

Fitting small-group learning in first-year engineering mathematics courses

N. Lohgheswary¹, Preethy. A^{2*}, Leelavathi. R³

¹School of Engineering and Computing, Regent College London, WC1R 4BH, United Kingdom.

²Centre for Sustainability in Advanced Electrical and Electronics Systems, Faculty of Engineering, Built Environment and Information Technology, SEGi University, 47810 Kota Damansara, Malaysia; ayappan@segi.edu.my (P.A.).

³School of Computing and Informatics, Albukhary International University, Malaysia.

Abstract: Even though the students were initially excited to study engineering, the course seemed to have sapped their enthusiasm. One of the factors contributing to the high first-year failure rate in the course is the difficulty of engineering mathematics. High school arithmetic is more straightforward than math in higher education. When students take drastically different courses at school and university, some first-year students find it difficult to transition. Class size is another element that affects the first-year student failure rate. Social isolation and peer competition are reduced when learning in small groups. It also promotes successful academic pursuits and positive interpersonal relationships. Students who work together to achieve learning objectives gain socially and intellectually. One challenge with small group learning is that, to avoid misunderstandings inside the group, the instructions for the group task must be clearly stated. There may be disagreements among students. Another challenge for small group learning arises when each member of the group is given the same mark for their work. Instead of one lecturer teaching over 100 students, there should be about five tutors to facilitate small group learning with the speaker. The 3P model encourages a more thorough examination of the factors affecting students' learning results. The model's prediction considers the student, the course, and the department's learning environment.

Keywords: *Small-group learning, Engineering Mathematics, Benefit, Challenge.*

1. Introduction

Since engineering requires calculations, mathematics is essential to engineering studies [1]. The reason for the high dropout rate among first-year engineering students is that the curriculum is "too dry and too boring." Students experience "academic disinterest and emotional disengagement" [2]. They don't feel that the engineering subject is relevant to them. Though the students were initially enthusiastic about studying engineering, the course somehow makes them feel less interested.

The difficulty of Engineering Mathematics is one of the reasons for the high first-year failure rate in the course. In mathematics, there is no easy transition from high school to college. Math in high school is simpler than math in higher education. Some first-year students find it difficult to adjust when their courses at school and university differ significantly. Another factor contributing to the first-year student failure rate is class size. A university lecture hall can hold up to 100 people during a session, but a classroom can only hold roughly 20 pupils. In small class settings as opposed to lecture halls, students are better able to focus.

An increasing number of courses are being offered online since the pandemic began. Digital technology will, on the one hand, have a significant impact on how mathematics courses are designed and delivered [3]. However, compared to other courses, fully online mathematics courses experience a higher rate of student attrition [4, 5].

Sophisms, paradoxes, and puzzles can stimulate pupils' emotions, imaginations, and problem-solving skills. Puzzles are recognized for their unconventional, irregular, and unstructured questions that are

presented in a fun manner. The situations in engineering are similar puzzles [6]. The parts of a puzzle that engineers are handed are market realities, professional or industrial codes, technology limitations, and customer expectations. To offer partial answers, engineers must create a process or product that satisfies every requirement. However, students frequently find it tedious to solve puzzles that require lengthy computations [7].

An undergraduate degree in general engineering does not include a single course on general thinking skills. The tutors' goal when they come to class is to help pupils improve their general thinking abilities so they can solve problems. Students frequently struggle to use their problem-solving abilities outside of the confines of the textbook. This generally happens because pupils frequently encounter common issues in their assignments. This can just be using a method you picked up in class to tackle an issue. Planning the solutions takes very little time [8]. Pupils work quickly to complete the questions and write the mathematical processes. This step could be used for a common issue. According to research, only 25% of above-average engineering students who applied complex algebraic techniques to address non-routine issues were able to pass [9]. Rather, students ought to utilize their calculus skills to address non-routine issues.

In a different study, engineering students who had to solve a "non-routine" problem had worse failure rates. Because of the question's peculiar phrasing, a 95% failure rate was reported [10]. The question was to "show that the truck should run approximately 28 km/h to minimize the total cost of a journey," rather than the typical "find the velocity that minimizes the total cost of a journey." Even the math needed to answer this problem was beyond the kids' comprehension. This demonstrates how students frequently find it difficult to answer indirect questions when they are presented with only direct question practice. Instead of learning the subject by heart, pupils who get it will be able to comprehend the questions and make plans for their answers, regardless of how they are rewarded. This is because they will connect the taught notion to the phases that need to be solved. After that, they will generalize the answer, which will then be applied to the issues.

Students in regular classrooms are typically seen as passive recipients. This is most likely caused by the instructor's tendency to talk a lot and the pupils' tendency to just listen and repeat what the teacher says [11]. This ends up being a barrier to pupils' comprehension of a vital topic like mathematics, which is composed of abstract ideas. Small group instruction is being used in the teaching and learning of engineering mathematics to address this issue. As early as the first engineering term, this project was launched.

Small group learning approaches include cooperative learning, collaborative learning, problem-based learning, peer-led learning, and team-based learning. A group is a collection of people who, although they are independent in their activity, perceive themselves as a cohesive social unit within a larger social structure [12, 13]. Cooperative learning entails students collaborating in small groups to optimize both their own and each other's learning [14]. While collaborate implies to work together, cooperate means to work or act together [15]. In cooperative learning, three to five students work together to study using a methodical and structured approach.

Conversely, collaborative learning entails providing students with the chance to participate in discussions, assume accountability for their own education, and develop their critical thinking skills [16, 17]. It also refers to students' mutual learning, which enables them to collaborate rather than only retain information from the teacher [18].

