

Exploring aspects of algorithmic thinking of informatics education students in solving linear equation systems

 Ariesta Kartika Sari^{1,4*},  Tatag Yuli Eko Siswono²,  Agung Lukito³

^{1,2,3}Doctoral Program in Mathematics Education, Surabaya State University (UNESA), East Java, Indonesia; ariesta.20002@mhs.unesa.ac.id (A.K.S.) tatagsiswono@unesa.ac.id (T.Y.E.S.) agunglukito@unesa.ac.id (A.L.).

⁴Department of Informatics Education, Trunojoyo University, Madura, Indonesia.

Abstract: Algorithmic thinking contributes significantly to education, particularly in problem-solving within mathematics and information technology. This study aims to explore the characteristics of algorithmic thinking among informatics education students when solving mathematical tasks. The focus is on three main components of algorithmic thinking: decomposition, abstraction, and algorithmization. Employing a qualitative case study approach, data were collected through tests and semi-structured interviews. The findings indicate that students engage in algorithmic thinking activities across these three aspects during mathematical problem-solving. Decomposition activities involve identifying relevant information and dividing tasks into sub-tasks. Abstraction activities include recognition, building with, and integrating prior knowledge. Algorithmization activities encompass planning, composing, and applying systematic steps. These aspects are interconnected, forming a cognitive process rather than functioning independently. The study's novelty lies in categorizing activities associated with each aspect of algorithmic thinking. It contributes to the theoretical understanding that algorithmic thinking is a relevant cognitive activity that facilitates mathematical problem-solving.

Keywords: *Abstraction, Algorithmic thinking, Algorithmization, Decomposition, Linear algebra, Mathematical problem solving.*

1. Introduction

Algorithmic thinking is a systematic and logical way of thinking to solve problems in a gradual and structured manner [1]. This ability is one of the key elements in developing digital literacy and information and communication technology literacy [2, 3]. In the context of informatics education, algorithmic thinking plays an important role in developing programming skills, data analysis, and the design of efficient and scalable technology-based solutions. Many studies have emphasized the importance of algorithmic thinking in supporting computational thinking skills, which are prerequisites for learning computer science Kallia et al. [4], Selby and Woollard [5], and Smith and Angeli [6]. Futschek [7] emphasized that algorithmic thinking is a basic skill that students must have in learning computer programming. In the context of ICT teacher candidate education, this ability is very relevant because it underlies the pedagogical and technological competencies that they will apply. Dogan [8] said that individuals with algorithmic thinking ability are able to model data, think systematically, and produce efficient solutions to various problems.

Although the significance of algorithmic thinking has been widely recognized in various literatures, its implementation in the context of higher education, especially in informatics education study programs, still faces various challenges. Previous studies have mostly focused on elementary school students [9, 10] and secondary level Stephens [11]. Johanning and Shockey [12] described teachers' teaching practices related to engaging students in algorithmic thinking related to fraction operations. While in-depth studies on prospective ICT teacher students are still limited. In contrast, students at this

level are expected not only to understand algorithmic concepts but also to be able to apply them in the context of learning and problem solving, especially in the field of mathematics, which is closely related to logic and programming.

In the domain of mathematics education, preliminary research findings concluded that out of 85 ICT teacher candidates, 67% of participants were inaccurate and unsystematic in solving algebraic math tasks (systems of linear equations). This study analyzed the participants' work in solving mathematical tasks. They presented the results with the categories of Model A (appropriate and systematic), Model B (appropriate and unsystematic), Model C (inappropriate and systematic), and Model D (inappropriate and unsystematic). Based on the analysis, this study found that 18% of the participants were categorized into Model A, 3% into Model B, 12% into Model C, and 67% into Model D. This indicates that the participants' algorithmic thinking skills are not optimal. Therefore, this finding recommends that further research is needed to explore the aspects and components of algorithmic thinking and how efforts can be made to improve these skills to enhance problem-solving and task completion.

Given the importance of algorithmic thinking skills and the limited studies at the university level, an approach is needed that can reveal in depth how informatics education students practice algorithmic thinking in real activities, such as solving mathematical problems. The context of mathematics is not without reason, considering that mathematics has structures and characteristics that are parallel to the way algorithms work: systematic, logical, and solution-oriented. This research aims to fill the gap by exploring how students' algorithmic thinking manifests when dealing with mathematical problems. Thus, this study seeks to explore in depth the characteristics of algorithmic thinking among informatics education students in solving math tasks.

2. Theoretical Framework

Algorithmic thinking does not only focus on the design stage of problem solving, as presented by Blannin and Symons [10], but also involves the process of analyzing the problem and applying solutions. It is revealed by Dogan [8] that algorithmic thinking is a way of thinking logically and involves a precise sequence of actions, which can be applied systematically and leads to the achievement of goals. Lockwood et al. [1] define algorithmic thinking as a logical and organized way of thinking used to break down complex goals into a series of steps (sequenced). Algorithmic thinking is more than just the implementation of a procedure or even an explanation of why a procedure works the way it does, but also involves: (a) planning and designing the steps; (b) having an overall understanding of what the algorithm can do; and (c) having the details to implement the algorithm successfully. Sadykova and Il'bahtin [13] defined "Algorithmic thinking" as a system of thinking methods that: (a) has a sequence of constructing the obtained results, (b) builds a sequence of actions, and (c) leads to the achievement of the goal.

