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Teacher support in collective argumentation to foster student understanding of parallelism based on the Toulmin model and Pirie-Kieren theory

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Abstract: Dialogue-oriented mathematics learning requires teachers not only to facilitate discussion but also maintain mathematical accuracy while preserving students' ownership of reasoning. This study investigates how teacher support shapes collective argumentation and students' understanding of parallelograms in a junior high school classroom. Using the Teacher Support for Collective Argumentation (TSCA) framework in combination with the full Toulmin argumentation model, the study examines how specific teacher moves contribute to the emergence, development, and coherence of students' argument structures, with particular attention to the coordination of warrants, qualifiers, and rebuttals. Students' conceptual development is further analyzed through the Pirie–Kieren framework to trace shifts from visual intuition to formal justification. Drawing on qualitative analysis of the recordings of class discussions, student artifacts, and interaction transcripts from geometry lessons, the findings indicate that teacher support enacted to probing questions, revoicing, and visual representation management sustains students' ownership of arguments while enhancing the coherence of mathematical argument structures. Integrating TSCA with the Toulmin model enables more precise identification of argument maturity, especially in the correspondence between warrant types and the levels of claim certainty. These findings suggest that collective argumentation functions as an important socio-epistemic pathway toward formal proof and highlight pedagogical implications for question design and teacher interventions in the geometry classroom.

Keywords: *Collective argumentation, Pirie-Kieren, Teacher support, Toulmin model, TSCA.*

1. Introduction

Contemporary mathematics curricula emphasize classrooms that support dialogic interaction between teachers and students, where mathematical ideas are articulated, debated, and refined through collective communication [1]. In such classrooms, mathematical meaning is negotiated through discussion grounded in evidence rather than transmitted unilaterally from teacher to student. Dialogue is therefore not positioned as a supplementary activity but as a central mechanism for learning. However, sustaining productive dialogue presents a significant pedagogical challenge. Teachers are expected to encourage independent student reasoning, conjecture, and exploration, while simultaneously ensuring that classroom discussions remain aligned with instructional goals and mathematically accurate [2]. Within this context, collective argumentation plays a critical role in sharpening mathematical discourse, supporting conceptual understanding, and making students' reasoning and contributions to problem solving publicly visible and accountable.

Conner's Teacher Support for Collective Argumentation (TSCA) framework conceptualizes teacher involvement in classroom argumentation through three forms of support: direct contribution to

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argument components, targeted questioning that elicits or probes student reasoning, and supportive actions such as revoicing or managing the flow of discussion [3]. This categorization enables researchers to examine not only how teachers scaffold students' reasoning during interaction, but also how instructional decisions influence whether students maintain ownership of arguments or shift it back to the teacher.

Beyond functioning as a communication strategy, TSCA also serves as an analytical tool for examining how teachers manage students' reasoning during classroom interactions. Central to TSCA analysis is the warrant, understood as the logical link that connects data to claims articulated in discussion [4]. Focusing on warrants allows researchers to examine whether students' conclusions are supported by explicit reasoning or remain implicit and unexamined. From this perspective, teacher actions involve continuous instructional decisions about when to respond, provide clarification, remain silent, or prompt students to articulate and refine their own reasoning [5]. These decisions shape the visibility and quality of reasoning within classroom discourse. The overarching aim of such actions is to preserve students' ownership of their arguments while gradually supporting the development of mathematically valid, well-structured reasoning.

Research on mathematical argumentation requires structural analytic tools capable of capturing both the form and certainty of reasoning, such as the Toulmin model with its six components: data, claim, warrant, backing, qualifier, and rebuttal. Applying the Toulmin scheme in its entirety allows researchers to examine the relationship between warrant types and levels of certainty expressed in claims [6]. These components align closely with students' progression towards formal proof, particularly through the use of qualifiers and rebuttals that regulate the strength and scope of the conclusion. Prior studies have shown that mismatches between warrant types (deductive and non-deductive) and levels of certainty weaken arguments and contribute to conceptual misunderstandings [7]. Accordingly, mathematics instructions should not aim for premature certainty but should support students in coordinating warrants, qualifiers, and rebuttals to reflect appropriate epistemic status. The full Toulmin model is therefore essential for capturing classroom argumentation, which often involves empirical and transitional forms of reasoning rather than purely deductive proof [8].

The TSCA framework and Toulmin model offer complementary analytic perspectives. TSCA focuses on how teachers facilitate the emergence of argument components and how responsibility for these components is distributed, whereas the Toulmin model enables evaluation of argument quality, particularly the relationships among warrants, qualifiers, and rebuttals [9]. In complex discussions, claims may generate sub-arguments, requiring components such as data or warrants to shift function. Accurate mapping of these shifts is critical, as a previously accepted claim may become the object of further justification. Together, TSCA and the Toulmin model provide a systematic means of examining both teacher support and the evolving ownership of reasoning in collective argumentation [10].

