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An alternative fuzzy time series forecast model combined generalized fuzzy logical relationship, natural partitioning, and adaptive defuzzification

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Abstract: This paper, we propose generalized fuzzy logical relationships based on natural partitioning and adaptive defuzzification. The proposed method provides a better approach to improve performance by producing a good evaluation of the forecasted value. This study aims to minimize forecasting errors for each data series. The general suitability of the proposed model was tested by implementing it in the fore- casting of student enrollments at the University of Alabama. In order to show the superiority of the proposed model over existing methods, the results obtained have been compared with evaluations such as MSE, RMSE, MAE, MAPE, and forecasting error errors for each time series data. Comparative studies show that the proposed method is superior to existing methods for all evaluations provided..
Keywords: *Adaptive defuzzification, Forecasting, Fuzzy time series, Generalized fuzzy logical relationships, Natural partitioning;*

1. Introduction

Forecasting continuously sequenced data that changes over time or time series is an important and exciting problem in various applications, such as predicting stock prices in the stock market, monitoring weather or air pollution in environmental protection, and estimating the number of student enrollments in universities. These problems are frequent. This problem has been extensively studied and studied comprehensively in the fields of statistics, signal processing and neural networks in the last decade.

Unclear and incomplete data phenomena make forecasting problems difficult to solve. [1], [2] introduced fuzzy time series to deal with the un- certainty of the data by representing the data as linguistic values in an uncertain environment. They predict the fuzzy time series for enrollment at the University of Albama with 4 procedures: (1) partitioning the universe of discourse into intervals of equal length. (2) define the universe of discourse, fuzzify the time series and model the fuzzy relations on each time series data. (3) forecasting (4) defuzzification of fore- casted output. Modeling fuzzy equation equations on each data series and reasoning estimates require a large processing time in fuzzy relations.

Since launching Song and Chissom's method, many researchers have competed to reduce forecasting errors and computational burden. Standard matrix multiplication operations on the Markov model [3], Simplified arithmetic operations in the forecasting process[4]. High-order fuzzy time series models [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19], [20], [21], [22], [23]. A heuristic approach for forecasting fuzzy time series[24], [25], [26], [27], [28].

The definition of a universe of discourses with arbitrarily selected parameters and was decomposed into the same interval length in step (1) greatly influences forecasting performance significantly[24]. The partitioning approach of the universe of discourse has been widely proposed. Interval length based on partition density [10], [29], [30], [31], [32], [33], [34], Automatic clustering techniques [8], [35], [36], [37], [38, p. 201], [39], [40], K-Means Clustering [41], [42], [43], [44]interval

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ratio[24], [45], [46], [46], [47]. Low forecasting error has been demonstrated through all partitioning approaches of the universe of discourse. All of these approaches identify a very large number of intervals. The more intervals identified, the smaller the forecast error achieved. However, too many intervals could result in fewer fluctuations in the fuzzy time series and complicate defuzzification. [48] proposed natural Partitioning (NP) to tackle the issue of interval length efficiently. The 3-4-5 natural partition rule based on the value range at the most significant digit (MSD) is applied to divide the universe of discourse naturally. NP produces small intervals compared to partition density, automatic clustering, K-Means Clustering, and interval ratio (University of Alabama enrollment data experiment 10 interval values for NP and 18 to 24 for other approaches). This paper used NP for partitioning the universe of discourses.

The formation of FLR by optimizing the intervals segmentation on the partition of a universe of discourse. FLR is one of the most critical factors affecting forecasting accuracy in fuzzy time series [31]. Generalized FLR or GTS (M, N) with M is a number of orders, and N is the hierarchies of principal fuzzy relationship applied to data forecasting at the University of Alabama, resulting in performance that GTS (M, N) is better than three conventional fuzzy time series models [37], [49]. In addition, forecasting with defuzzification. [50]proposed a Weighting Fuzzy Time Series (WFTS) based on chronological order in the fuzzy logical group to deal with repeated FLR problems in defuzzification.[51] adopted WFTS for enrollment forecasting resulting in competitive errors. Defuzzification of GTS (M, N) with WFTS for enrollment forecasting produces a small error. However, the forecasting results for each data series still found high errors. There exists defuzzification of fuzzy time series data is inappropriate when using WFTS. Therefore, this paper proposes an adaptive rule based on a minimum distance as an additional defuzzification rule

The proposed method selects the shortest distance between the actual data and the forecasting value based on the FLR. Adaptive rules based on a minimum distance can handle high error issues in each fuzzy time series defuzzification, especially the enrollment data for the University of Alabama, which is the benchmark for testing the fuzzy forecasting model.