Algorithms, as solutions to mathematical problems, are expressed in a clearly defined sequence of instructions. To handle algorithms successfully, algorithmic thinking requires other cognitive abilities, including decomposition and abstraction. As Stephens and Kadjevich [2] stated that to be successful in dealing with algorithms, algorithmic thinking requires different cognitive abilities, including decomposition and abstraction. The aspects of thinking in this study refer to Stephens and Kadjevich [2], who state that there are three foundations of algorithmic thinking, namely: (a) decomposition; (b) abstraction; and (c) algorithmization. Decomposition in this case is interpreted as breaking down the problem into sub-problems. Abstraction is used to make general statements that summarize specific examples of underlying concepts, procedures, relationships, and models. Algorithmization in this case is related to compiling algorithms [14].

Abstraction, being a mental activity, would be difficult to observe. Researchers found a way to make these mental activities observable, namely, observing through epistemic actions. Epistemic actions are mental actions by which knowledge is used or constructed [15]. The process of knowledge construction is expressed in the model through three observable and identifiable epistemic actions, namely recognition, building-with, and constructing. The three epistemic actions are hereafter abbreviated as

RBC (Recognition, Building-with, Constructing). This research will use the three epistemic actions approach in exploring the abstraction aspect of algorithmic thinking. Thus, in this study, algorithmic thinking is thinking that involves a precise sequence of actions that can be applied systematically to achieve a goal, involving the aspects of decomposition, abstraction, and algorithmization.

Table 1.
Theoretical Framework of Algorithmic Thinking Aspects in This Study.

No.	Aspect	Sub-aspect	Indicator: Each Aspect of Algorithmic Thinking
1	Decomposition	-	1. Identify the information contained in the task or the known and asked information (labelled: D1). 2. Divide the task/problem into sub-task/sub-problem components. (labelled: D2)
2	Abstraction	Recognition	1. Recognizing/recalling specific previous activities or knowledge constructions that are relevant or related to the mathematical task at hand. (labelled: Ar1) 2. Recognizing patterns in the sequence of steps in solving the mathematical task at hand. (labelled: Ar2)
		Building-With	Combining previously identified knowledge to produce solutions to sub-tasks/sub-problems. (labelled: Ab)
		Constructing	Assemble and integrate previously constructed knowledge to produce solutions to mathematical tasks and problems encountered. (labelled: Ac)
3	Algorithmization	-	1. Planning the steps to complete a mathematical task. (labelled: G1) 2. Compiling instructions consisting of clear and systematic steps to complete the task. (labelled: G2)

3. Research Method

3.1. Research Design

A qualitative approach was used in this study because it was naturalistic, with no treatment given to the subjects. As stated by Fraenkel and Wallen [16] ‘the natural setting is a direct source of data, and the researcher is the main instrument in qualitative research. Qualitative research was chosen by the researcher as the research approach because this study will explore the algorithmic thinking of computer science students in solving linear equation systems. This study is classified as a case study. It is also classified as exploratory descriptive research because the data collected is descriptive, including interview results, participant assignment results, and field notes. As stated by Creswell [17] in qualitative research involves creating a database of sentences or groups of sentences, where the data to be analyzed consists of segments of sentences (referred to as text segments). Subsequently, the meaning or interpretation of each group of sentences is provided.

3.2. Research Subject

The subjects involved were students registered in the Informatics Education study program. Purposive sampling was used to select subjects relevant to the research purposes. As stated by Merriam [18] purposive sampling is based on the researcher's desire to discover, understand, and gain insight. The subjects of this study were: (a) active students still enrolled in the computer science education study program; (b) students who had studied linear equation systems and had equivalent mathematical abilities. Mathematical ability was assessed using the Mathematics Ability Test (MAT), with categories of high (MAT score ≥ 75), medium ($60 \leq \text{MAT score} < 75$), and low (MAT score < 60). In this study, after conducting a mathematical ability test on 61 students, two male students with equivalent abilities (from the high MAT category) who were willing to participate as research subjects/participants were selected.

3.3. Instruments and Data Collection Techniques

This study utilized instruments such as the Mathematics Ability Test (MAT), a four-variable linear equation system (SLE4V) task, and interview guidelines. The SLE4V questions were used to explore

participants' algorithmic thinking activities based on three aspects/components (decomposition, abstraction, and algorithmization). Before use, the MAT test, SLE4V task, and interview guidelines were first validated by three lecturers who are experts in mathematics education and one lecturer who is an expert in language education. Using the Aiken V agreement index, it was concluded that the four-variable linear equation system (SLE4V) task had a high level of validity. From a linguistic perspective, the MAT and SLE4V instruments also have a high level of validity. For the interview guide instrument, the experts concluded that the guide could be used with revisions.

In addition to content validity checks by experts, the MAT questions were tested on 33 information technology students. Based on the validity test results (t-test), it was concluded that the MAT test questions were valid and could be used to assess students' mathematical abilities. The reliability test results (Cronbach's alpha) for the MAT questions indicated that the MAT test had a high level of reliability. After the validity test, a readability test was conducted on the SLE4V questions by students who were not subjects or participants in the study. Using the Aiken V index, it was concluded that the interpretation of the V agreement index was relatively high. Thus, the SLE4V questions instrument is suitable for use.

Question in Indonesia	Translate in English
<p>Berikut ini sistem persamaan linier (SPL) dengan empat variabel</p> $x_1 - 2x_2 + x_3 + x_4 = 3$ $x_1 + x_3 - x_4 = 0$ $2x_2 - x_3 - x_4 = 0$ $x_1 + 4x_2 + 2x_3 - x_4 = 3$ <p>Tentukan solusi untuk SPL tersebut!</p>	<p>The following is a system of linear equations (SPL) with four variables</p> $x_1 - 2x_2 + x_3 + x_4 = 3$ $x_1 + x_3 - x_4 = 0$ $2x_2 - x_3 - x_4 = 0$ $x_1 + 4x_2 + 2x_3 - x_4 = 3$ <p>Determine the solution for the SPL.</p>

Figure 1.
Questions in the System of Linear Equations with Four Variables (SLE4V).