At the international level, TSCA identifies a range of teacher talk moves, including requests for elaboration and justification, encouragement of conjecture, revoicing, summarization, and the use of visual representations [11]. These actions function to elicit and sustain student reasoning, particularly the articulation of warrants within classroom discourse. When aligned with Toulmin components, such talk not only sustains discussion but also directs attention to the structural relationship among data, claims, and warrants that underpin mathematical validity [12].

From a developmental perspective, argumentation must also be understood in relation to how students' conceptual understanding evolves. In geometry learning at the junior high school level, students' movement from visual reasoning to theoretical justification is inherently non-linear and involves continual interaction between concrete examples and generalization. The Pirie-Kieren theory conceptualizes this learning process as movement across layers of understanding, ranging from primitive knowing and image-making to property noticing and formalizing, with fold-back occurring when learners return to earlier layers to resolve cognitive obstacles or clarify meanings [13]. Within collective argumentation, moments of uncertainty or disagreement frequently emerge during these transitions. Such moments can be traced through Toulmin components such as qualifiers and rebuttals, which signal limits of certainty, invite further justification, and support the reorganization of reasoning toward greater coherence and mathematical formality [14].

There is a clear theoretical need to integrate TSCA, the Toulmin model, and the Pirie-Kieren framework within studies of classroom argumentation. Existing TSCA research has primarily focused on mapping forms of teacher support, with limited attention to the internal quality of students' argument structure. Conversely, studies employing the Toulmin model in mathematics education frequently rely on truncated schemes that overlook the interactions between warrants, qualifiers, and rebuttals [15]. Meanwhile, applications of the Pirie-Kieren theory have largely emphasized individual trajectories of understanding, with minimal consideration of how these trajectories unfold through collective argumentation. As a result, critical indicators such as mismatches between warrant types and levels of uncertainty remain underexplored, despite their pedagogical importance for guiding teacher intervention and supporting the transition from empirical generalizations to formal justification [16].

Recent studies on the TSCA framework suggest that teacher support through revoicing, requesting elaboration, and orchestrating discussions can distribute ownership of reasoning during the construction of Toulmin model-based argument structures [17]. However, few studies have examined how such support operates simultaneously at the level of argument structure and conceptual development. In the context of parallelogram learning, integrating TSCA with the full Toulmin model enables systematic identification of statements functioning as data, claims, or warrants, as well as the examination of how qualifiers and rebuttals strengthen mathematical validity beyond mere visual recognition. Viewed through the Pirie-Kieren framework, this collective negotiation of arguments reflects a process of conceptual refinement from primitive knowing to formalizing, in which teacher intervention mediates shifts between layers of understanding through socially constructed argumentation [18].

Accordingly, this study addresses three research questions: (1) How does the types and distribution of teacher support influence students' collective argumentation during learning about parallelograms?; (2) how does teacher support relate to the structure of student arguments, particularly in the use of warrants, qualifiers, and rebuttals during class discussions?; and (3) what role does teacher support play in facilitating students' movement from visual intuition to formal justification of the concept of parallelograms, as described by the Pirie-Kieren framework? These questions are designed to capture both the interactional dynamics of classroom discourse and the structural quality of students' reasoning as it develops over time. Theoretically, this study contributes by integrating the Teacher Support for Collective Argumentation (TSCA) framework with the full Toulmin model and the theory of understanding development, enabling a fine-grained analysis of how argumentation and conceptual growth intersect within collective classroom activity. This integration makes the development of reasoning visible not only as an individual cognitive process but also as a socially mediated practice shaped by teacher intervention. Practically, the findings offer guidance for designing teacher talk moves that align with specific argument components, particularly those that support the public articulation and refinement of warrants. By foregrounding the role of qualifiers and rebuttals, the study highlights instructional strategies for managing uncertainty productively, thereby supporting students' progression toward more coherent and formally grounded mathematical reasoning.

2. Methods

This study draws on three theoretical concepts that are applied consistently throughout the analysis: (1) the full Toulmin argumentation model, used to map the structure of mathematical arguments; (2) the Teacher Support for Collective Argumentation (TSCA) framework, employed to identify and assess variation in teacher support during collective argumentation; and (3) the trajectory of understanding development that emphasizes the progression from intuitive reasoning to formal justification through social negotiation of meaning in classroom interaction. The Toulmin Model is applied comprehensively, encompassing data (D); claim (C); warrant (W), backing (B), qualifier (Q), and rebuttal (R). Within this framework, data function as evidence, claims represent conclusions advanced in discussion, and warrants serve as logical bridges connecting data to claims, supported by appropriate backing. Warrants may be in the form of rules, definitions, or analogies. Qualifiers indicate the degree of certainty attached to claims, while rebuttals specify conditions under which claims may not hold. In contrast to the common practice in mathematics education research of employing truncated Toulmin schemes, the use of the full model in this study enables a more nuanced representation of classroom argumentation, including forms of non-deductive reasoning. This comprehensive approach foregrounds the management of uncertainty as a crucial stage in students' progression toward formal mathematical proof.

