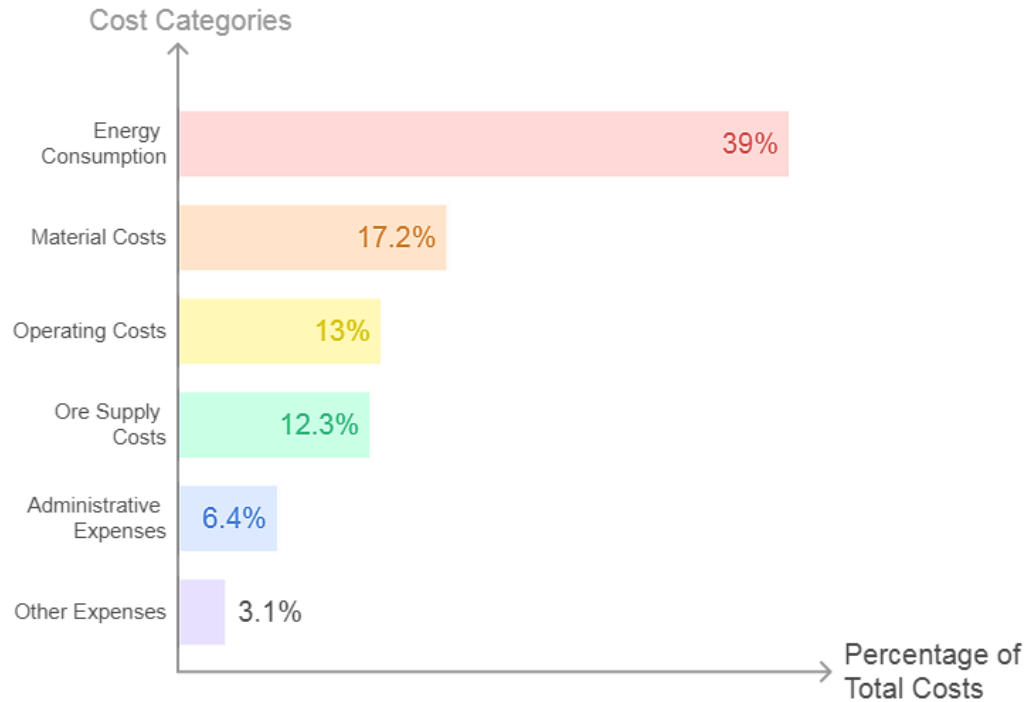


Figure 1.
Percentage distribution of total costs for the ore crushing process.

Figure 1 and analyses from numerous studies Jack and Alex [5] and Magdalena, et al. [6] show that electricity costs dominate other costs in the ore crushing process. Efficient use of electricity is particularly important in this energy-consuming process. Additionally, many factors impact the qualitative and quantitative indicators of the output material (crushed ore), resulting in changes to the electricity consumption of individual energy-consuming devices and, consequently, the entire process. These factors include technological features, equipment productivity, the degree of process automation, and production organization.

The final cost of the crushed material is most affected by electricity costs and the quality of the output material. The quality properties of the processed ore and the organization of the technological process determine the quality indicators of the output material. While the quality properties of the processed ore are practically uncontrollable, organizing the technological process is controllable and leads to changes in technological and energy characteristics.

As the above suggests, saving energy is key to improving the efficiency of the ore crushing process, which significantly impacts the cost, price, and quality of the final product. This paper examines various proposals for saving energy in ore crushing.

In Wu, et al. [7] a new grinding scheme is presented. An evaluation of the scheme's efficiency under production conditions revealed that the modified process fully satisfies production requirements, improves the granulometric composition of the finished product, and reduces overgrinding of the material. Consequently, energy savings of up to 21.44% can be achieved.

Preliminary processing of ore before crushing can reduce energy consumption. Several studies Shi, et al. [8]; Mishra, et al. [9] and Somani, et al. [10] have considered using various preliminary processing technologies in the crushing process to save energy. Article [11] addresses additional ore processing issues and the practical implementation of its results. An assessment of preliminary ore processing using electrical and ultrasonic methods shows a 24% and 66% reduction in energy consumption, respectively.