In general, positive interdependence, engagement, individual accountability, the application of interpersonal skills, and progress tracking are components of team-based learning [19]. The idea of positive independence holds that the group will succeed if each member contributes in some way to reaching its objectives. Peers support and encourage one another when there is constructive interaction between them. Each participant bears accountability and responsibility for the collective learning outcome. Members of the group must be able to handle disputes and have mutual respect and trust. Finally, a milestone must be used to track the group's progress to reach the goal.

Due to its social nature, academic learning involves active engagement in the communicative reasoning process [20, 21]. This involves taking part in scholarly and casual conversations in a small group setting. In general, researchers found that when students engage with one another, they learn more. Individual learning in small groups, which can consist of four to five students, is the main goal of small group learning. The group allocation strategy should consider differences in educational attainment, skill sets, and multiculturalism. A team-based approach to learning, or small group learning, is when students collaborate to meet learning goals.

The Jigsaw method, case studies, brainstorming sessions, role-playing games, problem-based and project-based learning, team teaching, team research, and debates are some of the technologies used in small group learning. In small group learning, students can employ many patterns and interactions, including debating, mentoring or being mentored by another student, observing, expressing oneself, reflecting, and making recommendations [18]. They can discuss a topic or issue with their peers through arguments. A student can impart knowledge to another student in the group once he has gained it.

Another way to promote engagement in small group learning is to be taught by a fellow student. Additionally, students learn by watching their peers as they acquire new information. When a student introduces and explains new information to other pupils, self-expression takes place. Students can improve their performance on subsequent assignments by correcting themselves through peer reflection. Peer commenting is another name for peer reflection. Peer feedback can take the form of recommendations, ideas, and well-reasoned arguments that support the opinions of other team members. Others should be receptive to accepting comments made by peers.

If the peer comment is untrue, the member should respond with arguments that have solid evidence behind them. Peer remarks ought to begin with a compliment about the team members and then offer advice on how to make them better. Team members will not misunderstand one another if they use a peer comment technique. This is because peer comments allow for the speech or writing of anything. To prevent misunderstandings within the team, it could be discussed whether there are any unclear points.

Students who attend in-person sessions find it easy to form social and professional bonds with their peers, which makes small group learning effortless for them. However, if they must meet virtually and live apart, the tutor will need to know certain techniques to support the pupils in the virtual learning environment. At the outset of the learning process, the instructor must arrange an introductory meeting or conversation. To respond to questions that the instructors publish, students must first register for a personal account on the blog or group website. The tutor can start by posting questions on the pupils' interests, pastimes, and perspectives on moral and social issues. Later on, the tutor will have all the time to devote to the course material.

Teachers could abide by some recommendations to guarantee that collaboration is effective. The assignment that the instructor assigns must relate to the goals of the course. Students are expected to understand that teamwork is essential to learning and achieving objectives. The purpose of team activities is to maintain student engagement outside of the classroom as well. Subsequently, the task ought to involve multiple phases and require assessment at each phase. Two team assignments every semester are the maximum amount of teamwork that students can be assigned.

We'll discuss the advantages of small group learning in the following paragraphs. The challenges faced by small group learning will be addressed next. Subsequently, there will be a further discussion on the use of small group learning in engineering mathematics classrooms for first-year students.

2. Benefits of Small-Group Learning

Students generally gain from small group instruction. Students' interpersonal, communication, and problem-solving skills are developed, all of which are critical for success in the workforce. When students converse and interact with others while working on mathematical problems, they acquire deeper and more complex mathematical concepts [22, 23]. By assigning a task that calls for group

collaboration and interaction, facilitators can help participants create fruitful mathematical conversations [24, 25].

Learning in small groups lessens peer competition and social isolation. It also encourages good interpersonal interactions and intellectual success. Students benefit academically and socially when they collaborate with others to accomplish learning objectives [26, 27]. Engaging with fellow students supports individual growth. Every student's input enhances the performance of the entire group. Through small group interaction, students improve their observational and communicative abilities. Additionally, they receive assistance from their classmates [28].

Small group activities help students develop new study habits, enhance their cognitive function, and strengthen peer relationships. Students' behavior and interpersonal interactions improved when they interacted with their peers [29, 30]. A study conducted on engineering mathematics students in Years 1 and 2 shows that while Year 2 students feel more at ease working in a group, Year 1 students prefer to work alone. Although they must adjust to the new learning environment, Year 2 pupils are more mature decision-makers and are fully committed to their work [28]. Compared to competitive and individualistic learning methodologies, small group learning yields more positive attitudes, according to additional research [31]. When compared to traditional learning methods, students' participation in academic debates was higher, which encourages higher individual success [32, 33].

Students engage in small group activities where they discuss topics other than their work, including themselves and other students. Mathematical learning is supported by students' social and academic discourse [24, 25, 34–37]. Students' conversations may center on persons (subjectifying), mathematical objects (mathematizing), or their characteristics (identifying) [37]. Talk that is subjectifying concentrates on individuals and their behaviors related to their tasks.

Comprehending the social interactions among students during off-task discourse could help make sense of on-task talk. Every student has a different set of learning possibilities, including peers who see themselves as the designated "teachers." While others follow their classmate, one student may guide the group in the right direction.

Tutors can provide scaffolding for students to learn how to use both academic and social discourse in communication. Mathematizing is supported in small group learning discussions only when suitable structuring is provided. The coaching helps kids' inquisitive conversations grow. If not, the conversation is either cumulative or disputatious [35]. Learning mathematics is positively correlated with the quantity and quality of mathematizing.