A central focus of employing the full Toulmin scheme in this study is the compatibility between warrants and qualifiers. Inglis, Mejia-Ramos, and Simpson identify two recurrent patterns of incompatibility that signal immaturity in argument structure: the use of absolute qualifiers with non-deductive warrants and the use of tentative qualifiers with deductive warrants [19]. Such mismatches reveal a gap between formal proof and the students' epistemic beliefs about the certainty of mathematical knowledge, a gap that requires instructional mediation. Accordingly, the instructional goal is not to eliminate non-deductive reasoning, but to support students in coordinating warrant types with appropriate levels of certainty expressed through qualifiers [20]. This focus is particularly important because inaccuracies in epistemic judgement may persist even when deductive proofs are formally complete, as evidenced by students' continued reservations about generalizing from proven statements.

The TSCA framework is used to analyze the dynamics of classroom interactions by identifying contributions from teachers, students, or both in the construction of a collective argument [21]. Within TSCA, teacher support is categorized into three forms: direct contributions to the argument components, questioning strategies that prompt elaboration and evaluation of student reasoning, and supportive actions such as revoicing and the use of visual representations. Through this analytic modeling, TSCA enables the identification of implicit argument components and the examination of how ownership of the reasoning is distributed during classroom discourse. This approach allows researchers to examine how student autonomy can be sustained while ensuring that collective argumentation remains oriented toward mathematically appropriate goals.

The theory of understanding development in mathematics learning provides a dynamic perspective for interpreting collective argumentation as a social mechanism through which students' reasoning is externalized and negotiated [22]. As students move from perception-based recognition toward noticing properties and constructing formal justifications, classroom interactions become a key space for negotiating the components of mathematical arguments. Within the TSCA framework, teachers employ a range of talk moves, including requests for justification, revoicing, summarizing, and the use of visual representations, to direct students' attention to warrants rather than to the conclusion alone [23]. Through prompts such as "Does this always apply?" or "Are there any exceptions?", teachers mediate the emergence of qualifiers and rebuttals that articulate the boundaries of certainty, often through the examination and testing of examples.

The integration of the full Toulmin model and TSCA framework provides a comprehensive theoretical platform for analyzing and supporting the quality of argumentation in the classroom. While the Toulmin model enables detailed analysis of the argument and sub-argument's structure, TSCA offers an analytic account of the teacher's role in initiating, shaping, and sustaining the emergence of these argumentative structures during interaction [24]. By combining these two frameworks, the study can trace how students' reasoning about parallelograms develops within classroom discourse, particularly as arguments evolve from intuitive responses to qualified claims open to testing and evaluation. This integrated approach allows cognitive development to be examined in relation to socially mediated instructional support, consistent with norms of mathematical reasoning and justification.

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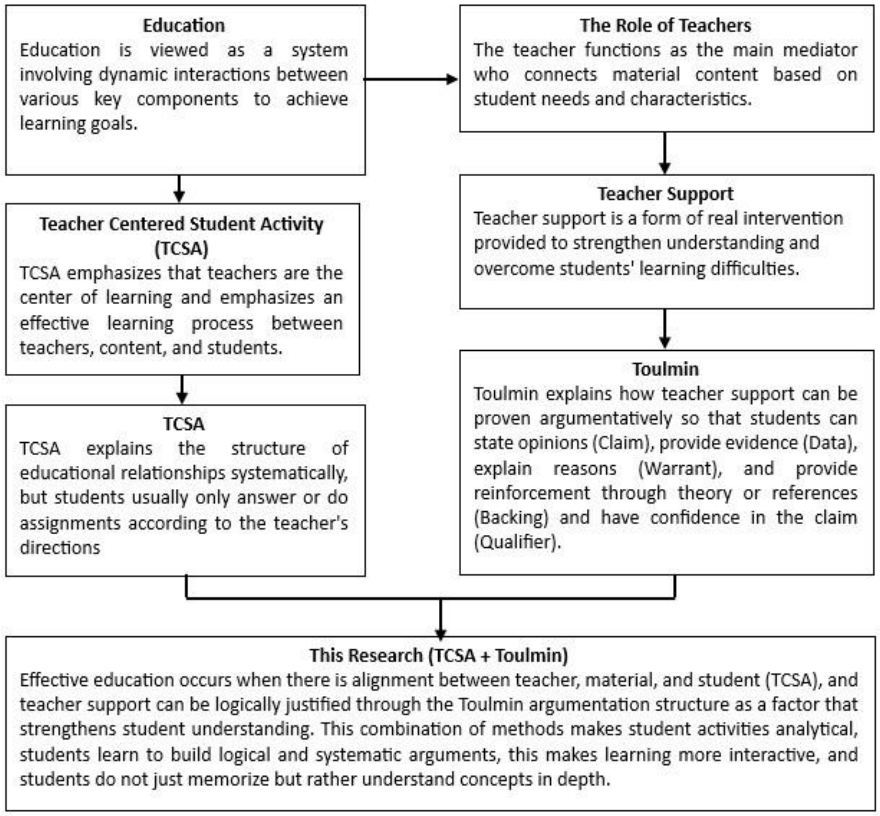