Testing techniques were used to collect data. Interview techniques were employed to verify test results and to explore information that was not obtained through the SLE4V questions. SLE4V questions are linear equation problems with four variables that have a single solution and are non-contextual.

3.4. Data Analysis Techniques

The data obtained in this study consisted of interview data, SLE4V test results, and field notes. There were three main activities in the analysis of this research data, namely: (1) data condensation, (2) data display, and (3) conclusion drawing/verification. As Miles et al. [19] suggest, there are three main activities in data analysis, namely: (1) data condensation, (2) data display, and (3) conclusion drawing/verification. The activities involved in condensation include: data selection, coding, theme development, category creation, and analytical note/memo creation.

The coding techniques used are first-cycle coding and second-cycle coding [20]. In this study, the data coding process was carried out using NVivo software. The research data were then categorized based on three aspects/components of algorithmic thinking: decomposition, abstraction, and algorithmization. Due to the difficulty of observing thinking activities, this study used three RBC epistemic actions (recognition, building-with, constructing) to explore algorithmic thinking activities in the aspect of abstraction.

Research Credibility. In order to build researchers' confidence in the credibility of research findings, researchers use the data source triangulation method, which involves examining the consistency of activities that are preliminary findings from the first and second data sources. In this study, the type of triangulation used is the 'person' data source. As Denzin and Lincoln [21] said that "Triangulation

means that researchers take different perspectives on an issue under study or, more generally speaking, in answering research questions. The concept of triangulation means that a research issue is considered or, in a constructivist formulation, is constituted from (at least) two points or perspectives [22].

4. Research Result

After conducting a mathematics ability test on 61 students from the computer science education program, two participants were selected as research subjects (referred to as Subject SL1 and Subject SL2) who had equivalent mathematical abilities (classified as high group), had programmed linear algebra courses, and were willing to participate in the research series. Subsequently, the subjects worked on the SLE4V questions and proceeded to the interview stage based on the results of the SLE4V completion and interview guidelines that referred to aspects of algorithmic thinking.

4.1. Results of Decomposition Aspects

The following is an excerpt from an interview with SL1 participants relevant to the aspect of decomposition.

Excerpt 01. Interview with SL1 Subjects Based on SLE4V Task (Decomposition Aspect)

- P : *What information did you get when reading the SLE4V task illustration? Can you explain it?*
 SL1 : *There are four equations. (D1.1)*
 P : *Yes, there are four equations. What else?*
 SL1 : *There are four different variables, namely x_1 , x_2 , x_3 and x_4 . (D1.2)*
 P : *What steps come to mind initially when encountering a problem like SLE4V? Could you list them?*
 SL1 : *I will solve the SLE4V problem using the OBE system. (D2.2)*
 P : *Okay, so in the OBE process, you are thinking about... forming a matrix?*
 SL1 : *Yes, creating a matrix. (D2.1)*
 P : *Are there any other thoughts or actions you plan to take?*
 SL1 : *Perhaps I will do this, ma'am. Typically, the pattern for a matrix involves a zero triangle, correct. (D2.3)*

The following is an excerpt from an interview with Subject SL2 relevant to the aspect of decomposition.

Excerpt 02. Interview with Subject SL2 Based on SLE4V Task (Decomposition Aspect)

- P : *First, for SLE4V, if you read the question, what information can you obtain from the SLE4V question?*
 SL2 : *For the SLE4V question, there are four variables. (D1.2)*
 P : *Four what? Four equations?*
 SL2 : *Yes, four equations. (D1.1)*
 P : *Before you start working on it, what comes to mind?*
 SL2 : *To solve it, we need to determine the values of x_1 , x_2 , x_3 , and x_4 , so the goal is to determine the value of each variable. (D2.1, D2.2)*
 P : *Then, what else needs to be done?*
 SL2 : *For the steps to solve it, we use OBE. (D2.2)*
 P : *What is OBE used to determine?*
 SL2 : *To determine the value of each variable. (D2.2)*
 P : *Here, you form a matrix, right?*
 SL2 : *Yes. (D2.1)*

The results of interviews with SL1 and SL2 participants demonstrated that they engaged in activities pertinent to the decomposition aspect, specifically identifying information (D1) and dividing the task into sub-tasks (D2). Analysis and coding using NVivo software yielded a coding map, as shown in Figure 2, for both participants. Figure 2 illustrates that data from subjects SL1 and SL2 can be coded and categorized under decomposition aspects. The arrow labeled 'child' indicates the hierarchical relationship between codes. The arrow labeled 'codes' signifies that the code originates from or is derived from source SL1 or SL2. The data analysis results suggest that there are similarities between the

activities of subjects SL1 and SL2 in identifying information, as both identified similarities and variables within the task. Additionally, both subjects divided the task into two subtasks.