Additionally, studies have demonstrated the superiority of small group learning techniques over conventional lecture-based training [38, 39]. The findings indicated that student achievement is positively impacted by small group learning. In both individualized and lecture-based classrooms, students received a 50% score; in small group learning environments, they received a 69% score. Four distinct approaches to small group learning over a thirty-year period yielded the improvement. These include peer-led team learning, problem-based learning, cooperative learning, and collaborative learning. Small group learning seems to be a good strategy for fostering academic achievement not just in STEM fields [40] but also in computer science, statistics, and engineering [38, 41, 42]. Students' academic proficiency improved and increased faster when they had the chance to participate in peer-supported learning.

Through small group instruction, "participants learn to learn" [43]. They acquire abilities in motivation, shared accountability, and critical thinking. Their teacher also emphasized the importance of developing self-worth and confidence. Students working in groups gain a deeper understanding of the material and are encouraged to think more critically. Team members experience less worry when sharing tasks, and their stress levels decrease when they have a sense of humor.

The following are some additional advantages of small group instruction. Students learn how to collaborate in a team despite having varying opinions, backgrounds, and personalities. They can critique their own interpretation of the issue and learn to value the opinions of others. They will have the chance to do work online if group learning is conducted online.

Giving open-ended problems and tasks supports group members in contributing to small group learning [44]. The key importance of developing group processes is the group members' responsibility toward each other. This includes giving justification, providing valid reasoning, and asking questions when they are confused or seek help from other members. Learning opportunities increase when they effectively ask for or give help in their small group learning.

3. Challenges of Small-Group Learning

One of the difficulties with small group learning is that the group work instructions need to be clearly laid out to prevent misunderstandings within the group. Every student will take a different approach to leading the learning process. Students might disagree with one another. Therefore, if the tutor has already provided clear instructions, this issue won't arise.

The instructor should select the group participants. It is important to consider both the diversity of the student body and their learning levels when classifying them. Learning will not be relevant if extremely fast or slow pupils are grouped. A diverse group of people will guarantee that the conversation is insightful.

When everyone in the group receives the same grade for their performance, it presents another difficulty for small group learning. Some or all the members may have contributed to the work. Therefore, it will be unjust for the students who completed the assignment to receive the same grades as the students who simply turned in their names without doing any work. Therefore, before giving everyone the same grade, the facilitator needs to make sure that every student is present for the session and participating in the group activity.

Teachers must recognize that students come to university from a variety of cultural and educational backgrounds. They may have various learning styles that work for them. Not all pupils may benefit from small group instruction. The tutor needs to give the group assistance if they tend to falter. The tutor must continue to provide the group with strong support, keep an eye on their development, and inculcate self-discipline. The tutor must clearly define the objectives. A team leader has the authority to appoint.

Sometimes, students with strong personality qualities can control collective decision-making. Their lengthy and loud speeches have the power to influence other kids. Maybe the other students aren't as good. At this stage, it's possible that the goal of providing each team member with an equal opportunity won't be accomplished [16]. In other cases, the more intelligent students could think that the slower group members are holding back their advancement. The intelligent kids' enthusiasm for learning will eventually wane. The instructor may then decide to use the conventional lecture-based approach.

The following are the additional difficulties. If someone in the group doesn't participate in the activities at all or reacts slowly, the experience as a team may suffer. The worst case occurs when there is group turnover. In groups, turnover is viewed as a negative experience for the group. When it comes to groups working on simple activities, turnover is more detrimental than when working on complex tasks [45]. The team comes up with creative ideas when working on challenging projects rather than relying on preexisting expertise. As a result, the group does not suffer greatly when a member leaves. When comparing groups that had turnover to those that did not, the productivity rates of the former were lower. However, the length of the task and the members' emotions are affected by member replacements and departures.

In most cases, students who experience difficulties turning in their own work should inform others that they require an extension. The other members were then informed that the job would be completed later. When there is a lack of communication and engagement among peers, peer collaboration is unproductive. The group's overall task will suffer from a late team member's silence.

4. Fitting Small Group Learning to Engineering Mathematics Courses

Including small group learning in first-year engineering mathematics classrooms is the solution to the given challenge. It is best to divide a larger lecture class of more than 100 students into smaller

groups. To enable small group learning with the speaker, there should be roughly five tutors rather than one lecturer instructing more than 100 pupils. First-year students benefit from small group learning because they retain the same sense of familiarity from their classroom studies.

Engineering mathematics can be learned more quickly and effectively with blended versions of in-person classes that include certain online exercises with in-person instruction [46, 47].

Using flipped learning, also known as inverted learning, is one method. This is a component of blended learning, which combines online lectures and readings with computer-based, tailored training to enhance group learning activities in the classroom [48-52]. According to student interviews, flipped learning in calculus classes using self-study videos and in-class small group activities was successful, but it had no influence on meeting learning objectives [53, 54].

This is because movies do not aid in the online study of mathematics [55]. Online tests contribute at least 10% to the improvement in calculus first-year courses [56]. Flipped learning involves having students complete online lectures and exercises to lay the groundwork for a chapter, then working collaboratively with a facilitator in class to apply what they have learned [57]. Nevertheless, the study did not track the pupils' performance. According to other research, students frequently have a negative opinion of blended learning [58, 59]. It will be more accurate to look at students' grades rather than conduct student interviews and surveys.

For a module designer, experimenting with blended learning experiences that include online and in-person learning experiences for mathematics is ideal [60]. Given that the school's majority of students had previously studied online, one of the greatest ways to study engineering mathematics is through blended learning. This entails considering the elements that affect learning results. Certain mathematical learning models fail to consider the learning environment, which is crucial for blended learning [61, 62]. However, the blended learning's precursor, process, and product are all considered by the 3P model of student learning [63, 64]. The 3P model of student learning is depicted in Figure 1.