Figure 1.
Parallel Concept Toulmin's Model and Pirie-Kieren Theory.

This study employs a qualitative descriptive research design to examine instances of naturally occurring argumentation in the eighth-grade geometry learning at a *madrasah tsanawiyah*. The design is intended to capture how teacher support operates within collective argumentation processes and how students' argument structures develop through classroom interaction. The research participants were selected purposively and included one mathematics teacher and three students who demonstrated active verbal participation and conceptual engagement during discussion. This selection was intended to capture variation in the authentic reasoning contributions and interactional dynamic characteristics of junior high school mathematics classrooms.

This study examines geometry learning focused on parallelograms, including classification tasks, the construction of hierarchical relationships among quadrilaterals, and the development of formal proofs concerning properties of opposite sides. Primary data were collected through audiovisual recordings of whole-class discussions and small-group interactions across three instructional sessions. These data were supplemented by student artifacts, including worksheets and reasoning charts produced during classroom activities. The researchers developed an observation protocol and analytic framework before data collection and served as non-participant observers throughout the instructional process to document classroom interaction while preserving the authenticity of teaching and learning practices.

Data analysis proceeded through three interrelated steps. Data condensation began with the identification of segments of classroom discourse in which mathematical claims were constructed, evaluated, or revised. These segments were then organized into episodes of argumentation based on topical coherence and continuity of verbal interaction. During the data display stage, each episode was analyzed using two complementary coding schemes. First, a complete Toulmin analytic framework, comprising claim, data, warrant, backing, qualifier, and rebuttal, was applied to examine the structure and mobility of argument components. Second, the TSCA framework was used to code forms of teacher support, including direct contributions, questioning strategies, and supportive actions such as revoicing, summarizing, and visual representation. The two coding schemes were temporally synchronized to examine the relationship between teacher support and the emergence and transformation of components of students' arguments over time.

During the interpretation stage, data were triangulated by cross-referencing annotated transcripts, Toulmin diagrams, TSCA coding maps, and student artifacts to enhance the consistency and credibility of analytic interpretations. A hybrid coding strategy was employed, combining *a priori* codes derived from the literature with inductive codes developed from recurrent interactional patterns observed in the data. To enhance analytic reliability, portions of the data were independently coded and compared, with discrepancies resolved through discussion. Trustworthiness was further supported by maintaining a systematic audit trail, peer debriefing, and the use of thick descriptions to document the relationship between evidence and interpretation. Ethical considerations were addressed through informed consent, anonymization of participant identities, and non-intrusive observation procedures to preserve the authenticity of the instructional process.

The research questions were operationalized through a systematic analytic procedure. First, instances of teacher support were identified and mapped with respect to their frequency, variation, and temporal positioning within discussion turns. Second, the relationship between students' argument structures and the presence of Toulmin argumentation components was examined, with particular attention to how these elements co-occurred within classroom interaction. This analysis was conducted in relation to teacher interventions embedded within unfolding argumentation sequences. Finally, students' conceptual progress was evaluated by examining shifts toward property noticing and formalization, as reflected in the use and coordination of warrants, qualifiers, and rebuttals across argumentation episodes. This multi-stage analytic process was designed to ensure a high level of transparency, thereby supporting analytic rigor and the study's replicability.

3. Results and Discussion

The Teacher Support for Collective Argumentation (TSCA) model indicates that teacher support enacted through revoicing, requests for elaboration, and orchestration of classroom discussions plays a central role in the construction of students' 'ownership of reasoning' and the development of argument structures align with the full Toulmin model. In the context of learning about parallelograms, integrating TSCA with Toulmin analysis allows students to identify their statements as either data or claims, and to examine the roles of warrants, qualifiers, and rebuttals in strengthening mathematical validity beyond visual recognition alone. Viewed through the Pirie-Kieren framework, this process of collective argumentation reflects students' trajectories of understanding from primitive knowing toward formalizing, in which strategic teacher intervention supports transitions between layers of understanding while remaining consistent with the dynamics of socially constructed argumentation in the classroom.