2. Brief of Some Concepts

2.1. Fuzzy Time Series

The basic concept of fuzzy time series (FTS) was introduced by [1], [2], where the values of FTS are represented by fuzzy sets. Let $L = p_1, p_2, p_3, \ldots p_n$ be the universe of discourse. The fuzzy set K_i of L is defined by $K_i = \frac{f_{K_i}(p_1)}{p_1} + \frac{f_{K_1}(p_2)}{p_2} + \ldots + \frac{f_{K_1}(p_n)}{p_n}$ where f_{K_i} is the membership function of fuzzy $f_{(K_i)}(p_i)$ degree of p_i in the fuzzy set K and $1 \leq a \leq n$. Let $Z(t)$ $(t = \ldots, 0, 1, 2, \ldots)$ is the universe of discourse from the predetermined

fuzzy set $f_i(t)$ if $F(t)$ is a set of $f_1(t), f_2(t)$ then $F(t)$ is a fuzzy time series defined by $Z(t)$ [5]. If $F(t)$ is caused only by $F(t-1)$ then the fuzzy logical relation is represented by $F(t) = F(t-1) * R(t, t-1)$ which is a fuzzy relation between $F(t)$ and $F(t-1)$ where is the operator. For simplicity, given $F(t-1) = K_i$ and $F(t) = K_j$ The fuzzy logic relation between $F(t)$ and $F(t-1)$ can be expressed as $K_i \rightarrow K_i$ where K_i is called the Left-Hand Side (LHS) and K_j is called the Right-Hand Side (RHS) of the Fuzzy Logic Relationship (FLR). Furthermore, fuzzy logic relations can be grouped into Fuzzy Logic Relationship Groups (FLRG) to build different fuzzy relations.

Given $F(t)$ fuzzy time series. If $F(t)$ is caused by $F(t-1), F(t-2), \ldots, F(t-m)$, then the fuzzy logic relations are expressed as $F(t-m), \ldots, F(t-2)F(t-1) \rightarrow F(t)$, and is called the m-order fuzzy time series forecasting model. Given $G(t)$ $(t = \ldots, 0, 1, 2, \ldots)$ a fuzzy time series, where a fuzzy set represents the value of $G(t)$ If $G(t)$ is caused by $G(t-1), G(t-2), \ldots, G(t-m)$, then

FLR will be represented as $G(t-m),\ldots,G(t-2),G(t-1)\rightarrow G(t)$ and is called the m-order FLR, $m=1$ and $m\geq 2$ are one order and high order, respectively.

2.2. Natural Partitioning

This section, interval formation will be carried out, where the universe of discourse partition intervals into equal-length sub-intervals uses the concept of a hierarchy based on natural partitioning (NP). This paper uses the 3-4-5 rule of NP, which recursively collects continuous values into uniform, intuitive or natural intervals [48]. The rule is based on the most significant digit (MSD) in an interval; the continuous value is partitioned into 3, 4, or 5 relatively equal sub-intervals. Table 1 informs the high-level to low-level recursively. From this table, partitioning intervals can use the 3 - 4 -5 rule where the need to produce a hierarchical concept in discretization variables. The impact of the interval length on the FTS can be investigated through the 3-4-5 rule.

Table 1.
The 3-4-5 rule of NP [48]

Distinct values of the value range at MSD	Number of intervals segmented
3, 6, 9	3 equiwidth intervals
7	3 intervals in the grouping of 2-3-2
2, 4, 8	4 equiwidth intervals
1, 5, 10	5 equiwidth intervals

2.3. The Higher Order Fuzzy Time Series Based on Generalized Fuzzy Logical Relationship

Higher-order forecasting is based on general fuzzy logical relationships, namely the process of creating a relationship matrix and finding out the fluctuation pattern of a time series based on basic fuzzy rules that are easy to understand. Let L partitioned into n equal intervals of length l_{i_n} then $\mu_{K_i}(t)$ is the degree of membership defined to calculate the weight of fuzzy set K_i by the following equation.

$$\mu_{K_i}(x_t)=\begin{cases}1, i=1\wedge x_t\leq m_1\\ 1, i=n\wedge x_t\leq m_n\\ \max\left\{0,\left(1-\dfrac{|x_i-m_i|}{2\times l_{i_n}}\right)\right\} otherwise\end{cases} \quad (1)$$

at time $t_i(i=1,2,3,\ldots,n)$, where x_t is the time series value at time t with m_1 is middle value at time t_1, m_n is middle value at time t_n and l_{i_n} is interval of length. Suppose $G_K(t-M)=(\mu_1(t-M),\mu_2(t-M),\ldots,\mu_n(t-M))$ and $f_K(t)=(\mu_1(t),\mu_2(t),\ldots,\mu_n(t))$. If $\overline{\mu_i}^{(t-M)}$ and $\overline{\mu}_j^t$ are the maximum values of $(\mu_1(t),\mu_2(t),\ldots,\mu_n(t))$, respectively, then $\overline{K}_i^{(t-M)}\rightarrow\overline{\mu}_j^t$ is called the M-order first principal fuzzy relationship, denoted as GT $S(M,1)$ (generalized fuzzy logical relationships). If $\overline{K}_i^{t-M}$ is the N^{th} maximum value of $(\mu_1(t-M),\mu_2(t-M),\ldots,\mu_n(t-M))$, then $\overline{K}_i^t\rightarrow\overline{K}_j^{t+1}$ is called the relation M^{th} order N^{th} main fuzzy logic, denoted as $GTS(M,N)$ that is grouped into matrix $M\times N$ and expressed as $R^{(k,l)}(k=1,2,\ldots,M;\ l=1,2,\ldots,N)$[49]

2.4. Weighted Fuzzy Time Series

Weighted Fuzzy Time Series (WFTS) is the development of the FTS, which handles the problem of recurring fuzzy logic relations and gives weight to each fuzzy relation to describe the difference in the importance of the sequence of fuzzy relations [50]. The FLR with the same LHS can be grouped into FLRG by placing all of their RHS together as the RHS in FLGR. For example, $A_i\rightarrow A_j, A_i\rightarrow A_k. A_i\rightarrow A_k, A_i\rightarrow A_F$ be grouped $A_i\rightarrow A_j, A_k, A_k, A_p$ The second FLR in which there is an intermediate