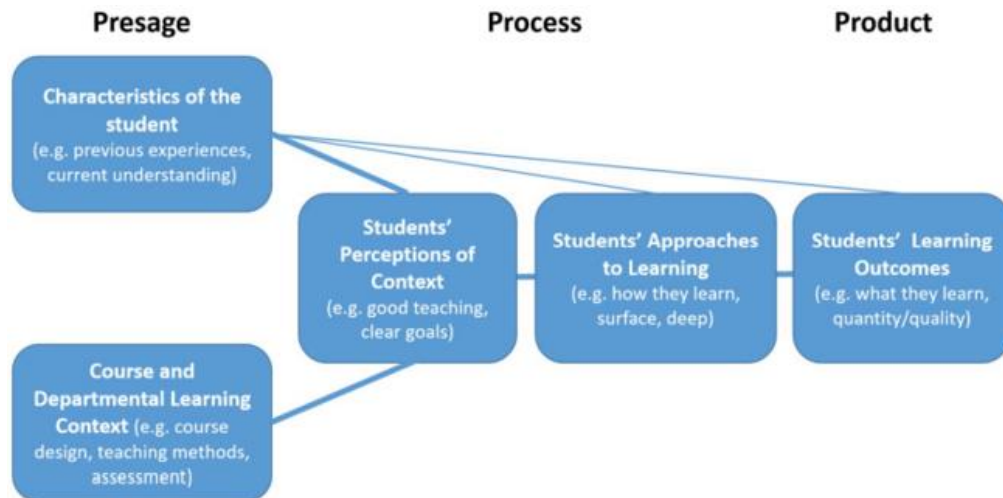


Figure 1.
The 3P model of student learning.

A student's learning can be divided into three categories: presage, process, and product. Presage refers to activities that take place prior to learning. Conversely, a process demonstrates how students approach learning, but a product refers to the results attained [63, 64]. These phases are dynamic, requiring students to make quick judgments about how to approach learning in the classroom based on prior experiences [64]. The sequence of aim, approach, and outcome is likewise followed in the learning of mathematics [65]. A more comprehensive analysis of the variables influencing students' learning outcomes is encouraged by the 3P model.

The student's attributes and the departmental and course learning context are covered by the presage mode. The course design, instructional strategies, and assessment are covered in the departmental and course learning contexts. First-year engineering mathematics courses are usually provided in person via computer with lectures, tutorials, and practical sessions. The purpose of textbooks and reference books is to direct students. Instead of providing in-depth knowledge, the syllabi are created to cover the material and eventually encourage surface-level student interaction [66]. While the lectures are available on video, the key gestures made by the lecturer and his or her spontaneous computations using drawings on whiteboards are rarely recorded [67]. Students' mathematical thinking may not be supported by the recorded lectures [55]. The expectations for students' learning are established by these teaching and learning arrangements. First-year students are unfamiliar with the university setting and are subject to certain expectations regarding their learning style.

Prescience also refers to the attributes of the learner [63], which include prior knowledge and present comprehension. Some anxious students exhibit avoidance behavior, which leads them to finish their arithmetic assignments in the final moments [68]. Mathematical attitudes and math anxiety are related. Students with a "fixed mindset," or the belief that intelligence is innate, are more anxious when it comes to mathematics than students with a "growth mindset," or the belief that intellect is developed through work, feedback, and reflection [69]. When students learn from their errors in an online setting, math anxiety can be conquered [68].

Grit, tenacity, and a desire for long-term goals are additional traits of students associated with success [61]. While persistence has been linked to retention in engineering programs [70, 71], grit is thought to be a more accurate indicator of first-year student success [72]. According to a study, engineering students' identities change during their first year, making it an ideal moment to help them come to terms with who they are [73]. Students who use an online system that questions their preconceptions about how they study mathematics and corrects their errors during the problem-solving process will exhibit more favorable traits.

Prior to learning in a new setting at the institution, students can self-correct their understanding through online tests. One of the best examples of the new learning environment is small group instruction. It has been demonstrated that first-year engineering students' proficiency in mathematics is a predictor of their performance [74]. This is accomplished through online tests that give students feedback on their mathematical proficiency and online study aids that allow them to customize their education [75] and enhance student learning results [56].

The way students approach learning and how they perceive the context are both part of the 3P model's process stage. Students' approaches to learning are influenced by their perceptions of learning [63]. When mathematical problems are placed within engineering contexts, students recognize the significance of what they are learning [76]. At this stage, it's essential to use real, complex engineering challenges to scaffold students' learning and help them get unstuck. You can accomplish this through small-group instruction. With the use of online scaffolding, students can become ready for increasingly difficult in-person problem-solving exercises [77]. The degree of readiness, the goals, and the organization are the other elements of the learning context [63]. Students' success in mathematics courses at the postsecondary level is influenced by how much time they devote to mathematical assignments [78]. The requirement of 10 hours per week for each topic when students begin their studies in engineering mathematics, along with the learning activities they participate in, encourages them to dedicate sufficient time to their studies. Technology can be used to automate conversation-based learning [56, 79, 80] and create individualized learning experiences that inspire students to take charge of their own mathematical education [81]. More difficult application problems can be answered by students if they have learned the fundamentals of engineering mathematics [57]. Online summative tests provide students with the opportunity to grasp mathematical concepts, discover areas of knowledge gaps, and enhance learning outcomes [56, 75].

