

Optimal power density estimation in Sub-Saharan Africa regions based on Weibull distribution parameters

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Abstract: The production of wind energy requires knowledge of certain wind speeds and directions. Several tools are used for this purpose to characterize the wind power, including the Weibull distribution function with two parameters, shape factor k , and the scale factor c . In this paper six methods Graphical Method (GP), Empirical Method of Justus (EMJ), Empirical Method of Lysen (EML), Energy Pattern Factor (EPF), Maximum Likelihood (ML), and Moroccan Method (MMa) are used to estimate Weibull distribution parameters to evaluate the wind potential and its power density. Twelve sites in West African sub-region such as Abuja, Accra Kotoka, Bamako Senou, Conakry Gbessia, Cotonou Cadjehoun, Kano Mallam Aminu, Lagos Ikeja, Lome Tokoin, Niamey, Niamtougou, Ouagadougou, Tambacounda were selected as case studies. For each selected site, hourly wind speed data collected at 10 m height for the twelve years from January 2011 to December 2023 are used. The evaluations of each method were carried out every month and statistical criteria to provide a more complete analysis. The results show that the EPF, EMJ, EML, and ML provide highly desirable better performance while the GP and MMa showed poor performance for all stations. For all sites, the EPF was recognized as the most appropriate method except for the Lagos site where the EMJ ranks first on the others. The methods that ranked second after EPF varied among the sites.

Keywords: Numerical estimation methods, Power density, Weibull distribution, Wind speed.

1. Introduction

The problem of electrification in West Africa has been growing in recent years. Additionally, the depletion of fossil fuels and the environmental impacts have led to a shift towards other sources of electrical energy production. In fact, West Africa has significant hydroelectric potential, strong wind potential for the deployment of wind energy, high solar radiation, particularly in desert areas, and considerable hydrocarbon resources, accounting for about half of the continent's reserves [1]. In terms of wind power, only 43 megawatts of installed capacity were deployed, with another 230 megawatts being installed in the region by 2011. The only wind farm operational on a commercial scale is Cabeolica in Cape Verde, which has the largest installed capacity at over 28 MW. Consequently, West Africa lags behind with few projects in the wind sector. For example, a project to build and operate a 25.2 MW wind power plant in Lome is planned but has not yet been implemented. However, the West Africa region is well positioned especially in its coastal areas to take advantage of its wind energy potential.

Other aspects to consider in a wind farm project include comprehensive assessments of the wind resource, as well as the power and energy densities required to verify the financial viability of this potential. The power density of a wind turbine is an important factor to consider when assessing wind resources and implementing wind farm projects, as it helps in selecting the optimum turbines for a particular area [2]–[4]. Power density is calculated from the amount of energy available, which can be converted into electricity using wind turbines. There are two approaches to calculating the power density of a wind turbine. In the first approach, wind power density is calculated empirically from measured wind speed data. The wind energy at a given location depends on the cube of the wind speed. Thus, the power density for the time series of actual wind speed data can be calculated using the parameters of the Weibull distribution [5].

Several methods have been proposed in the literature to estimate the parameters of the Weibull distribution with two parameters k and c , respectively the shape and scale factors [2], [6]–[9]. The Graphical method, Justus' empirical method, Lysen's empirical method, the Energy pattern factor method, the Moroccan method, the Maximum likelihood method, the Modified maximum likelihood method, the Method of moments are commonly used [10], [11]. It can be concluded that the relevance of the methods may vary depending on the sample size of the data, the distribution of the sample data, the sample data format and the precision of the fit tests. In this paper, the GP, EMJ, EML, EPF, ML, and MMA are used to compute of the Weibull parameters of twelve sites in West Africa. The aim is to determine the most accurate for wind power density estimation in this region. The rest of the study is organized as follows: in section 2 the study background is presented, section 3 depicts the study area and data description, Sections 4 and 5 exhibits respectively the wind speed and wind potential based on Weibull distribution and evaluation metrics. The case study results and discussions of twelve West African sites are presented in Section 6 concludes with section 5.