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chronological order A_i and $A_j, A_k, \dots, A_p$ Next, the computation to determine the weight is formulated as follows [51]:

$$\begin{aligned} \omega \quad &= [\, \omega_1, \omega_2, \dots, \omega_n \,] \\ &= \left[\frac{j}{j+k+l+m}, \frac{k}{j+k+l+m}, \dots, \frac{p}{j+k+l+m} \right] \\ &= \left[\frac{c_1}{c_1 + c\_2 + c_3 + c_4}, \frac{c_2}{c_1 + c\_2 + c_3 + c_4}, \dots, \frac{c_4}{c_1 + c\_2 + c_3 + c_4} \right] \\ &= \left[\frac{c_1}{\sum_{h=1}^{n} c_h}, \frac{c_2}{\sum_{h=1}^{n} c_h}, \dots, \frac{c_n}{\sum_{h=1}^{n} c_h} \right] \qquad (2) \end{aligned}$$

3. The Proposed Method

Forecasting results on each time series data found a high error. The weighting based on chronological order in the fuzzy logical group to proposed by [50] has yet to provide the best solution for defuzzification. This paper proposes a weighted generalized fuzzy logic relationship based on natural partitioning and adaptive defuzzification on high-order fuzzy time series. The workflow of the proposed method is shown in Figure 1. This paper applied the proposed method to forecasting enrollments at the University of Alabama is as shown in [4]. The following is the proposed method algorithm:

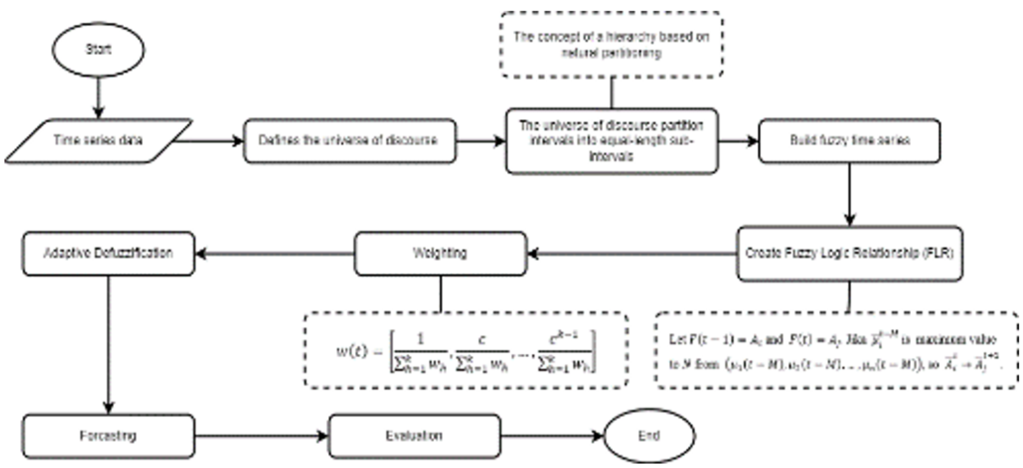

Figure 1.
Workflow of proposed method.

Step 1: Define the universe of discourse and partition intervals. Supposed S_{max} and S_{min} are actual maximum and minimum enrollment data, respectively. Select ε_1 and ε_2 are appropriate positive integer numbers such that $\ell = (S_{max} + \varepsilon_1 - (S_{min} - \varepsilon_1))$ partitioned with the digital position of ℓ MSD. Next, the universe of discourse is defined to be $S = [S_{min} - \varepsilon_1, S_{max} + \varepsilon_2]$. In Table 2, $S_{max} = 19337$, then $\varepsilon_1 = 3055$ and $\varepsilon_2 = 663$. Hence, $S = [10000, 200000]$.

Step 2: the 3-4-5 rule of NP is implemented to S and five equiwidth intervals at the first level are obtained due $to\ (20000 - 10000)\ 10000 = 1$: $[10000, 12000, [12000 - 14000], [14000 - 16000], [16000 - 18000]$, and $[18000 - 20000]$. However, $[10000 - 12000]$ needs to be deleted due to without covering any historical value. In the following, play the NP rule to the remaining four intervals, respectively. For example, $[12000, 14000]$ will be partitioned into four equal subintervals with $(14000 - 12000)\ 10000 = 2$. Thus, a second level interval partition can be obtained as shown in

Table 3. Three Intervals, i.e. $[12000,12500], [12500,13000]$, and $[195000 - 20000]$ are deleted because of there exist no actual data are covered. Thus, the obtaining intervals are

Step 3: Build fuzzy sets. From Table 4, the triangular fuzzy sets is presented.

Table 2.
The interval partition of actual enrollment data using the NP Rule.

The one-level interval partition	The 2nd -level interval partition
[12000, 14000]	[12000, 12500] [12500, 13000] [13000, 13500] [13500, 14000]
[14000, 16000]	[14000, 14500] [14500, 15000] [15000, 15500] [15500, 16000]
[16000, 18000]	[16000, 16500] [16500, 17000] [17000, 17500] [17500, 18000]
[18000, 20000]	[18000, 18500] [18500, 19000] [19000, 19500] [19500, 20000]

Table 3.
Partition of the universe of discourse.