For students to achieve desired learning outcomes, their methods of learning are essential. Based on their objectives, students use either surface-level or deep learning approaches [63]. High school math

classrooms use five different environments to measure student engagement in small group problem-solving: standing at a whiteboard, sitting at a flip-chart paper on a table, standing at a whiteboard, and individual students working in a notebook at a table. These environments are used to measure student workspaces. According to the study's findings, having students stand up at the whiteboard created the optimal setting for them to participate in activities [82]. The board tutorial [83, 84] is used in this study. The tutorial rooms have whiteboards mounted on their walls. To answer questions written on the board, students work in pairs. It is encouraged for students to talk about the methods they use to address challenges. They are free to observe the strategies used by other groups as well. Through inquiry and coaching, the tutor helps the learning process move forward. After the session, answers are given. Board tutorials can foster sophisticated problem-solving activities and enhance students' learning results when implemented in first-year engineering mathematics courses. This has been demonstrated to be an effective in-person teaching and learning strategy that enhances spoken communication and problem-solving [83].

5. Conclusions

In summary, the paradigm of traditional-based education has changed in the twenty-first century, leading to the adoption of alternative teaching strategies suitable for the circumstances. A technique identified is small group instruction. Though there are some drawbacks to this approach, they are outweighed by its advantages and positive aspects. Small group learning techniques can therefore be incorporated into the curriculum for first-year engineering mathematics students. Compared to traditional teacher-based instruction, there will be greater opportunities for students to interact with one another.

For first-year engineering mathematics classes, small group instruction should use the 3P model of student learning. The learning environment within the department, the course, and the student are all considered in the model's presage. Certain students might possess prior knowledge of the subjects being taught and be able to connect it to their current understanding of the subject. Students with prior experiences can assist in imparting their knowledge to other group members. The course learning objectives, instructional strategies, and course assessment are all included in the 3P model's presage stage. When engineering mathematics is taught in small groups using this format, the learning process becomes systematic, and the facilitator may effectively guide the group.

The way that students approach learning and how they perceive the context are considered in the process stage of the 3P model. Pupils studying engineering mathematics in small groups place a high priority on productive peer discussions, which also establish the course objectives. Students' learning styles in small groups, whether superficial or deep, impact their knowledge and comprehension. It goes without saying that first-year engineering mathematics students who are unfamiliar with the material will need to study it thoroughly. The small group's conversation and involvement will guarantee that learning occurs smoothly.

Students' learning outcomes are the focus of the 3P model of student learning's product stage. The completion of the job or assignment is the final product in the small group engineering mathematics learning process. The tasks will demonstrate students' learning from both small-group and peer learning experiences. The knowledge students gain from small group instruction will undoubtedly be beneficial.

It is therefore anticipated that over time, this approach will encourage students to engage in the learning process and that the student attrition rate will decline. University enrollment is the result of careful selection. As a result, the system needs to guarantee that every one of them leaves the university and graduates on schedule.

Transparency:

The authors confirm that the manuscript is an honest, accurate, and transparent account of the study; that no vital features of the study have been omitted; and that any discrepancies from the study as planned have been explained. This study followed all ethical practices during writing.

Copyright:

© 2026 by the authors. This article is an open-access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

References

- [1] D. Harris, L. Black, P. Hernandez-Martinez, B. Pepin, J. Williams, and W. t. T. Team, "Mathematics and its value for engineering students: What are the implications for teaching?," *International Journal of Mathematical Education in Science and Technology*, vol. 46, no. 3, pp. 321-336, 2015. <https://doi.org/10.1080/0020739X.2014.979893>
- [2] K. S. Blondal and S. Adalbjarnardottir, "Student disengagement in relation to expected and unexpected educational pathways," *Scandinavian Journal of Educational Research*, vol. 56, no. 1, pp. 85-100, 2012.
- [3] M. C. Borba, P. Askar, J. Engelbrecht, G. Gadanidis, S. Llinares, and M. S. Aguilar, "Blended learning, e-learning and mobile learning in mathematics education," *ZDM*, vol. 48, pp. 589-610, 2016. <https://doi.org/10.1007/s11858-016-0798-4>
- [4] G. G. Smith and D. Ferguson, "Student attrition in mathematics e-learning," *Australasian Journal of Educational Technology*, vol. 21, no. 3, pp. 323-334, 2005.
- [5] S. Trenholm, J. Peschke, and M. Chinnappan, "A review of fully online undergraduate mathematics instruction through the lens of large-scale research (2000-2015)," *Primus*, vol. 29, no. 10, pp. 1080-1100, 2019. <https://doi.org/10.1080/10511970.2018.1472685>
- [6] B. Parhami, "A puzzle-based seminar for computer engineering freshmen," *Computer Science Education*, vol. 18, no. 4, pp. 261-277, 2008.
- [7] P. Gnädig, G. Honyek, and K. Riley, *200 Puzzling physics problems, with hints and solutions*. Cambridge: Cambridge University Press, 2001.
- [8] A. Schoenfeld, *Mathematical problem solving*. New York: Academic Press, 1985.
- [9] A. Selden, J. Selden, and S. Hauk, "Why can't calculus students access their knowledge to solve non-routine problems?," *Research in Collegiate Mathematics Education*, vol. 4, pp. 128-153, 2000.
- [10] S. Klymchuk, T. Zverkova, N. Gruenwald, and G. Sauerbier, "University students' difficulties in solving application problems in calculus: Student perspectives," *Mathematics Education Research Journal*, vol. 22, pp. 81-91, 2010. <https://doi.org/10.1007/BF03217567>
- [11] M. Galton, L. Hargreaves, C. Comber, D. Wall, and T. Pell, "Changes in patterns of teacher interaction in primary classrooms: 1976-96," *British Educational Research Journal*, vol. 25, no. 1, pp. 23-37, 1999.
- [12] S. G. Cohen and D. E. Bailey, "What makes teams work: Group effectiveness research from the shop floor to the executive suite," *Journal of Management*, vol. 23, no. 3, pp. 239-290, 1997. <https://doi.org/10.1177/014920639702300303>
- [13] R. A. Guzzo and M. W. Dickson, "Teams in organizations: Recent research on performance and effectiveness," *Annual Review of Psychology*, vol. 47, no. 1, pp. 307-338, 1996.
- [14] D. W. Johnson and R. T. Johnson, "Making cooperative learning work," *Theory into Practice*, vol. 38, no. 2, pp. 67-73, 1999. <https://doi.org/10.1080/00405849909543834>
- [15] The Australian, *The Australian concise Oxford dictionary*. Melbourne, Australia: Oxford University Press, 1997.
- [16] A. A. Gokhale, "Collaborative learning enhances critical thinking," *Journal of Technology Education*, vol. 7, no. 1, pp. 22-30, 1995.
- [17] S. Totten, T. Sills, A. Digby, and P. Russ, *Cooperative learning: A guide to research*. New York: Garland, 1991.
- [18] O. Sumtsova, T. Aikina, L. Bolsunovskaya, C. Phillips, O. Zubkova, and P. Mitchell, "Collaborative learning at engineering universities: Benefits and challenges," *International Journal of Emerging Technologies in Learning (iJET)*, vol. 13, no. 1, pp. 160-177, 2018.
- [19] R. T. Johnson and D. W. Johnson, *An overview of cooperative learning*. Baltimore: Brookes Press, 1994.
- [20] J. Andriessen, *Arguing to learn*, In K. Sawyer (Ed), *The Cambridge handbook of the learning sciences*. Cambridge: Cambridge University Press, 2006.
- [21] S. Lerman, "Cultural, discursive psychology: A sociocultural approach to studying the teaching and learning of mathematics," *Educational Studies in Mathematics*, vol. 46, no. 1, pp. 87-113, 2001. <https://doi.org/10.1023/A:1014031004832>
- [22] D. Y. White, "Promoting productive mathematical classroom discourse with diverse students," *The Journal of Mathematical Behavior*, vol. 22, no. 1, pp. 37-53, 2003.