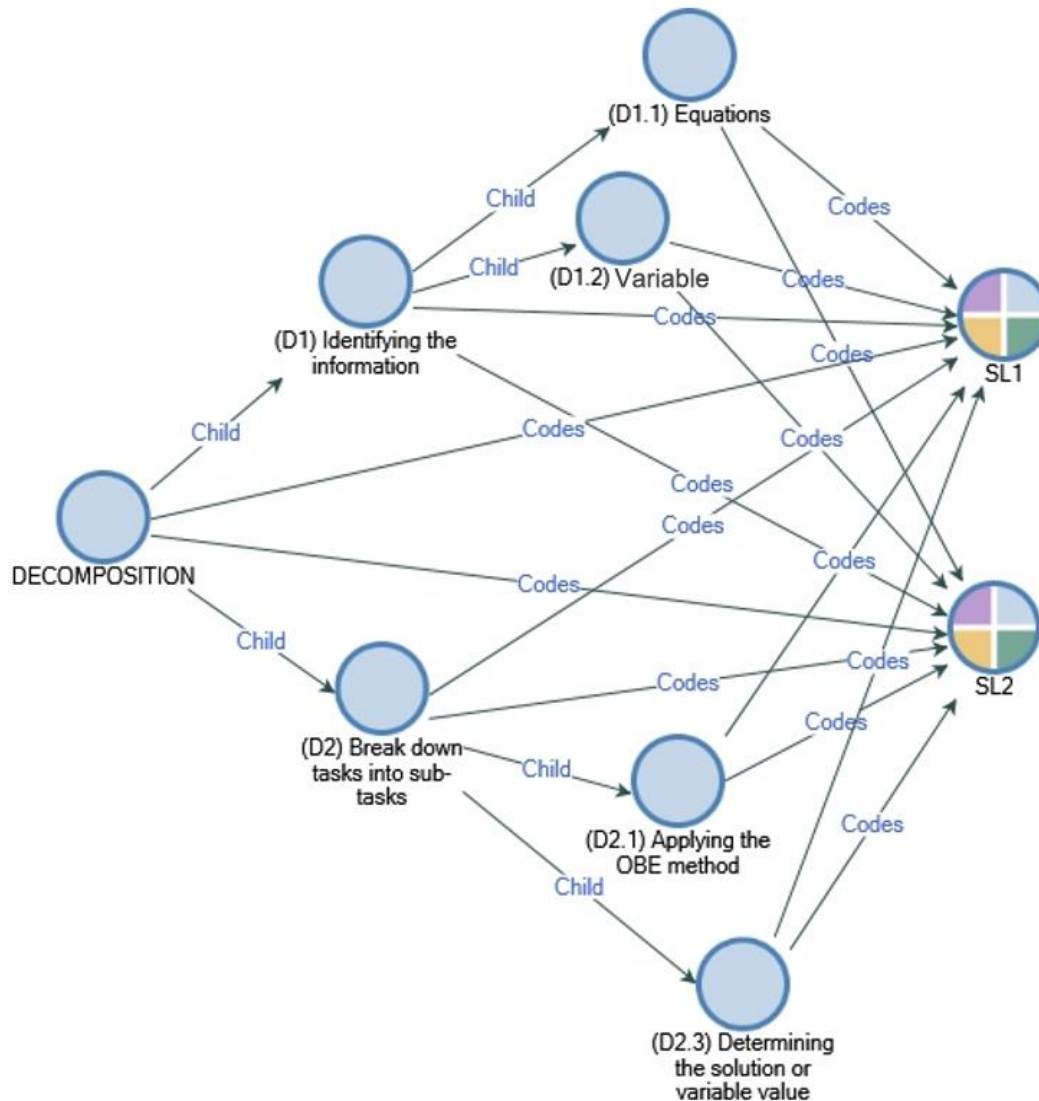


Figure 2.
Coding Project Map of Research Data from SL1 and SL2 for Decomposition Aspects (Sumber: luaran project map dalam NVivo).

4.2. Results of Abstraction Aspects

The aspect of abstraction in algorithmic thinking activities in this study was examined through three sub-aspects, namely recognition (Ar), building with (Ab), and constructing (Ac). Based on the interview results, subjects SL1 and SL2 exhibited activities relevant to the sub-aspect of recognition, namely: recognition of relevant knowledge and recognition of solution patterns. Some examples of recognized knowledge include: the elimination method (Ar1.1), the substitution method (Ar1.2), elementary row operations (Ar1.3), lower triangular matrices (Ar1.4), variables and variable coefficients (Ar1.5), arithmetic (Ar1.6), main elements (Ar1.7), and matrices for OBE (Ar1.8). Subjects SL1 and SL2 recalled a pattern in solving Task SLE4V, namely, using the OBE method (Ar2.1). In using the OBE

method, the subjects recall a pattern of creating lower triangular matrices and making the lower triangular elements zero. The following excerpt 03 is an example of a quote from an interview with subject SL1 that is relevant to the sub-aspect of recognition in completing Task SLE4V.

Excerpt 03. Interview with Subject SL1 Based on SLE4V Task (sub-aspect Recognition)

- SL1 : *I will solve the problems from SLE4V using the OBE system (Ar2.1)*
P : *Okay, so during the OBE process, what did you think about?*
SL1 : *Yes, creating a matrix (Ar1.8)*
P : *Is there anything else you have in mind, or anything else you will do?*
SL1 : *Maybe I will do this, because usually the pattern of the matrix is a zero triangle. (Ar1.4, Ar2.1, Ar2.2)*
P : *Perhaps there is experience, for example, SPL with another method like that.*
SL1 : *We have learned about elimination substitution. (Ar1.1, Ar1.2)*
P : *So, that means you also combine knowledge from before. Earlier, there was OBE, then there was knowledge of elimination, substitution, and the OBE procedure, right?*
SL1 : *Yes. (Ar1.3)*
P : *What is the purpose of subtracting row b2 from row b1?*
SL1 : *To make 'leading 1' here (Ar1.7)*
P : *How can you arrange this augmented matrix?*
SL1 : *The elements in the first column are the coefficients of the variable x_1 . (Ar1.5)*

Figure 3 presents a scheme of codes in the first cycle coding stage obtained from the data of subjects SL1 and SL2. The scheme shows that there is data coded from subjects SL1 and SL2 that is relevant to recognition activities, so that these codes will then be categorized into sub-aspects of recognition.