Index	Interval	Index	Interval
s1	[13000,13500]	s8	[16500,17000]
s2	[13500,14000]	s9	[17000,17500]
s3	[14000,14500]	s10	[17500,18000]
s4	[14500,15000]	s11	[18000,18500]
s5	[15000,15500]	s12	[18500,19000]
s6	[15500,16000]	s13	[19000,19500]
s7	[16000,16500]		

From Table 2 and 4, the value of membership degree by Eq.(1). Next, the fuzzification process is carried out by selecting 2 values of the maximum degree of membership (Jilani et al., 2010). In detail, The value of the membership degree and the fuzzification results are shown in Table 5.

Table 4.
Triangular fuzzy sets.

Fuzzy	Interval fuzzy
A1	[13000, 13250, 13500]
A2	[13500, 13750, 14000]
A3	[14000, 14250, 14500]
A4	[14500, 14750, 15000]
A5	[15000, 15250, 15500]
A6	[15500, 15750, 16000]
A7	[16000, 16250, 16500]
A8	[16500, 16750, 17000]
A9	[17000, 17250, 17500]
A10	[17500, 17750, 18000]
A11	[18000, 18250, 18500]
A12	[18500, 18750, 19000]
A13	[19000, 19250, 19500]

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Step 4 Build FLR based on generalized fuzzy logical relationships or $\mathrm{GTS(M,N)}$ with $M = 3$ and $N = 2$. The fuzzy logical relationships of the enrollments as shown in Table 7. Next, create the FLRG for each $\mathrm{GTS(M,N)}$. The groups identified for the enrollments is shown in Table 8.

Step 5: Determine weight by using weight matrix proposed by using Eq. (2). The weight matrix identified for the enrollments is presented in Table 9.

Step 6: Play adaptive defuzzyfication. Let $A_i \rightarrow A_j, A_k, \ldots, A_p$ is FLRG and the corresponding weight for $A_j, A_k, \ldots, A_p$ are $\omega_1, \omega_2, \ldots, \omega_n$. Mid value of $A_j, A_k, \ldots, A_p$ are $m_j, m_k, \ldots, m_p$. The final defuzzification is denoted as the multiplication of the defuzzification matrix and the transpose of the weighting matrix as follows:

$$\varrho(t) = M(t) \times W(t)^T$$
$$= [m_j, m_k, \ldots, m_p] \times [\omega_1, \omega_2, \ldots, \omega_n] \quad (3)$$

Thus, the adaptive defuzzyfication denoted by T that is formulated as follows:

$$T(t) = \min[(m_j - x(t)), (m_k - x(t)), \ldots, (m_p - x(t)), (\varrho(t) - x(t))] \quad (4)$$

Table 5.
The value of the membership degree and the fuzzification results.

Year(t)	Actual (x)	max(μ_(K_i))		Fuzzified enrollment
		1	2	
1971	13055	0.55	0.05	A1;A2
1972	13563	0.93	0.56	A1;A2
1973	13867	0.87	0.63	A2;A1
1974	14696	0.80	0.69	A3;A4
1975	15460	0.96	0.64	A5;A4
1976	15311	0.81	0.61	A5;A4
1977	15603	0.89	0.60	A5;A6
1978	15861	0.86	0.63	A6;A5
1979	16807	0.86	0.63	A8;A7
1980	16919	0.91	0.58	A8;A7
1981	16388	0.88	0.61	A7;A6
1982	15433	0.93	0.56	A5;A4
1983	15497	0.99	0.50	A5;A4
1984	15145	0.85	0.64	A4;A5
1985	15163	0.83	0.66	A4;A5
1986	15984	0.98	0.51	A6;A5
1987	16859	0.85	0.64	A8;A7
1988	18150	0.85	0.65	A10;A11
1989	18970	0.97	0.53	A12;A11
1990	19328	0.82	0.67	A13;A12
1991	19377	0.87	0.62	A13;A12
1991	18876	0.87	0.62	A12;A11

Based on Eq. (4), if $T(t) = (m_j - x(t))$, then $T(t) = m_j$. Suppose from Table 7, $A1$ is the fuzzification of $x(1971) = 13055$ and $x(1972) = 13563$. FLR in $R(1,1)$ used for bulid of FLRG as $A_1 \rightarrow A_1, A_2$. If $F(t-1) = A_1$, then forcasting value is A_1, A_2. From Eq. (3) is obtained $\varrho(1972) = 13583$ and Eq. (4) is obtained $T(1972) = \min(313, 187, 20) = 20$. Hence, $T(1972) = 20$, then $T(t) = 13583$. Next, we will calculate the forecasting results at $R^{(1,2)}, R^{(2,1)}, R^{(2,2)}, R^{(3,1)}$ and $R^{(3,2)}$ in the same away analogously. The results of enrollment data forecasting using generalized FLR with $M = 3$ and $N = 2$ are presented in Table 10.

Step 7: Evaluation of forecasting results. The calculations of rrror evaluation used Mean Absolute Error (MAE), Root Mean Squared Error (RMSE) and Mean Absolute Percentage Error (MAPE).

$$\text{MAE} = \frac{1}{n}\sum_{t=1}^{n} |At - Ft| \tag{5}$$

$$\text{RMSE} = \sqrt{\frac{\sum_{t=1}^{n}(A_t - F_t)^2}{n}} \tag{6}$$

$$\text{MAPE} = \frac{\sum_{t=1}^{n}\left|\frac{A_t - F_t}{At} \times 100\%\right|}{n} \tag{7}$$

From Eq. (5), (6) and (7), we calculated the evaluation error of the enrollment data forecasting. The comparison results of enrollment data forecasting errors are presented in Table 10.