- [23] T. Woods, G. Williams, and B. McNeal, "Children's mathematical thinking in different classroom cultures," *Journal for Research in Mathematics Education*, vol. 37, no. 3, pp. 222-255, 2006. <https://doi.org/10.2307/30035059>
- [24] J. Boaler, "Promoting 'relational equity' and high mathematics achievement through an innovative mixed-ability approach," *British Educational Research Journal*, vol. 34, no. 2, pp. 167-194, 2008.
- [25] R. Hunter, "Scaffolding small group interactions," In: J. Watson and K. Beswick (Eds.), *Mathematics: Essential research, essential practice*, in *Proceedings of the 30th Annual Conference of the Mathematics Education Research Group of Australasia*, 2007.
- [26] W. Johnson and R. T. Johnson, "Cooperative learning and social interdependence," *Theory and Research on Small Groups*, vol. 4, pp. 9-35, 2002.
- [27] R. E. Slavin, "Research on cooperative learning and achievement: What we know, what we need to know," *Contemporary Educational Psychology*, vol. 21, no. 1, pp. 43-69, 1996.
- [28] H. Othman, I. Asshaari, H. Bahaludin, N. M. Tawil, and N. A. Ismail, "Student's perceptions on benefits gained from cooperative learning experiences in engineering mathematics courses," *Procedia-Social and Behavioral Sciences*, vol. 60, pp. 500-506, 2012. <https://doi.org/10.1016/j.sbspro.2012.09.414>
- [29] D. W. Jordan and J. L. Métais, "Social skilling through cooperative learning," *Educational Research*, vol. 39, no. 1, pp. 3-21, 1997.
- [30] R. M. Gillies, "The effects of cooperative learning on junior high school students during small group learning," *Learning and Instruction*, vol. 14, no. 2, pp. 197-213, 2004. [https://doi.org/10.1016/S0959-4752\(03\)00068-9](https://doi.org/10.1016/S0959-4752(03)00068-9)
- [31] A. O. Akinbobola, "Enhancing students' attitude towards Nigerian senior secondary school physics through the use of cooperative, competitive and individualistic learning strategies," *Australian Journal of Teacher Education (Online)*, vol. 34, no. 1, pp. 1-9, 2009.
- [32] E. Dugan, D. Kamps, B. Leonard, N. Watkins, A. Rheinberger, and J. Stackhaus, "Effects of cooperative learning groups during social studies for students with autism and fourth-grade peers," *Journal of Applied Behavior Analysis*, vol. 28, no. 2, pp. 175-188, 1995. <https://doi.org/10.1901/jaba.1995.28-175>
- [33] D. W. Johnson, R. T. Johnson, and K. A. Smith, "Cooperative learning returns to college what evidence is there that it works?," *Change: the Magazine of Higher Learning*, vol. 30, no. 4, pp. 26-35, 1998.
- [34] E. Cohen, *Designing groupwork: Strategies for the heterogeneous classroom*. New York: Teachers College Press, 1994.
- [35] N. Mercer, R. Wegerif, and L. Dawes, "Children's talk and the development of reasoning in the classroom," *British Educational Research Journal*, vol. 25, no. 1, pp. 95-111, 1999. <https://doi.org/10.1080/0141192990250107>
- [36] N. M. Webb and A. Mastergeorge, "Promoting effective helping behavior in peer-directed groups," *International Journal of Educational Research*, vol. 39, no. 1-2, pp. 73-97, 2003.
- [37] M. B. Wood and C. A. Kalinec, "Student talk and opportunities for mathematical learning in small group interactions," *International Journal of Educational Research*, vol. 51, pp. 109-127, 2012. <https://doi.org/10.1016/j.ijer.2011.12.008>
- [38] S. A. Kalaian and R. M. Kasim, "Small-group vs. competitive learning in computer science classrooms: A meta-analytic review. In Innovative teaching strategies and new learning paradigms in computer programming." Hershey, PA: IGI Global Scientific Publishing, 2015.
- [39] A. K. Sema, M. K. Rafa, and K. N. Julia, "Effectiveness of small-group learning pedagogies in engineering and technology education: A meta analysis," *Journal of Technology Education*, vol. 29, no. 2, pp. 20-35, 2018.
- [40] L. Springer, M. E. Stanne, and S. S. Donovan, "Effects of small-group learning on undergraduates in science, mathematics, engineering, and technology: A meta-analysis," *Review of Educational Research*, vol. 69, no. 1, pp. 21-51, 1999. <https://doi.org/10.3102/00346543069001021>
- [41] S. A. Kalaian and R. M. Kasim, "A meta-analytic review of studies of the effectiveness of small-group learning methods on statistics achievement," *Journal of Statistics Education*, vol. 22, no. 1, 2014.
- [42] S. A. Kalaian, R. M. Kasim, and J. K. Nims, "Effectiveness of small-group learning pedagogies in engineering and technology education: A meta-analysis," *Journal of Technology Education*, vol. 29, no. 2, 2018.
- [43] O. Gol and N. Andrew, "Collaborative learning in engineering education," *Global Journal of Engineering Education*, vol. 11, no. 2, pp. 173-180, 2007.
- [44] J. Boaler, "How a detracked mathematics approach promoted respect, responsibility, and high achievement," *Theory into Practice*, vol. 45, no. 1, pp. 40-46, 2006. https://doi.org/10.1207/s15430421tip4501_6
- [45] L. Argote, C. A. Insko, N. Yovetich, and A. A. Romero, "Group learning curves: The effects of turnover and task complexity on group performance 1," *Journal of Applied Social Psychology*, vol. 25, no. 6, pp. 512-529, 1995.
- [46] R. Ellis and P. Goodyear, *Students' experiences of e-learning in higher education: The ecology of sustainable innovation*. London: Routledge, 2013.
- [47] M. M. Patchan, C. D. Schunn, W. Sieg, and D. McLaughlin, "The effect of blended instruction on accelerated learning," *Technology, Pedagogy and Education*, vol. 25, no. 3, pp. 269-286, 2016. <https://doi.org/10.1080/1475939X.2015.1013977>
- [48] J. L. Bishop and M. A. Verleger, "The flipped classroom: A survey of the research, In D. Riley (Ed.)," presented at the 120th ASEE Annual Conference & Exposition, Atlanta, Georgia, 2013.