The influence of grinding time, mixer rotation speed, bulk density of the solid material, and feedstock size on grinding efficiency, as characterized by specific energy consumption during ore grinding, was studied [12]. The required particle size of 1 μm could be achieved with a maximum mixer rotation speed of 500 rpm and a solids concentration of 33.3%. Grinding took 17 hours. The specific energy consumption was approximately 1,225 kWh/t [12].

Kołodziej, et al. [13] presents methods for modeling and optimizing the grinding process in a real limestone plant. Regression models based on artificial neural networks were developed using process data obtained from the SCADA system and experimental results. The parameters of the controlled process were used as input data. Bayesian and genetic algorithms were then employed to determine the optimal operating parameters of the plant, thereby reducing energy consumption.

Paper Ogonowski [14] examines issues relating to the operational control of an electromagnetic mill. The paper proposes an optimization algorithm whose effectiveness is tested on a simulation model and a semi-industrial dry grinding and classification scheme.

Particular attention is paid to using reactive power compensation capabilities to reduce electricity consumption during ore crushing. During this process, synchronous and asynchronous motors operate simultaneously; the former generates reactive energy, and the latter consumes it. This setup enables significant electricity consumption savings at a low cost. The authors of Baghdasaryan and Davtyan [15] proposed a mathematical model for the optimal distribution of reactive power generated by synchronous motors. This model ensures a power factor close to unity on the common power bus and minimizes the total active power losses of motors that generate reactive power. Power factor values on the common power bus were estimated for technological schemes with different structures used in the ore crushing process. The optimal values for reactive power distribution of synchronous motors and the share of electricity savings were determined using a genetic decision-making algorithm.

An analysis of existing work shows valuable progress in reducing electricity consumption in ore crushing processes. These studies primarily consider saving electricity by accounting for technological factors. However, a comprehensive account of technological and energy factors is paramount for proposing new, effective solutions. This is because the ore crushing process is driven by powerful electric motors whose uneven operation affects the entire technological process. The objective of this study is to propose a method of minimizing the specific energy consumption of the ore crushing process while ensuring the stability of electric motor operation within the electric drive system.

2. Materials and Methods

Figure 2 shows a general view of the structural diagram of the mechanical components of the electric grinding system of the ore mill. As the mill drum rotates, the grinding bodies rise to a certain height due to friction forces. Then, they roll down or fall freely. Material to be ground is continuously fed into the mill through one of the rollers. As the material passes through the mill, it is exposed to the grinding bodies and ground by impact, friction, and crushing. The ground material is continuously unloaded from the other roller. The material is transported into the mill by the pressure of the continuous feed.

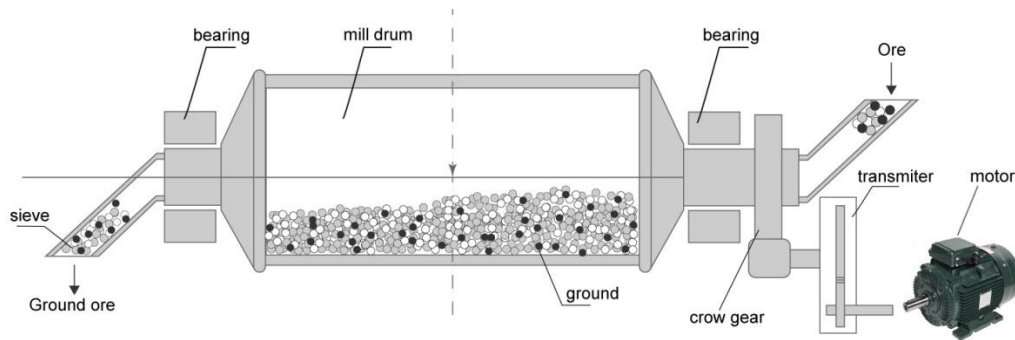


Figure 2.
Structural diagram of the mechanical components of the mill's electric drive system.