3.1. Typology of Teacher Support Through the Integration of Probing Questions and Supportive Actions

The research data show that teachers consistently used probing questions to direct students' attention toward relevant mathematical objects and to reduce terminological ambiguity. For example, when students responded to the question "What is the length of the sides of a parallelogram?" by referring to the diagonals, the teacher immediately redirected the discussion by stating: "The question is about the length of the sides, not the diagonals. Aren't they the same?" This intervention served a dual purpose: clarifying mathematical terminology and distinguishing between data and claims within the emerging argument structure, specifically whether the observation concerned side length or diagonal

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length. A similar pattern was observed when students equated "fold symmetry" with parallelograms. Rather than correcting the statement directly, the teacher asked, "What is fold symmetry?" This prompted students to articulate a working definition and examine the consistency of their claim. In both cases, teacher interventions clarified definitions while simultaneously using the structure of argumentation to organize facts and claims. From the TSCA perspective, such interventions are classified as supportive actions and probing questions that strategically redirect discussion, ensuring that emerging claims are grounded in mathematically sound reasoning.

During the phase of classifying the relationship among quadrilaterals, the teacher employed revoicing and visual representation to redirect the discussion toward "the nature that determines inclusion." When students stated that rectangles belong to the set of parallelograms and subsequently questioned this claim based on the "equal length" of the diagonals, the teacher initiated a sequence of probing questions that prompted students to consider whether the characteristic was relevant to the membership claim and whether it constituted a valid refutation. As the data indicate, students' interactions around the competing statements "the diagonals are equal in length" and "they are not equal in length" reflect an evaluation of the argumentative language mediated through revoicing. By instructing students to "use proper sentences, because it is not a parallelogram," the teacher explicitly reframed the argument in the form "X is not a parallelogram because property Y is not fulfilled." This intervention made explicit the need for a clear, publicly accountable connection between data and claims within the classroom argumentation process.

Additional forms of supportive action were observed through the instructional strategy "look at properties, if there are likenesses, then include them," which emerged during students' work on constructing hierarchical relationships among quadrilaterals using the metaphor of "siblings". The teacher responded to this metaphor by adapting it and subsequently translating it into more formal mathematical criteria for set inclusion. Classroom artifacts and instructional prompts, such as "Look at the properties; if there are similarities, include them" and the invitation to "create arguments like concept maps," indicate deliberate orchestration toward more explicit visual and structural representations. Through these representations, relationships among claims, data, and warrants could be mapped and examined. This pattern of support aligns with the TSCA principle that effective teacher intervention involves reorganizing the medium and flow of contributions to make the components of the argument explicit, rather than directly supplying connecting reasons.

Teachers' direct contributions to the argument components appeared selectively, particularly when discussions reached an impasse due to terminological confusion or difficulties in linking mathematical properties. In a whole-class discussion, the teacher guided students to identify diagonals AC and BD and their intersection point O, followed by a sequence of probing questions that led to the formulation "*the diagonals intersect and divide in half*". At this point, the teacher consolidated the argument by articulating the appropriate warrant-claim relationship as a shared reference for subsequent reasoning. From the perspective of the TSCA framework, such provision of argument components is considered didactically appropriate when it serves to establish a valid reasoning bridge that the class can collectively adopt. Notably, this direct intervention occurred sparingly in comparison with probing and clarifying questions, reflecting adherence to the TSCA principle of maintaining students' reasoning ownership whenever possible.

3.2. Analysis of Students' Argument Structure through Data-Claim Matching and Component Mobility

Mapping classroom discourse using the Toulmin scheme indicates that claim (C) concerning the defining properties of parallelograms is initially grounded in empirical data (D), while warrants (W) are often implicit or imprecise. During the early exploration phase, students articulated the claim "*two pairs of parallel/equal sides*" simultaneously, yet supported it with data that did not clearly distinguish between the properties of sides and diagonals. Teacher intervention, prompted by the question "that means they are the same, right?", encouraged students to reconsider their reasoning, leading to the revised response "*they are not the same*". This shift improved the coherence of the *data-claim* relationship within the

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Toulmin structure (D to C) and facilitated a more accurate articulation of the warrant linking side parallelism and equality to the defining characteristics of parallelograms.