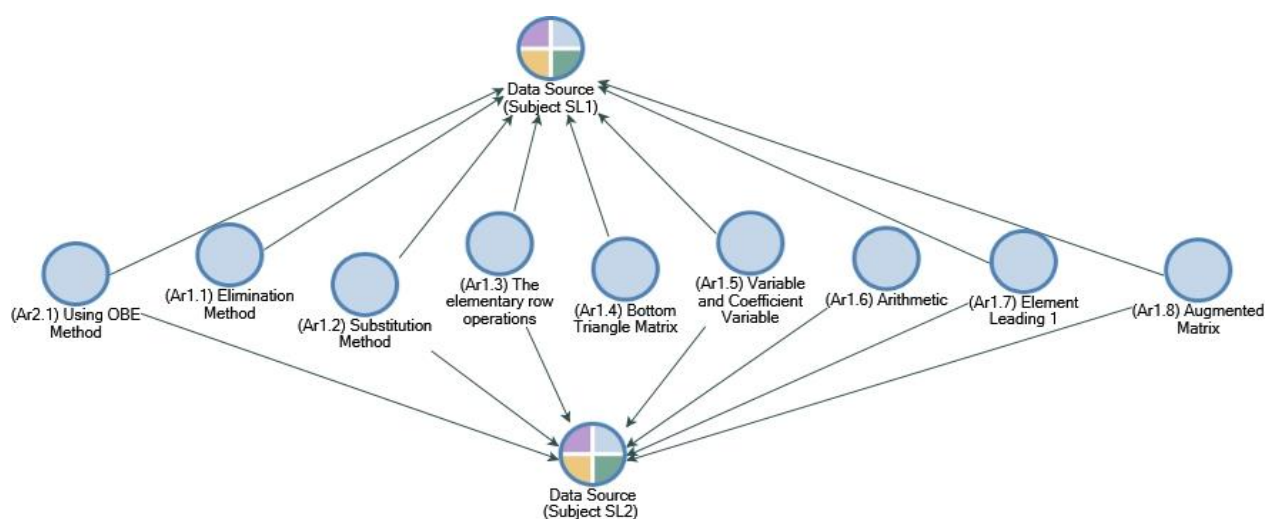


Figure 3.

Schematic diagram of the relationship between codes and data sources in the first cycle code stage for the sub-aspect of recognition (source: created by researchers using the NVivo application).

Regarding the sub-aspect activity of 'building with,' subjects SL1 and SL2 performed activities that combined prior knowledge to construct new knowledge, including: making the lower triangle elements zero (Ab.1), creating elements called 'main ones' (Ab.2), creating zero elements other than the 'leading 1' (Ab.3), creating a matrix containing variable coefficients (Ab.4), and creating upper triangular elements equal to 0 (Ab.5).

From the results of interviews with subjects SL1 and SL2, related to the sub-aspect activity of "constructing", the subjects carried out activities, namely integrating previous knowledge constructions to obtain problem solving, including: (a) Solutions for solving SPL or values of variables x_1 , x_2 , x_3 and

x_4 (Ac.1). Subjects SL1 and SL2 gained knowledge that the SLE4V task had a single solution; (b) strategies for obtaining the SLE4V solution. Subjects SL1 and SL2 used the OBE method in their strategy to obtain the solution (Ac3). Some of the knowledge integrated was related to knowledge in the building aspect, including: compiling a matrix containing the coefficients and constants of SLE4V, making the main diagonal elements into 'leading 1' elements, using elementary row operations, and making elements other than 'leading 1' into 0 elements.

Based on interview excerpts and categorization results, subjects SL1 and SL2 demonstrated activities relevant to recognition, building-with, and constructing. Thus, subjects SL1 and SL2 performed activities relevant to the abstraction aspect of algorithmic thinking. Referring to Saldana [20], the following is a diagram of the relationship between data sources, categories, and themes in the abstraction aspect presented in the following subjects. The diagram pertains to the second cycle coding stage.