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Table 6.
The FLR of enrollments based on generalized fuzzy logical relationships.

Year (t)	Actual (x)	Fuzzified enrollment	k=1		k=2		k=3	
			l=1	l=2	l=1	l=2	l=1	l=2
1971	13055	A1;A2	-	-	-	-	-	-
1972	13563	A1;A2	A1→ A1	A2→ A1	-	-	-	-
1973	13867	A1;A2	A1→ A2	A2→ A2	A1→ A2	A2→ A2	-	-
1974	14696	A2;A3	A2→ A3	A1→ A3	A1→ A3	A2→ A3	A1→ A3	A2→ A3
1975	15460	A3;A4	A3→ A5	A4→ A5	A2→ A5	A1→ A5	A1→ A5	A2→ A5
1976	15311	A5;A4	A5→ A5	A4→ A5	A3→ A5	A4→ A5	A2→ A5	A1→ A5
1977	15603	A5;A6	A5→ A5	A4→ A5	A5→ A5	A4→ A5	A3→ A5	A5→ A5
1978	15861	A6;A5	A5→ A6	A6→ A6	A5→ A6	A4→ A6	A5→ A6	A4→ A6
1979	16807	A8;A7	A6→ A8	A5→ A8	A5→ A8	A6→ A6	A6→ A8	A4→ A8
1980	16919	A8;A7	A8→ A8	A7→ A8	A6→ A8	A5→ A8	A5→ A8	A4→ A8
1981	16388	A7;A6	A8→ A7	A7→ A7	A8→ A7	A5→ A7	A7→ A7	A6→ A7
1982	15433	A5;A4	A7→ A5	A6→ A5	A8→ A5	A7→ A5	A6→ A5	A5→ A5
1983	15497	A5;A4	A5→ A5	A4→ A5	A7→ A5	A6→ A5	A4→ A4	A7→ A5
1984	15145	A4;A5	A5→ A4	A4→ A4	A5→ A4	A4→ A4	A4→ A4	A7→ A4
1985	15163	A4;A5	A4→ A4	A5→ A4	A5→ A4	A4→ A4	A4→ A5	A6→ A4
1986	15984	A6;A5	A4→ A6	A5→ A6	A4→ A6	A5→ A6	A5→ A8	A6→ A6
1987	16859	A8;A7	A6→ A8	A5→ A8	A4→ A8	A5→ A8	A5→ A10	A4→ A8
1988	18150	A10;A11	A8→ A10	A7→ A10	A6→ A10	A5→ A10	A7→ A12	A5→ A10
1989	18970	A12;A11	A10→ A12	A11→ A12	A8→ A12	A7→ A12	A11→ A13	A5→ A12
1990	19328	A13;A12	A12→ A13	A11→ A13	A10→ A13	A11→ A13	A11→ A13	A5→ A13
1991	19377	A13;A12	A13→ A13	A12→ A13	A12→ A13			

Table 7.
The generalized fuzzy logical relationship group.

Group	k=1		k=2		k=3	
	l=1	l=2	l=1	l=2	l=1	l=2
G1	A1→ A1, A2	A1→ A3	A1→ A2, A3	A1→ A5	A1→ A3, A5	A1→ A5
G2	A2 → A3	A2→ A1, A2	A2→ A5	A2→ A2, A3	A2→ A5	A2→ A3, A5

G3	A3→ A5		A3→ A5		A3→ A5	
G4	A4→ A4, A6	A5→ A8, A4, A6, A8	A4→ A6, A8	A5 → A8, A6, A8, A10	A4→ A8, A10	A5→ A5, A5, A10, A12, A13
G5	A5→ A5, A5, A6	A6→ A6, A5	A5→ A5, A6, A8, A4	A6→ A8, A5	A5→ A6, A8, A8, A4, A6	A6→ A7, A4, A6
G6	A6→ A8, A8	A7→ A8, A7, A10	A6→ A8, A10	A7→ A7, A5, A12	A6→ A7, A12	A7→ A5, A4, A13
G7	A7→ A5	A11→ A12, A13	A7→ A5	A11→ A13, A13	A7→ A4	A11→ A12
G8	A8→ A8, A7, A10	A12→ A12, A13	A8→ A7, A5, A12	A12→ A13	A8→ A13, A5, A6	-
G9	A10→ A12		A10→ A13	-	A10→ A13	-
G10	A12→ A13		A12→ A13	-	A12→ A12	-
G11	A13→ A13, A12		-	-	-	-

Table 8.
The weight matrix of enrollments.