The main requirements for the ore crushing process are:

- High productivity while ensuring the technical parameters of the finished material are met;
- Minimum power consumption per ton of finished material;
- Maximum reliability of the control system;
- Minimum operating costs of the system.

This work implements the aforementioned requirements for the grinding process according to the following algorithm:

- Definition and evaluation of the main criteria affecting the specific energy consumption;
- Definition of the variables, target and limit functions of the problem of optimizing the specific energy consumption.
- Development of a mathematical model for optimizing specific energy consumption;
- Evaluation of the obtained results.

Figure 3 illustrates the structure of the algorithm used to optimize electricity consumption.

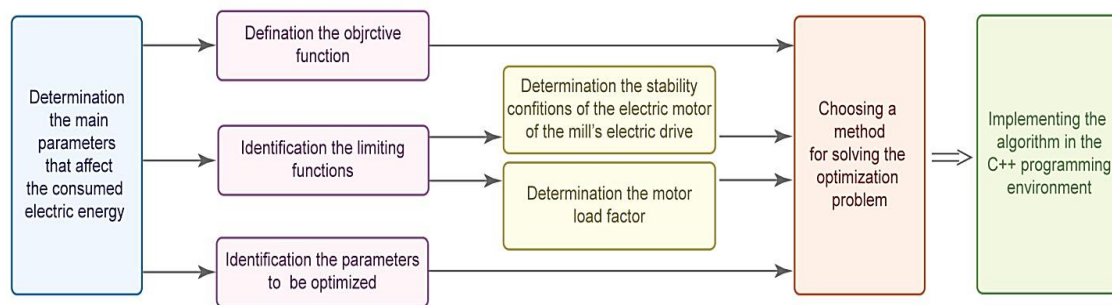


Figure 3.
Structural diagram of the algorithm for minimizing electricity consumption.

The quality criteria required to solve the problem of minimizing specific energy consumption are selected in accordance with the main requirements of the grinding process. Observations confirm that the relationship between the technological and energy quality criteria does not permit minimizing the specific energy consumption while also ensuring stable motor operation and the required load factor value. To address this issue, a target function is formed using a function that characterizes the specific energy consumption. Restrictions are formed using functions that characterize the permissible limits of the load factor and the stability conditions of the synchronous motor.

The efficiency of the grinding process is determined by a large number of factors, and it is important to assess their impact on the consumed electric power. Figure 4 illustrates the influence of various factors, such as the size of the ore being ground, productivity, bulk density, the degree of filling of the grinding drum, the relative rotation speed, and the wear of the protective layer, on the electric power consumption. These data were obtained by averaging the results of surveys and tests conducted among specialists.

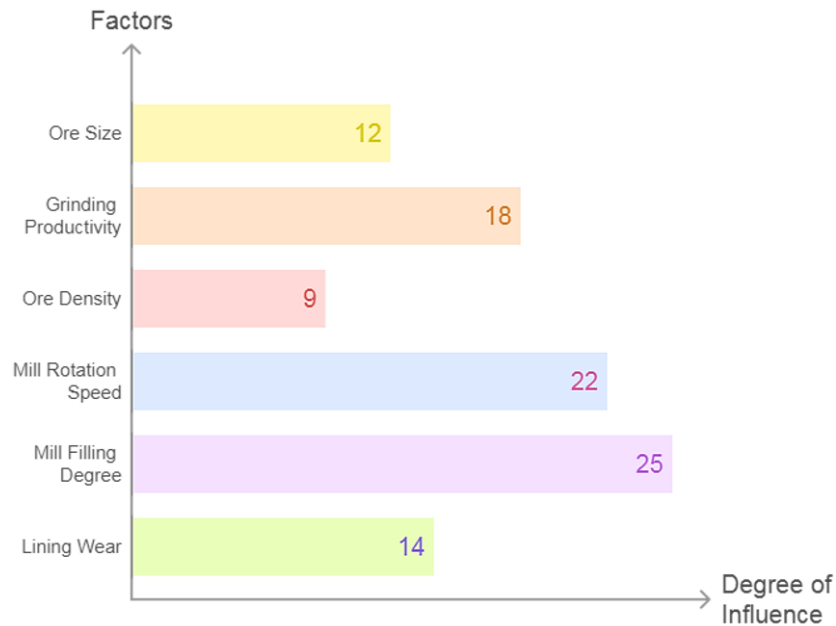


Figure 4. Evaluation of the influence of various factors on electric power consumption in the ore grinding process.