A recurring instructional pattern was observed in addressing incorrect claims about the nature of diagonals, such as the assertion, "two diagonals intersect at equal lengths." Students' responses of "just measure it" followed by comparisons of 4cm vs 2.5cm measurements illustrate reliance on perceptual evidence to support an inadequate warrant. In this situation, the teacher deliberately refrained from directly correcting the misconceived warrant and instead redirected attention to the distinction between objects and their properties by stating, "The question is about the length of the sides, and not the diagonals." This intervention was intended to stimulate the emergence of independent warrants through collective negotiation of meaning. This pattern is further corroborated by student artifacts, such as the statement "A quadrilateral with diagonals that intersect perpendicularly is not a parallelogram because a parallelogram has diagonals of equal length". This statement reflects a generalization grounded primarily in visual data and an inaccurate warrant, underscoring the urgency for instructional tasks that explicitly challenge and refine students' reasoning bridges.

Structurally, the mobility of argument components can be observed in shifts of a question's function from primary *claims* to claims within sub-arguments. For example, once the classroom community accepts the statement "rectangles are parallelograms" as a claim, justifying this statement requires the construction of a sub-argument that links specific data about rectangles to a more general warrant, such as "if the sides of a quadrilateral are parallel, then it is a parallelogram." From a TSCA perspective, this process is a normal feature of instructional argumentation, in which any claim that is questioned becomes the focus of a subordinate argument that requires renewed argumentation. In this context, the use of integrated TSCA/Toulmin representations plays an important role in identifying how argument components are redistributed across levels of argumentation and in tracing who contributes which components at different stages of the discussion.

The adoption of the full Toulmin scheme in this study is grounded in the assumption that including qualifiers (Q) and rebuttals (R) enables a more precise categorization of the quality of mathematical arguments. Consistent with the perspectives of Inglis, Mejia Ramos, and Simpson, the instructional goal is not to eliminate non-deductive warrants but to support students in coordinating them with appropriate qualifiers, while inviting rebuttals that help reduce uncertainty on the path toward formal proof. In classroom interaction, qualifiers often emerged implicitly through student practices, such as examining multiple visual representations before drawing conclusions. For example, students' repeated measurement of diagonals across different figures, followed by tentative statements accompanied by questions such as "*what if they are the same*," reflects the use of provisional qualifiers and the emergence of potential rebuttals. These observations support the view advanced by Inglis et al. that the dynamics of mathematical reasoning cannot be adequately captured when qualifiers and rebuttals are excluded from analytic consideration.

The quality of a mathematical argument can be examined through instances of mismatch between warrants and qualifiers, as documented by Inglis, et al. [19]. Pairing an inductive warrant with an absolute qualifier constitutes a problematic configuration, as it prematurely forecloses the space for refutation. Conversely, a mismatch also occurs when a deductive warrant is paired with a tentative qualifier, as illustrated by expressions such as "*does this apply to large numbers?*", which weakens the epistemic status of the argument without sufficient justification. Together, these patterns indicate that effective coordination between warrants and qualifiers should be treated as an explicit instructional competence in mathematical argumentation, extending beyond the mere presentation of formally valid evidence.

Through the Pirie-Kieren framework, students' trajectories of understanding progress from primitive knowing grounded in perceptual experience to property noticing, as students begin to name, organize, and summarize properties into a coherent list. This progression may extend to the formalizing stage, when the class can articulate generalized reasoning in the form of a warrant. Traces of primitive knowing are evident in repeated utterances such as "*just measure it*" and in actions such as comparing

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diagonal lengths ("4cm vs 2.5cm"), which function as perceptual backing for initial assumptions. The transition to property noticing is observed when students generate lists of properties, including "two pairs of sides are equal in length and parallel; two pairs of opposite angles are equal in size; two diagonals intersect and are equal in length, with no fold symmetry," despite persistent inaccuracies related to diagonal properties. During this phase, the teacher strategically refrained from direct correction and instead employed differential questioning and revoicing to support students in clarifying and refining their concepts independently.

At the formalizing stage, teachers guided students beyond merely naming properties toward activities that required them to demonstrate why those properties hold. Artifact data reveal an explicit instructional emphasis through prompts such as "*show how*" regarding diagonal properties. At the same time, in the whole-class discussion, the teacher modeled a representation that highlighted the diagonal intersection point as a stimulus for the emergence of the warrant "*dividing in half*." This stage marks a transition from reliance on visual information to the articulation of reasons that must be examined and validated by the class community through testing. Importantly, this progression of understanding is non-linear. Fold-back moments occurred when terminological constraints prompted students to return to earlier stages of observation or measurement to re-examine assertions, such as those concerning "equal length diagonals," before eventually advancing toward more stable and valid generalizations.