Group	k=1		k=2		k=3	
	l=1	l=2	l=1	l=2	l=1	l=2
G1	(1, 3/2)	(1)	(2/5,3/5)	(1)	(3/8,5/8)	(1)
G2	(1)	(1, 3/2)	(1)	(2/5,3/5)	(1)	(3/8,5/8)
G3	(1)	(5/24,5/24,5/24,5/24,4/24)	(1)	(5/24,5/24,4/24,6/24,4/24)	(1)	(6/30,8/30,8/30,8/30)
G4	(4/10, 6/10)	(8/26,4/26,6/26,8/26)	(6/14,8/14)	(8/32,6/32,8/32,10/32)	(8/18,10/18)	(5/45,5/45,10/45,12/45,13/45)
G5	(5/25, 5/25, 6/25, 5/25, 4/25)	(6/11, 5/11)	(5/29,6/29,8/29,4/29,4/29)	(8/13, 5/13)	(6/32,8/32,8/32,4/32,6/32)	(7/17,4/17,6/17)
G6	(8/16,8/16)	(8/25,7/25,10/25)	(8/18,10/18)	(7/24,5/24,12/24)	(7/19,12/19)	(5/22,4/22,13/22)
G7	(1)	(12/25,13/25)	(1)	(13/26,13/26)	(1)	(1)
G8	(8/25,7/25,10/25)	(12/25,13/25)	(7/24,5/24,12/24)	(1)	(13/24,5/24,6/24)	-
G9	(1)	-	(1)	-	(1)	-
G10	(1)	-	(1)	-	(1)	-
G11	(13/25,12/25)	-	(1)	-	-	-

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Table 9.
The results of the actual forecasting enrollment data.

Year (t)	Actual (x)	k=1		k=2		k=3	
		l=1	l=2	l=1	l=2	l=1	l=2
1971	13055	-	-	-	-	-	
1972	13563	13583	13583	-	-	-	
1973	13867	13750	13750	13950	13950	-	
1974	14696	14250	14250	14250	14250	14875	14875
1975	15460	15250	15250	15250	15250	15250	15250
1976	15311	15283	15250	15250	15250	15250	15250
1977	15603	15750	15250	15657	15750	15250	15250
1978	15861	15750	15750	15750	15750	15750	15750
1979	16807	16750	16750	16750	16750	16750	16750
1980	16919	17010	17010	16750	16875	16750	16750
1981	16388	16250	16250	16250	16250	16250	16250
1982	15433	15250	15523	15250	15250	15250	15250
1983	15497	15283	15250	15250	15250	15250	15250
1984	15145	15250	15167	15250	15208	14750	15250
1985	15163	15350	15250	15250	15208	14750	15250
1986	15984	15750	16212	15750	15750	16125	15750
1987	16859	16750	16750	16750	16875	16750	16750
1988	18150	17750	17750	17750	17750	17750	17894
1989	18970	18750	18990	18750	18750	18750	18750
1990	19328	19250	19250	19250	19250	19250	19250
1991	19377	19250	19250	19250	19250	19250	19250
1991	18876	18750	18750	18750	18750	18750	18750

Table 10.
The comparison results of enrollment data forecasting errors.

Evaluation	Model	Proposed Method			[49]		
		M = 1	M = 2	M = 3	M = 1	M = 2	M = 3
MAE	GTS(M,1)	152.17	147.52	168.95	473	471	501
	GTS(M,2)	142.06	138.02	138.62	372	375	390
RMSE	GTS(M,1)	185.9	184.1	209.7	625	632	643
	GTS(M,2)	186.4	180.1	165.0	447	449	460
MAPE	GTS(M,1)	0.00937	0.00903	0.01039	0.0293	0.0291	0.0307
	GTS(M,2)	0.00882	0.00846	0.00844	0.00227	0.00227	0.0023

4. The Comparison Study

Forecasting of the University of Alabama enrollment data has been done in a fuzzy time series. The existing methods aim to achieve the smallest evaluation values (MAE, RMSE, MAPE). Based on Table 11, it can be seen that the proposed method produces evaluation values of all orders of the smallest compared to the evaluation values [49]. The lower the RMSE value, the more variation in the value generated by the forecasting model is closer to the interpretation. The proposed method obtains an average MAPE value of less than 10%, so the model's forecasting performance is very good. Moreover, the proposed method can handle high forecasting errors for each data series by using adaptive defuzzification with Eq. (4). The proposed method can obtain small errors in each data series forecast. The comparison results of forecasting errors for each enrollment data series is shown in Figure 2. The figure shows that the proposed model, namely GTS (1.1), GTS (1.2), GTS (2.1), GTS (2.2), GTS(3,1)

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and GTS(3,2), produces an error of less than 500 in each data series and GTS(1.2) is the model that has the smallest error compared to other models. Correspondingly, Qiu's method obtained GTS (1,2) as the best model with the smallest MSE [37]

We compare the proposed method with the existing methods methods [6], [8], [12], [15], [16], [49], [52], [53] for enrollment data forecasting.