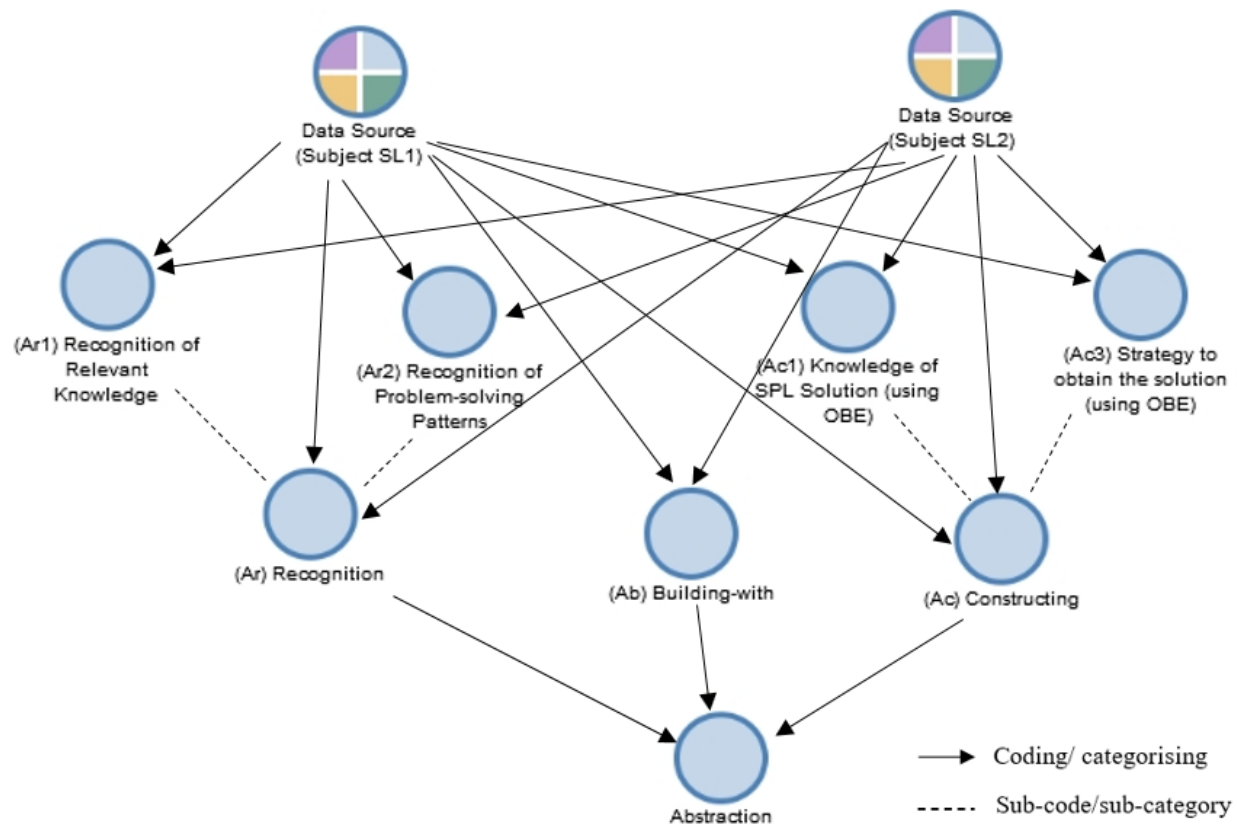


Figure 4. Schematic diagram of the relationship between data sources and categories/sub-categories in the second cycle stage for the aspect of abstraction in male subjects (source: created by researchers using NVivo12)

4.3. Results of Algorithmization Aspects

The algorithmization aspect relates to the activity of planning steps to complete a math assignment and arranging the steps to complete the assignment clearly and systematically. The following is an excerpt from an interview with an SL1 subject based on the results of solving SLE4V problems relevant to the algorithmization aspect.

Excerpt 06. Interview with SLE4V-Based SL1 Subjects (for Algorithmization Aspect)

P : Can you describe the sequence of steps you have decided on? What is your first step?
 SL1 : Analyze the problem. (G2.1)

- P : *You analyze it first, how do you analyze it?*
 SL1 : *How many equations are there, and how many variables are in the problem (G2.1)*
 P : *What then?*
 SL1 : *We try to solve the problem in SLE4V using elimination or OBE methods. (G1.1, G1.2)*
 P : *You try again?*
 SL1 : *Yes, using the OBE method (G2.3)*
 P : *I'll go back to the steps. So you arrange the matrix with specific specifications. After that, what do you do?*
 SL1 : *After that, make the numbers in the lower triangle above equal to 0. (G2.4)*
 P : *How do you change 1 to be equal to 3?*
 SL1 : *For example, in the first row, this 1 belongs to x_1 (G2.5)*

Based on the analysis of the interview results and work results, subjects SL1 and SL2 performed activities relevant to the algorithmization aspect, including planning the steps for solving the problem (G1) and determining the steps for solving the problem in sequence (G2). Subjects SL1 and SL2 were both planned using the OBE method. Subjects SL1 and SL2 also determined the solution steps (G2), as follows: analyzing the problem, determining the solution method, applying the solution method, and determining the solution to the linear equation system. In applying the solution method, subjects SL1 and SL2 both used the OBE method. Some examples of activities in applying the OBE method include: compiling a matrix containing variable coefficients, making other row elements (apart from the main element) zero, making the upper/lower triangle elements equal to 0, making the elements on the main diagonal equal to 1, and determining the 'leading 1' in each row.

In solving SLE4V mathematics problems, Subjects SL1 and SL2 performed activities relevant to the decomposition aspect, namely identifying information in the task (D1) and dividing the task into sub-tasks (D2). Subjects SL1 and SL2 also performed activities relevant to sub-aspects of the abstraction aspect (recognition, building-with, and construction). From an algorithmization aspect, Subjects SL1 and SL2 performed activities relevant to the algorithmization aspect, namely planning the steps for completion (G1) and determining the steps for completion (G2). Thus, there was consistency in the decomposition, abstraction, and algorithmization activities between Subject SL1 and Subject SL2 (convergent in nature). Furthermore, there is relevance between the activities of decomposition, abstraction, and algorithmization with the conceptual framework of algorithmic thinking. Thus, it is concluded that the findings/results of the study are credible and relevant to algorithmic thinking.

5. Discussion

5.1. Decomposition Aspects

The research subjects/participants divided tasks/problems into sub-tasks/sub-problems to make the task completion process easier. This decomposition activity involved two main aspects, namely identifying important information from the task/problem (D1) and dividing the task into sub-tasks (D2). Some examples of information obtained from the research task questions included the number of variables and equations. The research subjects divided the task into sub-tasks so that they could be handled sequentially, such as compiling matrices and applying elementary row operations, as well as determining the solution to the system of linear equations.