In this phase, the pedagogical roles of qualifier and rebuttal become particularly salient. When the teacher asks, "Aren't they all the same, all four of them?", the question externalizes a qualifier by prompting students to consider the scope and conditions under which a statement holds. Conversely, questions such as "Are the differences too great?" prompt students to consider deeper visual similarities when determining set inclusion, thereby encouraging rebuttals. The consistent presence of both types of questions across classroom transcripts and student artifacts indicates that the management of uncertainty constitutes an integral component of mathematical reasoning in the classroom, rather than extraneous 'noise' to be eliminated at an early stage of learning.

A notable finding in the data concerns students' attempts to construct genealogical relationships among quadrilaterals through metaphors such as "brother", "descendant", and analogies such as "sheep-goat". Although these metaphors initially appeared to shift the discussion away from formal mathematical registers, teachers strategically appropriated them as bridges to property-based classification criteria. Artifact analysis reveals student statements such as "rectangles are parallelograms; a rhombus is not a parallelogram; a square is not a parallelogram" and tentative justifications such as "because the sides are of equal length" or "because they have different properties." These claims were subsequently unpacked by the teachers through probing questions such as "are there too many differences?" and "look at the properties," which functioned as formal filters for inclusion. When students encountered cognitive resistance, for example, rejecting rhombi as parallelograms on the grounds that "all sides are equal in length", the teacher refrained from providing direct answers and instead scaffolded students' identification of determining properties. Consistent with TSCA principles, this approach sustained students' ownership argumentation and supported their eventual self-construction of inclusion and exclusion criteria.

3.3. Focus on Argument Maturity and the Role of Teachers

The use of qualifier and rebuttal components, informed by the perspectives of Inglis, Mejia-Ramos, and Simpson, enables a more comprehensive assessment of students' mathematical arguments' maturity. Students who rely on visual intuition are therefore not positioned as making absolute errors, but as operating within an earlier epistemic stage of reasoning. From an instructional perspective, a more productive approach is to guide students to pair inductive warrants with appropriate qualifiers, such as "*possibility*," "*in that picture*," or "*if the lines...*", thereby marking the conditional scope of their claims. This process includes testing potential rebuttals through instructional counterexamples, such as diagrams of a parallelogram with visually similar diagonals. In this way, learning frames the reduction of uncertainty as an essential practice on the path towards proof, rather than treating uncertainty as an epistemic deficiency. This is in line with the main principle that the goal of learning is not to erase traces of mom-deductive reasoning, but to ensure that it is properly qualified.

The TSCA framework enables close examination of whether the teacher's orchestration effectively supports the emergence of argumentative components from students' contributions. For instance, when the teacher poses the question "*what is fold symmetry*?", students are prompted to provide conceptual backing in the form of operational definitions rather than isolated examples. Similarly, when the teacher emphasizes that "*the question is about the sides, not the diagonals*," students are directed to re-examine the data relevant to the claim as a prerequisite for establishing an accurate warrant linking data (D) to claim (C). during whole-class discussion, the teacher's formulation "*diagonals divide each other in half*" functioned to stabilize the collective argument by anchoring it in a key defining properties of a parallelograms. Through this integration of TSCA and the Toulmin model, the significance and proportionality of the teacher's actions in shaping the dynamics of classroom argumentation become analytically visible.

3.4. Didactic Implications of Designing Support that Fosters Understanding

These findings point to two fundamental implications for mathematics education. Pedagogically, the dominance of teacher support enacted through probing questions and supportive actions was effective in sustaining students' ownership of argumentation while simultaneously increasing the rigor of classroom discourse. This pattern aligns with the TSCA principle that effective teacher support involves not merely stimulating discussion, but attentively tracking who contributes to the argument and how non-contributory actions prompt the emergence of subsequent argument components. Epistemically, the full application of the Toulmin model offers heightened analytic sensitivity to argument quality, particularly in evaluating the degree of certainty associated with claims, examining limiting conditions, and ensuring alignment between warrants and the epistemic status of claims. Accordingly, the coordination between warrants and qualifiers should be explicitly framed as an instructional focus and systematically embedded in learning materials (LKPD) and feedback rubrics, rather than remaining an implicit expectation within discussion-oriented pedagogy.

In practice, teachers can develop a repertoire of instructional questions explicitly aligned with Toulmin components. For example, questions such as "Which part is the data, which part is the conclusion?" support differentiation between data (D) and claims (C), while "What reasons connect the data to your conclusion?" activates warrants (W). Questions such as "Is your conclusion always true or only under certain conditions?" serve to externalize qualifiers (Q), and "Are there any examples that invalidate your statement?" invites rebuttals (R). in addition, revoicing strategies that reformulate students' statements into the Toulmin-consistent structures, together with visual representation highlighting features such as the midpoint of the diagonals, help direct attention toward valid warrant candidates (e.g., "half and half") rather than misleading visual cues (e.g., "looks the same length"). The data indicate that such forms of teacher support were already present in classroom interaction, although occasional direct intervention was necessary to reorient argumentation toward mathematically relevant properties. From a TSCA perspective, the proportional use of these support types can be systematically monitored and evaluated over time.