Observations show that the greatest influence on the amount of electricity consumed is exerted by the degree to which the grinding drum is filled with ore (K), the relative rotation speed (ψ), and the productivity of the ore feed (Q). We adopted these as variables.

2.1. Determining the specific energy consumption

The formula for determining the specific energy consumption of the electromechanical system that ensures the machine's operation is as follows:

$$w = \frac{P_o + P_l + P_2}{Q\eta_m\eta_d} \quad (1)$$

Where η_m is the efficiency of the mechanical transmission, η_d is the efficiency of the motor, Q is the productivity of feeding ore into the mill, P_o is the useful power spent on grinding, P_l is the power of no-load losses, P_2 is the additional power losses arising in the bearings.

The powers P_o, P_l, P_2 are determined respectively as follows:

$$P_o = 3.15KR^{3/2}L\gamma g^{3/2}\psi\left(f\sqrt{\ell_x^2 + \ell_y^2 + \ell_z^2}\right), P_l = 6L\psi\sqrt{R}, P_2 = CP_l.$$

Where γ is the specific density of the material being crushed, R is the radius of the crushing drum, L is the length of the crushing drum, g is the acceleration of gravity, ψ is the relative speed of rotation of the crushing drum, K is the degree of filling of the drum with ore, C is a coefficient determined by the dimensions of the crushing drum and the degree of its filling, ℓ_x, ℓ_y, ℓ_z are the distances of the center of gravity from the axes x, y and z , respectively (Figure 5).

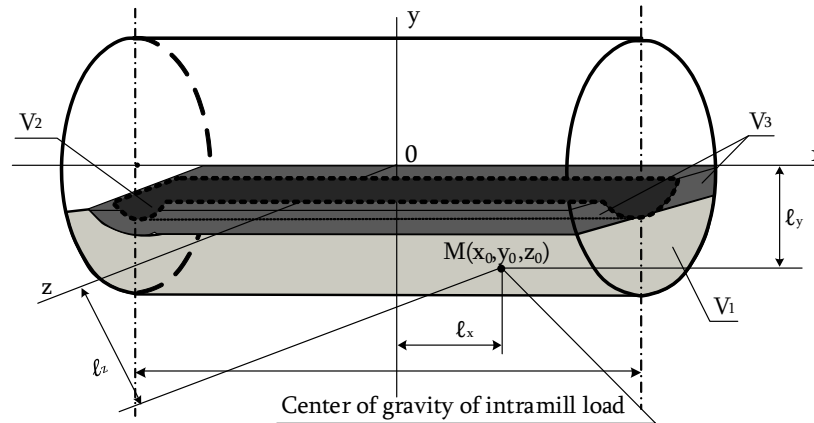


Figure 5.
Calculation model.

2.2. Algorithm for obtaining the stability condition of an electric drive motor

A study of the operating modes of the ore-grinding complex reveals that the synchronous motor of the electric drive operates not only in normal synchronous mode but also in asynchronous mode during operation. The asynchronous mode differs from the normal mode in that the motor operates with non-zero slip for a certain period of time, which leads to additional energy losses [16, 17].

Therefore, ensuring stable motor operation is important for saving electricity. A differential equation is used to determine the stability condition of the electric drive motor for the mill and express the dependence of the mill's mechanical characteristics [17].

$$T_m \frac{ds}{dt} + M_s \sin \theta + M_{as} = M, \quad (2)$$

where T_m is the moment of inertia, s is the slip, θ is the phase angle between the vectors of the main EMF and the network voltage, M is the resistance torque of the mill, M_s is the synchronous torque of the motor, M_{as} is the asynchronous component of the torque of the synchronous motor, which is determined by the formula:

$$M_{as} = \frac{2M_k(\omega_s - \omega)}{s_k \omega_s}, \quad (3)$$

where ω_s is the synchronous angular velocity of the motor, ω is the angular velocity of rotation of the motor shaft, M_k is the critical torque, s_k is the critical slip.