- [49] N. Hamdan, P. McKnight, K. McKnight, and K. M. Arfstrom, *The flipped learning model: A white paper based on the literature review titled a review of flipped learning*. Amsterdam: Flipped Learning Network/Pearson/George Mason University, 2013.
- [50] J. E. McLaughlin, P. J. White, J. Khanova, and E. Yuriev, "Flipped classroom implementation: A case report of two higher education institutions in the United States and Australia," *Computers in the Schools*, vol. 33, no. 1, pp. 24-37, 2016. <https://doi.org/10.1080/07380569.2016.1137734>
- [51] R. M. Clark, A. Kaw, and M. Besterfield-Sacre, "Comparing the effectiveness of blended, semi-flipped, and flipped formats in an engineering numerical methods course," *Advances in Engineering Education*, vol. 5, no. 3, p. n3, 2016.
- [52] A. Wedding, A. Cousins, and D. Quinn, *Transitioning staff, students and course materials to blended and online learning environments, Blended learning in engineering education: Recent developments in curriculum, assessment and practice*. London, UK: CRC Press, 2018.
- [53] S. Kadry and A. El Hami, "Flipped classroom model in calculus II," *Education*, vol. 4, no. 4, pp. 103-107, 2014.
- [54] J. McGivney-Burelle and F. Xue, "Flipping calculus," *Primus*, vol. 23, no. 5, pp. 477-486, 2013. <https://doi.org/10.1080/10511970.2012.757571>
- [55] S. Trenholm, L. Alcock, and C. L. Robinson, "Mathematics lecturing in the digital age," *International Journal of Mathematical Education in Science and Technology*, vol. 43, no. 6, pp. 703-716, 2012. <https://doi.org/10.1080/0020739X.2011.646325>
- [56] C. Varsavsky, "Can online weekly quizzes contribute to learning in mathematics," in *Proceedings of the 9th asian Technology Conference in Mathematics (pp. 161-168)*. Singapore: ATCM Inc, 2004.
- [57] Z. Sun, K. Xie, and L. H. Anderman, "The role of self-regulated learning in students' success in flipped undergraduate math courses," *The Internet and Higher Education*, vol. 36, pp. 41-53, 2018. <https://doi.org/10.1016/j.iheduc.2017.09.003>
- [58] R. A. Bjork, J. Dunlosky, and N. Kornell, "Self-regulated learning: Beliefs, techniques, and illusions," *Annual Review of Psychology*, vol. 64, no. 1, pp. 417-444, 2013.
- [59] L. Deslauriers, L. S. McCarty, K. Miller, K. Callaghan, and G. Kestin, "Measuring actual learning versus feeling of learning in response to being actively engaged in the classroom," *Proceedings of the National Academy of Sciences*, vol. 116, no. 39, pp. 19251-19257, 2019.
- [60] D. Quinn and J. Aarão, "Blended learning in first year engineering mathematics," *Zdm*, vol. 52, no. 5, pp. 927-941, 2020. <https://doi.org/10.1007/s11858-020-01160-y>
- [61] E. Dubinsky and P. Lewin, "Reflective abstraction and mathematics education: The genetic decomposition of induction and compactness," *The Journal of Mathematical Behavior*, vol. 5, no. 1, pp. 55-92, 1986.
- [62] D. Tall, "The transition to formal thinking in mathematics," *Mathematics Education Research Journal*, vol. 20, pp. 5-24, 2008. <https://doi.org/10.1007/BF03217474>
- [63] J. B. Biggs, "Approaches to the enhancement of tertiary teaching," *Higher Education Research and Development*, vol. 8, no. 1, pp. 7-25, 1989. <https://doi.org/10.1080/0729436890080102>
- [64] K. Trigwell and M. Prosser, "Towards an understanding of individual acts of teaching and learning," *Higher Education Research & Development*, vol. 16, no. 2, pp. 241-252, 1997. <https://doi.org/10.1080/0729436970160210>
- [65] A. Reid, L. N. Wood, G. H. Smith, and P. Petocz, "Intention, approach and outcome: University mathematics students' conceptions of learning mathematics," *International Journal of Science and Mathematics Education*, vol. 3, no. 4, pp. 567-586, 2005. <https://doi.org/10.1007/s10763-004-5818-0>
- [66] J. Biggs, "What the student does: Teaching for enhanced learning," *Higher Education Research & Development*, vol. 18, no. 1, pp. 57-75, 1999. <https://doi.org/10.1080/0729436990180105>
- [67] A. Weinberg, T. Fukawa-Connelly, and E. Wiesner, "Characterizing instructor gestures in a lecture in a proof-based mathematics class," *Educational Studies in Mathematics*, vol. 90, no. 3, pp. 233-258, 2015. <https://doi.org/10.1007/s10649-015-9623-1>
- [68] E. M. Marshall, R. V. Staddon, D. A. Wilson, and V. E. Mann, "Addressing maths anxiety and engaging students with maths within the curriculum," *MSOR Connections*, vol. 15, no. 3, pp. 28-35, 2017.
- [69] T. F. Smith and G. Capuzzi, "Using a mindset intervention to reduce anxiety in the statistics classroom," *Psychology Learning & Teaching*, vol. 18, no. 3, pp. 326-336, 2019. <https://doi.org/10.1177/1475725719836641>
- [70] A. L. Duckworth, C. Peterson, M. D. Matthews, and D. R. Kelly, "Grit: perseverance and passion for long-term goals," *Journal of Personality and Social Psychology*, vol. 92, no. 6, pp. 1087-1101, 2007.
- [71] D. San Choi, B. Myers, and M. C. Loui, *Grit and two-year engineering retention*. Indianapolis, Indiana: Frontiers in Education, 2017.
- [72] D. Vardin, A. Godwin, A. Kirn, L. Benson, and G. Potvin, *Understanding how engineering identity and belongingness predict grit for first-generation college students*. Virginia, United States: School of Engineering Education Graduate Student Series, 2018.
- [73] P. Hernandez-Martinez, J. Williams, L. Black, P. Davis, M. Pampaka, and G. Wake, "Students' views on their transition from school to college mathematics: Rethinking 'transition' as an issue of identity," *Research in Mathematics Education*, vol. 13, no. 2, pp. 119-130, 2011. <https://doi.org/10.1080/14794802.2011.585824>