This finding is in line with the definition of decomposition in algorithmic thinking, according to Futschek [7] and Lockwood et al. [1], namely, the process of breaking down complex problems into simpler and more logical steps. Dogan [8] also mentions that individuals with algorithmic thinking skills will tend to think systematically and be able to break down problems into small parts that can be handled in a structured manner. Bacelo Polo and Gómez Chacón [23] and Tupouniua [24] explain that decomposition involves "breaking down complex problems into smaller ones," which allows students to identify problems explicitly and separate their elements. This is in line with Kadijevich et al. [25], who emphasize that decomposition is a key skill in mathematics learning and helps students understand the

structure of problems. Thus, the results of this study reinforce previous findings that decomposition skills are an important foundational component in solving problems that require logical and systematic steps, such as solving linear equation systems using the OBE method.

5.2. Abstraction Aspects

The aspect of abstraction in algorithmic thinking shows a significant difference in the exploratory approach between this study and previous studies. Blannin and Symons [10] state that abstraction is a mental process for interpreting available data, including recognizing patterns and simplifying complex ideas into basic concepts. However, they do not explicitly explain how this abstraction can be observed in the thinking process of students. In contrast, this study explores aspects of abstraction using the three epistemic actions framework as proposed by Hershkowitz et al. [15] namely recognition, building with, and constructing. This approach provides a more in-depth description of how knowledge is used and constructed in thinking activities. Participants performed activities relevant to the sub-aspects of abstraction (recognition, building with, and constructing).

Recognition. Subjects performed two main activities relevant to recognition, namely the recognition of prior knowledge relevant to the task and the recognition of solution patterns. The prior knowledge recognized includes: knowledge of the elimination method, elementary row operations, lower triangular matrices, variables and variable coefficients, arithmetic concepts, principal entries, and matrices to be used in the application of OBE (augmented matrices). The research subjects also recognized patterns for solving linear equation systems, including: using the OBE method and the elimination-substitution method.

Building-with. The building-with activity is reflected in the subjects' mental activities when combining several pieces of knowledge to build understanding/knowledge in solving sub-tasks from the decomposition aspect. The research subjects combined their prior knowledge to build knowledge, for example: making the lower triangle elements zero, creating elements called 'leading 1', making elements other than 'leading 1' zero, creating a matrix containing variable coefficients, and making the upper triangle elements zero.

Constructing. The activity of constructing is seen in the process of integrating knowledge that produces new understanding, namely, how to apply the OBE method comprehensively so that participants/subjects can achieve the completion of linear equation system tasks/problems. In this study, the integration of prior knowledge construction resulted in several pieces of knowledge/understanding, including: knowledge of solutions or variable values, namely the knowledge that the assigned linear equation system problem has a single solution, and the strategy for obtaining the solution to the task using elementary row operations. Thus, these findings enrich the literature related to abstraction in algorithmic thinking. The results of this study provide a framework for identifying and describing students' cognitive activities, not only as data simplification, but also as construction activities aimed at achieving solutions.

5.3 Algorithmization Aspects

Both research subjects carried out activities, including planning solution steps and determining solution steps systematically. Both research subjects planned the solution steps, namely, using the elementary row operation method. Although in determining the solution to the linear equation system, the first subject (SL1) and the second subject (SL2) provided different solutions to the linear equation system, both subjects determined the solution steps in sequence. These solution steps included activities such as analyzing the problem, determining the solution method, applying the solution method, and determining the solution to the linear equation system. These findings support Kallia et al. [4] that algorithmic thinking includes the ability to organize a series of problem-solving steps that can be executed by humans or machines. The results of this study indicate that computer science students have the ability to develop systematic procedures or steps for solving mathematical problems related to linear/algebraic equation systems.

6. Conclusion

The findings of this study indicate that the informatics education students used algorithmic thinking by involving three main aspects in solving mathematical problems/tasks, namely decomposition, abstraction, and algorithmization. The decomposition aspect activities of the students included: identifying task information and dividing tasks into subtasks to facilitate/simplify the application of solution steps, or obtaining solutions. Students perform abstraction activities, which include: recognition activities, building activities, and construction activities. Recognition activities are manifested in the recognition of relevant knowledge and the recognition of patterns in the sequence of steps to complete the task. Building activities are carried out by combining several pieces of knowledge to build understanding/knowledge to complete several subtasks from the decomposition aspect. Construction activities are seen in the process of integrating knowledge that produces new understanding, namely, how to apply the OBE method and knowledge about single solutions from linear equation systems. Meanwhile, the algorithmic aspect is seen in the activities of planning and applying solution steps so that male subjects can solve linear equation system tasks/problems accurately and systematically.

The three aspects of algorithmic thinking do not appear separately but are interrelated to form logical and systematic thinking activities in solving mathematical tasks. This shows that algorithmic thinking is not only a skill related to constructing algorithms but also contributes as a relevant cognitive activity in solving mathematical tasks/problems or algorithmic tasks/problems for computer science students. In solving linear equation systems with four variables, students have not yet obtained a single correct solution, even though they have carried out systematic algorithmization activities. This can be used as a recommendation for further research, namely, to find ways to improve algorithmic thinking skills so that students can find solutions to mathematical problems logically, accurately, and systematically.

Transparency:

The authors confirm that the manuscript is an honest, accurate, and transparent account of the study; that no vital features of the study have been omitted; and that any discrepancies from the study as planned have been explained. This study followed all ethical practices during writing.

Acknowledgments

This research is a part of the final dissertation project of the doctoral programme in mathematics education, Faculty of Mathematics and Natural Sciences, Surabaya State University. Therefore, the researcher would like to thank all leaders, promoters, and co-promoters for their support in conducting this research. We would also like to thank all participants from the informatics education programme, Faculty of Teacher Training and Education, Trunojoyo University, Madura, Indonesia.