The given value of the mill resistance torque is determined as follows:

$$M = \frac{1000K\pi^2 R^{3/2} \psi L \gamma g \left(f \sqrt{\ell_x^2 + \ell_z^2} + \ell_y \right)}{0,105 \omega \eta_m \eta_d}, \quad (4)$$

where γ is the specific density of the material being crushed, g is the acceleration of gravity, K is the degree of filling of the crushing drum, f is the coefficient of friction.

Substituting (3) and (4) into (2), and also taking into account that at $s=0$ the angle θ of the motor is constant, and the slip is proportional to its change, we obtain the following differential equation:

$$\frac{d^2\theta}{dt^2} = \frac{1}{T_m} \left[(K_k - M) \frac{d\theta}{dt} + M_s \sin \theta - M \right] \quad (5)$$

where $K_k = \frac{2M_k}{s_k}$.

Applying the Maclaurin series to the third term on the right-hand side of equation (5) yields a differential equation that enables us to determine the condition for the stable operation of a synchronous motor.

$$\frac{d^2\theta}{dt^2} + a_1 \frac{d\theta}{dt} + a_2 \theta = 0, \quad (6)$$

where $a_1 = \frac{1}{T_m} (K_k - M)$, $a_2 = \frac{M_s}{T_m}$.

Replacing $\theta = e^{kx}$, we obtain the characteristic equation

$$k^2 + a_1 k + a_2 = 0 \quad (7)$$

Let us denote the roots of the characteristic equation (7) by k_1 and k_2 . The stability or instability of the equation (6) solution is determined by the nature of the k_1 and k_2 roots:

$$k_{1,2} = -\frac{a_1}{2} \pm \sqrt{\frac{a_1^2}{4} - a_2}$$

The stability of the system will be ensured if all the coefficients are positive, and the roots are either real, or negative, or complex with a negative real part [18, 19].

Let us consider all possible cases:

The first case: the roots of the characteristic equation are real and negative. This occurs when the equation's parameters satisfy the following conditions:

$$\begin{cases} a_1^2 - 4a_2 > 0 \\ a_1 > 0 \\ a_2 > 0 \end{cases} \quad (8)$$

System (8) is applicable when the product of the roots of the characteristic equation is positive and the sum is negative.

$$\begin{cases} k_1 k_2 > 0 \\ k_1 + k_2 < 0 \end{cases} \quad (9)$$

By substituting the values of the roots of the characteristic equation into expression (9) and making some adjustments, we can derive the following stability condition:

$$\begin{cases} \left[\frac{1}{T_m}(K_k - M) - 2\sqrt{\frac{M_s}{T_m}} \right] \left[\frac{1}{T_m}(K_k - M) + 2\sqrt{\frac{M_s}{T_m}} \right] \geq 0 \\ \frac{M_s}{T_m} < 0 \\ \frac{1}{T_m}(K_k - M) > 0 \end{cases} \quad (10)$$

The second case: the roots of the characteristic equation (6) are complex with a negative real part. This is possible if the parameters of the characteristic equation satisfy the following conditions:

$$\begin{cases} \left[\frac{1}{T_m}(K_k - M) \right]^2 - \frac{4M_s}{T_m} > 0 \\ \operatorname{Re} \left[-\frac{1}{2T_m}(K_k - M) \right] \pm \sqrt{\frac{1}{4T_m^2}(K_k - M)^2 - \frac{M_s}{T_m}} < 0 \end{cases} \quad (11)$$