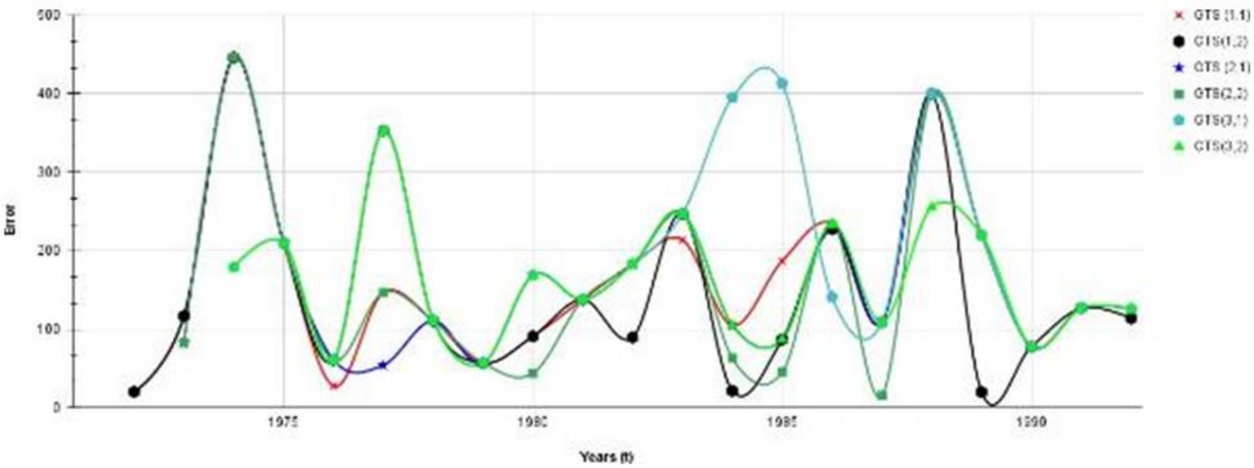

Figure 2.
The comparison results of forecasting errors for the different of GTS(m,n)

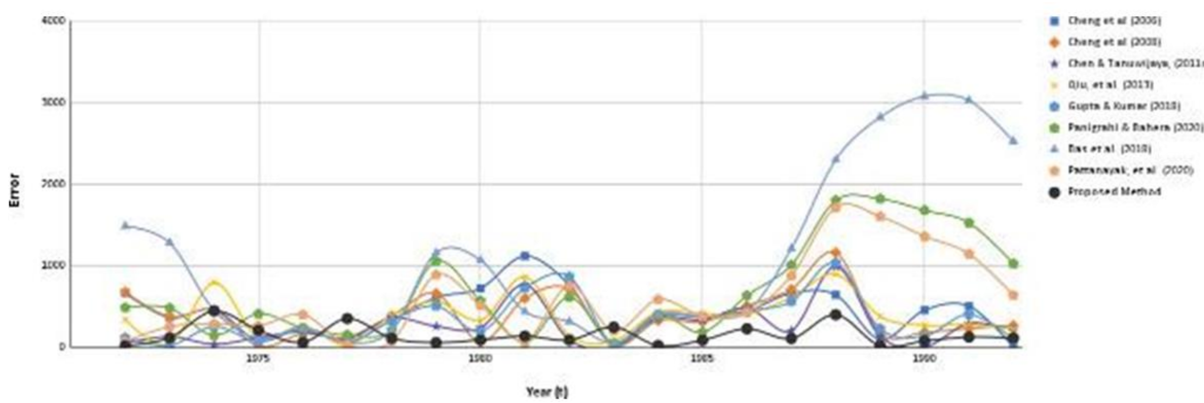

Figure 3.
The comparison results of forecasting errors for each enrollment data series.

A comparison of the Mean Square Error (MSE) of the proposed method with existing methods is shown in Table 12. The MSE of the proposed method is the smallest compared to existing methods. We also compare the forecasting error values for each data series provided in Figure 3.

Similarly, the MSE value, the error value of the proposed method, is smaller than the existing methods. Meanwhile, the comparison of all evaluations on data enrollment forecasting is shown in Figure 4. The proposed method is superior compared to existing methods from all the evaluations provided (MAE, RMSE, MAPE). The comparative study explained above clearly shows that the proposed method can overcome the problem of forecasting error for each time series data. Based on the evaluation, the proposed method is superior to existing methods. Therefore, the proposed method has a good chance of being developed in forecasting with different data.