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References

- [1] E. Lockwood, A. F. DeJarnette, A. Asay, and M. Thomas, "Algorithmic thinking: An initial characterization of computational thinking in mathematics. In M. B. Wood, E. E. Turner, M. Civil & J. A. Eli (Eds.)," in *Proceedings of the 38th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 1588-1595). Tucson, AZ, 2016.
- [2] M. Stephens and D. M. Kadijevich, *Computational/Algorithmic thinking*. In S. Lerman (Ed.), *Encyclopaedia of Mathematics Education*. Cham, Switzerland: Springer Nature Switzerland AG, 2020.
- [3] L. Zsakó and P. Szlávi, "ICT competences: Algorithmic thinking," *Acta Didactica Napocensia*, vol. 5, no. 2, pp. 49-58, 2012.

- [4] M. Kallia, S. P. van Borkulo, P. Drijvers, E. Barendsen, and J. Tolboom, "Characterising computational thinking in mathematics education: a literature-informed Delphi study," *Research in Mathematics Education*, vol. 23, no. 2, pp. 159-187, 2021. <https://doi.org/10.1080/14794802.2020.1852104>
- [5] C. Selby and J. Woollard, "Computational thinking: The developing definition," in *Proceedings of the ITiCSE Conference 2013*, 2013.
- [6] J. M. Smith and C. Angeli, *Computational thinking*. In A. Tatnall (Ed.), *Encyclopedia of Education and Information Technologies*. Cham, Switzerland: Springer Nature Switzerland AG, 2019.
- [7] G. Futschek, "Algorithmic thinking: The key for understanding computer science," presented at the International Conference on Informatics in Secondary Schools-Evolution and Perspectives, 2006.
- [8] A. Dogan, "Algorithmic thinking in primary education," *International Journal of Progressive Education*, vol. 16, no. 4, pp. 286-301, 2020.
- [9] H. Y. Mumcu and S. Yıldız, "The investigation of algorithmic thinking skills of 5th and 6th graders at a theoretical dimension," *MATDER Matematik Eğitimi Dergisi*, vol. 3, no. 1, pp. 41-48, 2018.
- [10] J. Blannin and D. Symons, *Algorithmic thinking in primary schools*. In A. Tatnall (Ed.), *Encyclopaedia of Education and Information Technologies*. Cham, Switzerland: Springer Nature Switzerland AG, 2019.
- [11] M. Stephens, *Embedding algorithmic thinking more clearly in the mathematics curriculum*. In Y. Shimizu & R. Vithal (Eds.), *ICMI Study 24: School Mathematics Curriculum Reforms – Challenges, Changes and Opportunities*. Cham, Switzerland: Springer, 2018.
- [12] D. I. Johanning and K. S. Shockey, "Fraction operation algorithmic thinking: Leveraging students' use of equivalence as a tool," presented at the 34th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education (NAR PME), Kalamazoo, MI, 2012.
- [13] O. V. Sadykova and G. G. Il'bahtin, "The definition of algorithmic thinking," in *Proceedings of the International Session on Factors of Regional Extensive Development (FRED 2019) (Advances in Economics, Business and Management Research, Vol. 113, pp. 419-422)*. Atlantis Press, 2020.
- [14] A. Pyzara, "Algorithmization as mathematical activity and skill," *Didactica Mathematicae*, vol. 40, pp. 97-117, 2018.
- [15] R. Hershkowitz, B. B. Schwarz, and T. Dreyfus, "Abstraction in context: Epistemic actions," *Journal for Research in Mathematics Education*, vol. 32, no. 2, pp. 195-222, 2001. <https://doi.org/10.2307/749673>
- [16] J. R. Fraenkel and N. E. Wallen, *How to design and evaluate research in education*, 7th ed. New York: McGraw-Hill, 2009.
- [17] J. W. Creswell, *Educational research: Planning, conducting, and evaluating quantitative and qualitative research*, 4th ed. Boston, MA: Pearson Education, 2012.
- [18] S. B. Merriam, *Qualitative research*, 2nd ed. United States of America: USA: John Wiley & Sons, 2009.
- [19] M. B. Miles, A. M. Huberman, and J. Saldana, *Qualitative data analysis: A methods sourcebook*, 3rd ed. United States of America: USA: SAGE Publication, 2014.
- [20] J. Saldana, *The coding manual for qualitative researchers*, 4th ed. Los Angeles: SAGE Publication, 2021.
- [21] N. K. Denzin and Y. S. Lincoln, *The SAGE handbook of qualitative research*, 5th ed. Thousand Oaks, CA: SAGE Publications, 2018.
- [22] N. K. Denzin, Y. S. Lincoln, M. D. Giardina, and G. S. Cannella, *The SAGE handbook of qualitative research*, 6th ed. United States of America: SAGE Publication, 2023.
- [23] A. Babelo Polo and I. M. Gómez Chacón, "Characterising algorithmic thinking: A university study of unplugged activities," *Thinking Skills and Creativity*, vol. 48, p. 101284, 2023. <https://doi.org/10.1016/j.tsc.2023.101284>
- [24] J. G. Tupouniua, "Differentiating between counterexamples for supporting students' algorithmic thinking," *Asian Journal for Mathematics Education*, vol. 1, no. 4, pp. 475-493, 2022. <https://doi.org/10.1177/27527263221139869>
- [25] A. Kadijevich, D. M. Stephens, and M. Rafiepour, *Emergence of computational/algorithmic thinking and its impact on the mathematics curriculum*. In: Shimizu, Y., Vithal, R. (eds) *Mathematics curriculum reforms around the world. New ICMI study series*. Cham: Springer, 2023.