Grouping possible cases (10) and (11) results in the following condition, which ensures the motor operates stably:

$$\frac{1}{T_m}(K_k - M) > 0 \quad (12)$$

3. Results

The following mathematical formula is given to optimise the specific energy consumption of an electromechanical system:

$$\begin{aligned} w(Q, K, \psi) &\rightarrow \min \\ \left\{ \begin{aligned} k(Q, K, \psi) &\geq k_p, \quad \frac{1}{T_m}(K_k - M) > 0, \\ Q^{\min} \leq Q \leq Q^{\max}, \quad K^{\min} \leq K \leq K^{\max}, \quad \psi^{\min} \leq \psi \leq \psi^{\max} \end{aligned} \right\}, \end{aligned} \quad (13)$$

where Q^{\max} , K^{\max} , ψ^{\max} are the maximum limit values of the controlled parameters, respectively, Q , K , ψ ; Q^{\min} , K^{\min} , ψ^{\min} are the minimum limit values of the controlled parameters, respectively, k_p are the permissible values of the motor load factor.

The solution to the presented optimization problem can be implemented using known numerical methods (see references [20, 21]). Considering the features of these numerical methods and the mathematical description of the problem, a random search method was employed [22]. Random search methods differ from regular optimization methods in that they introduce a random element, which provides opportunities for organizing target control. These methods are general, free from conditions imposed on the target function (such as differentiability and convexity), simple to implement, and easily allow for restrictions in the search process. Various strategies exist for introducing randomness into the search process. The proposed problem solution is based on multiple random sampling to introduce randomness. The algorithm is as follows: first, a random step is performed from the starting point. Then, the value of the function at this point is calculated and compared with the initial value [23]. If the trial step is unsuccessful, random samples are taken from the starting point until a successful sample is found, after which a working step is performed in this direction. In the best-sample algorithm, m random samples are taken and then a working step is performed with the best sample. These samples are taken within a hypersphere, where the radius determines the size of the trial steps and the density of

the random point distribution on the hypersphere remains constant. This algorithm does not accumulate the results of previous attempts and therefore does not learn during the search.

The initial values of the variables (i.e. the controlled parameters), the permissible values of the constraints, the maximum number of experiments and the minimum and maximum values of the working step are provided as input data. The described algorithm is implemented in the C++ programming environment. The optimization calculation data are given in Table 1. Observations were carried out on a drum mill with a radius of 1.8 m and a length of 2.7 m, with crushed ore densities of 2.8 and 3.2 t/m³. The results show that the specific energy consumption can be reduced by 34.4% at $\gamma = 2.8$ t/m³ and by 31.7% at $\gamma = 3.2$ t/m³.

Table 1.
Dynamics of changes in specific electricity costs before and after optimization.

Specific density of crushed ore, γ (t/m ³)	Specific power consumption, W, (kW/t)	Minimum specific energy consumption, not taking the stability condition into account (kW/t)	In the event that stability conditions are taken into account	
			Minimum value of specific energy consumption, (kW/t)	Optimized variable values
2.8	9.63	7.14	6.32	K = 0.38
				$\Psi = 0.87$
				Q = 42.65 t/h
3.2	9.71	7.31	6.63	K = 0.35
				$\Psi = 0.81$
				Q = 41.12 t/h

4. Conclusion

We have proposed a new hypothesis to reduce the electric power consumed by an ore-grinding mill. The key feature of this hypothesis is that it takes into account the steady state of the electric drive system, which ensures the operation of the drum mill. This is achieved by adapting the stability condition in the mathematical formulation of the optimal solution problem. Using the stability condition as a criterion for optimizing electric power consumption enables the mill's electric drive motor to operate stably under dynamic loads, preventing undesirable phenomena and the associated unnecessary energy costs. The advantage of this approach is that it considers the possibility of operating the synchronous electric drive motor in abnormal modes. Other studies assume that the motor remains in synchronous mode throughout the entire operating period. Data obtained from optimization shows that depending on the density of the material being crushed, energy savings range from 34.4 to 31.7 percent, which is a good indicator. Using the stabilization condition to optimize specific energy consumption can be successfully applied to the optimal control system of the electromechanical mill system, thereby improving system operational efficiency.

Transparency:

The authors confirm that the manuscript is an honest, accurate, and transparent account of the study; that no vital features of the study have been omitted; and that any discrepancies from the study as planned have been explained. This study followed all ethical practices during writing.

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