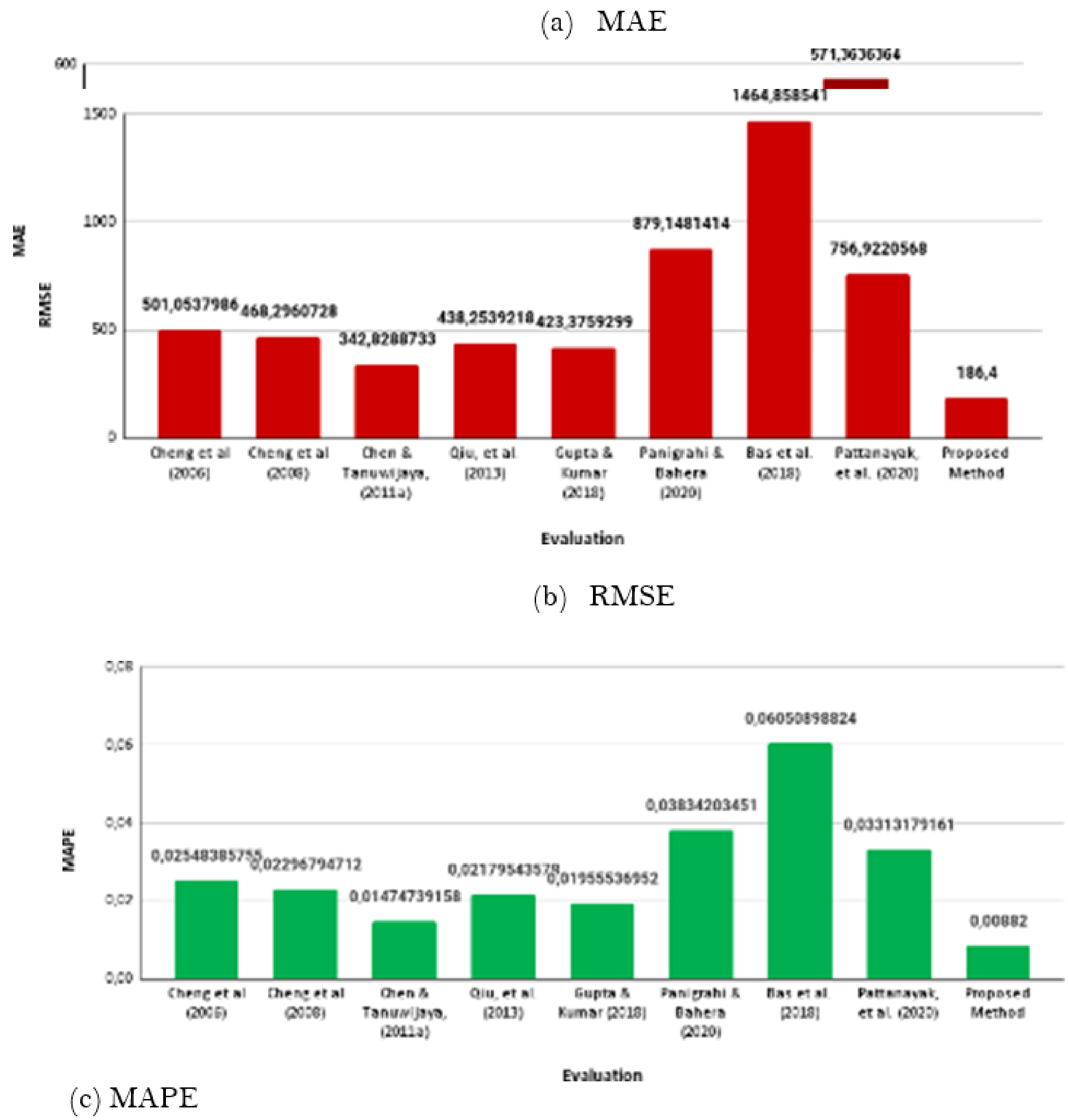

Figure 4.
The comparison of all evaluations on data enrollment forecasting.

5. Conclusion

This paper proposes a new defuzzification method with adaptive rules by considering the smallest distance between the actual value and the defuzzification result as the selected forecasting value. The proposed method can minimize errors for each data series. The proposed method is also tested for the efficiency of forecasting fuzzy time series enrollment data at the University of Alabama and provides a comparative study with existing methods [6], [8], [12], [15], [16], [49], [52], [53]. From Table 11, we see that in forecasting the enrollments at the University of Alabama, our proposed method outperforms the method proposed by the existing methods. Although this study made great improvements in dealing with the high error problem for each data series forecaster, there is a limitation of the proposed method: adaptive defuzzification is applied to partitioned time series data with universes of discourse into equal-length sub-intervals. If the interval lengths differ and the partitions increase, the defuzzification calculation process becomes more complicated.

Table 11.
A comparison of the Mean Square Error (MSE) of the proposed method with existing methods.

Year	Actual (x)	The existing method								
		[52]	[53]	[8]	[49]	[12]	[15]	[6]	[16]	Propose
1971	13055	-	-	-	-	-	-	-	-	-
1972	13563	14230	14242	13512	13902	13680.75	14049	15049	13637	13583
1973	13867	14230	14242	13998	13902	13844.43	14349	15149	14120	13750
1974	14696	14230	14242	14658	13902	14951.36	14549	15149	14408	14250
1975	15460	15541	15474.3	15341	15576	15532.34	15049	15349	15195	15250
1976	15311	15541	15474.3	15501	15576	15533.19	15549	15549	15712	15283
1977	15603	15541	15474.3	15501	15576	15533.19	15449	15549	15635	15750
1978	15861	16196	15474.3	15501	16246	15533.19	15649	15649	15786	15750
1979	16807	16196	16146.3	17065	16246	16298.77	15749	15649	15918	16750
1980	16919	16196	16988.3	17159	17251	17113.79	16349	15849	16406	17010
1981	16388	17507	16988.3	17159	17251	17113.79	16449	15949	16466	16250
1982	15433	16196	16146.3	15341	15576	16298.77	16049	15749	16190	15523
1983	15497	15541	15474.3	15501	15576	15533.19	15549	15549	15698	15250
1984	15145	15541	15474.3	15501	15576	15533.19	15549	15549	15731	15167
1985	15163	15541	15474.3	15501	15576	15532.34	15349	15549	15550	15250
1986	15984	15541	15474.3	15501	15576	15532.34	15349	15549	15559	16212
1987	16859	16196	16146.3	17065	16246	16298.77	15849	15649	15982	16750
1988	18150	17507	16988.3	17159	17251	17113.79	16349	15849	16433	17750
1989	18970	18872	19144	18832	18591	18741.35	17149	16149	17366	18890
1990	19328	18872	19144	19333	19596	19190.44	17649	16249	17967	19250
1991	19377	18872	19144	19083	19094	18972.15	17849	16349	18230	19250
1992	18876	18872	19144	19083	19094	18972.15	17849	16349	18236	18890
MSE		251055	219031	117532	192067	179247	772901	2145801	572931	34759

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