- [74] A. Dekkers, N. Adams, and S. Elliott, *Using technology to provide a supportive mathematical pathway into university*, In *18th delta conference on the teaching and learning of undergraduate mathematics and statistics*. New Zealand: Rotorua, 2011.
- [75] N. A. Gordon, *Enabling personalized learning through formative and summative assessments*, In J. O'Donoghue (Ed.), *Technology-supported environments for personalized learning: Methods and case studies*. Lancashire: IGI Global, 2010.
- [76] J. Herrington, T. C. Reeves, and R. Oliver, *Authentic learning environments*, In *handbook of research on educational communications and technology*. New York: Springer, 2014.
- [77] R. Azevedo and A. F. Hadwin, "Scaffolding self-regulated learning and metacognition—Implications for the design of computer-based scaffolds," *Instructional Science*, vol. 33, no. 5/6, pp. 367-379, 2005.
- [78] T. Thiel, S. Peterman, and M. Brown, "Addressing the crisis in college mathematics: Designing courses for student succes," *Change: The Magazine of Higher Learning*, vol. 40, no. 4, pp. 44-49, 2008.
- [79] L. S. Vygotsky, "Thought and language," *Annals of Dyslexia*, vol. 14, no. 1, pp. 97-98, 1994.
- [80] P. Ramsden, *Learning to teach in higher education*. London: Routledge, 2003.
- [81] G. E. Davis and M. A. McGowen, "Formative feedback and mindful teaching of undergraduate mathematics," in *Proceedings of the 30th International Conference for the Psychology of Mathematics Education (Vol. 1, p. 241)*, 2006.
- [82] P. Liljedahl, M. Santos-Trigo, U. Malaspina, and R. Burder, *Problem solving in mathematics education*. New York: Springer, 2016.
- [83] K. A. Seaton, D. M. King, and C. E. Sandison, "Flipping the maths tutorial: A tale of n departments," *Australian Mathematical Society Gazette*, vol. 41, no. 2, pp. 99-113, 2014.
- [84] G. Williams, "Blackboard tutorials in first year mathematics," *Overview-University of Wollongong Teaching & Learning Journal*, vol. 2, no. 1, pp. 11-12, 1994.