The limitations of this study relate primarily to its scope, which is confined to a single classroom, a single mathematical topic, and a small number of focal participants; therefore, the findings should be generalized with caution. Nevertheless, the choice of parallelograms as the instructional focus constitutes a critical case for examining the integration of TSCA and the full Toulmin model, as this topic requires learners to move from visual observation to property identification and from measurement-based reasoning to formal justification. Methodologically, the decision to infer implicit argument components is consistent with the TSCA framework, which acknowledges that certain components of argumentation must be reconstructed from interactional context and represented analytically through diagrams. Future research could build on these findings by designing contrastive instructional tasks to challenge erroneous visual warrants related to diagonal properties, and by incorporating warrant-qualifier alignment indicators into assessment rubrics to provide teachers with shared evaluative language for assessing argument maturity.

In conclusion, this study reaffirms that collective argumentation functions as a socio-epistemic trajectory toward formal mathematical proof. Drawing on Krummheuer's perspective within the TSCA framework, a distinction can be made between purely deductive analytical arguments and substantial arguments, in which claims are constructed through the presentation of background information, responses to others' ideas, discussions of qualifiers, and the articulation of defenses. Both forms of argumentation hold pedagogical value in mathematics learning, with substantive arguments providing a critical foundation for the development of analytical proofs. The research data illustrate how this foundation is established through strategic questioning, precise revoicing, and the focused use of visual representations, enabling the classroom community to reduce uncertainty collectively. These findings support the recommendation that substantive arguments should not be marginalized, but rather systematically integrated as a foundation for proof by carefully managing the relationship among warrants, qualifiers, and rebuttals throughout the learning process.

Overall, the three strands of findings are mutually reinforcing. Teachers' probing questions and supportive actions sustained collective attention on relevant mathematical objects and precise language; Toulmin-based mapping argument structure made visible the compatibility or incompatibility between warrants and the epistemic status of claims; and shifts in students' layer of understanding emerged when the classroom community collectively articulated reasons that extended beyond visual representations, specified conditions and limits, and invited potential nullifying cases. Through this process, claims were progressively refined and epistemically strengthened. The integration of TSCA and the Toulmin model thus renders the dynamic of classroom argumentation not only observable but also systematically documented and deliberately re-teachable within mathematics instruction.

4. Conclusion

This study demonstrates that teacher support, as conceptualized through the Teacher Support for Collective Argumentation (TSCA) framework, particularly through probing questions and supportive actions such as revoicing and the strategic use of representations, fosters students' conceptual understanding of parallelograms. Integrating TSCA with the full Toulmin model reveals that instructional strategies that stimulate discussion, rather than supply direct answers, effectively shift ownership of reasoning to students, enabling warrants as bridges of reasoning to emerge through social interaction in the classroom. Viewed the Pirie–Kieren framework, appropriately calibrated teacher support facilitates students' progression from primitive knowing to property noticing and ultimately to formalizing. This progression supports the systematic management of uncertainty through productive fold-back, as the students revisit empirical instances to clarify meanings before constructing more robust and coherent mathematical arguments.

In practical terms, this study suggests that the organization's geometric arguments should be explicitly considered when mapping students' reasoning onto the elements of Toulmin's model of argumentation: data (D), claim (C), warrant (W), qualifier (Q), and rebuttal (R). Supportive instructional actions, such as visualizing the intersection point of diagonals, were shown to focus students' attention on valid warrant candidates while reducing reliance on misleading visual warrants. At a theoretical level, the primary contribution of this study lies in the full integration of TSCA and the Toulmin model as an analytic apparatus for rendering the development of mathematical reasoning observable and analyzable. By foregrounding the relationships between warrants and qualifiers, this study positions argumentation as a legitimate and necessary precursor to proof in junior high school mathematics learning.

Although the scope of this study is limited to a single classroom and a single geometric topic, it provides a foundation for replication across other areas of geometry and with a different student population. Future research is encouraged to incorporate contrastive instructional tasks designed to test the strength of warrant-qualifier coordination and to examine the medium-term effects of such

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interventions on students' autonomy in mathematical reasoning. Overall, this study verifies that balanced teacher intervention constitutes an active didactic practice rather than merely a piecemeal attempt to enliven classroom discussion. By establishing instructional conditions in which reasons are made public, appropriately qualified, and systematically tested, classroom communities can progress from tentative assumptions toward mathematically sound reasoning while retaining collective ownership of the argumentation process.

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