2. Study Background

Two parameters of the Weibull distribution are known as form (k) and scale (c) parameters. As mentionné, l'estimation de ces paramètres de la distribution de la vitesse du vent est faite grâce à plusieurs méthodes proposées dans la littérature. For example, [11] compared the performance of five methods for calculating the shape and scale parameters of the Weibull function to characterize the wind speed distribution. The results indicate that the maximum likelihood method outperforms the other methods in terms of representing the wind speed distribution. In [12], the authors evaluated the performance of four parameter estimation methods of the Weibull function for the monthly wind speed distribution modeling in Halabja, Pakistan. [13] compared the performance of six different methods to compute shape and scale parameters for estimating the wind speed distribution. According to the results, the maximum likelihood method followed by the modified maximum likelihood method showed the highest performance, while the graphical methods had the lowest performance. The study in [14] evaluated the Weibull parameters to represent the distribution of wind speed in Garoua, Nigeria. Their results showed that the use of the energy pattern factor has more aptitude than the other examined methods. [2] has evaluated the performance of six numerical methods as GP, EMJ, EML, EPF, ML, and MML to determine the k and c parameters of the Weibull distribution function to evaluate wind energy density at four stations distributed in the province of Canada, Alberta. À cet égard, la densité d'énergie éolienne estimée à l'aide de la fonction de Weibull est comparée à celle obtenue avec les données de vent mesurées. The daily and monthly results indicated that the accuracy changed with numerical methods compared to empirical results. It was found that the EMJ, EML, EPF, and ML methods provided the best performance than the GP method which ranks last for all the considered stations. Another observation was that the EMJ and EML methods are very close in terms of efficiency. A study conducted in [15], analyzed wind characteristics using wind speed data collected from five meteorological sites in Lebanon. The authors found that the power density method gives the best estimate of the measured distribution for all sites except Quaraoun where Justus' empirical method gives the best estimate. In the Northeast region of Brazil [13], the EPF and GP are efficient in fitting

Weibull distribution for wind speed data from the coastal area of Ceará, based on data collected from the cities of Camocim and Paracuru and analyzed using statistical tests. The authors in [16], have determined for the Lome site, the best model that corresponds to la forte fréquence des vents calmes observee to accurately estimate the amounts of recoverable wind energy. They propose using the Weibull hybrid distribution approach. Two other traditional approaches are often used in this context to evaluate the la pertinence de cette approach. They found that if the frequencies of calm winds are relatively high, the Weibull distribution is inadequate. In this case, the hybrid Weibull distribution function is the best solution. The same authors in [17] also presented the characterization and evaluation of the wind potential at annual and monthly scales of the Lome site and specified the characteristics of the wind turbines to be installed on this site. The wind speed data collected over two years at a height of 10 meters above the ground show that the mean annual speed is 2.9 m/s. These studies also showed that February, March, April, July, August, and September have a monthly average speed close to 4 m/s. Il est ressorti que that wind turbines with low nominal speeds of the order of 6 m/s to 8 m/s will be the most suitable for optimal exploitation of electrical energy from wind energy on the Lome site from a height of 25 meters above ground. Other approaches as, Rayleigh distribution [18], mixture hybrid Weibull distribution [19], le machine learning [20] are also used in the literature.

The methods for assessing wind resources make them suitable for installing commercial wind turbines. However, understanding the wind characteristics and potential in West Africa has gaps due to a lack of specifically documented information to assess the wind potential of study sites and select appropriate commercial turbines. Additionally, no studies have directly addressed the wind regimes and unique atmospheric conditions prevailing in West Africa over various months. The relevance of power density in evaluating wind resources and selecting suitable turbines is crucial. Further research is needed to obtain specific information for wind project feasibility studies. The study used the Weibull distribution method, with the best numerical fit, to analyze monthly wind patterns and assess energy potential

3. Study Area and Data Analyses

For this study, twelve sites Abuja, Accra Kotoka, Bamako Senou, Conakry Gbessia, Cotonou Cadjehoun, Kano Mallam Aminu, Lagos Ikeja, Lome Tokoin, Niamey, Niamtougou, Ouagadougou, Tambacounda were selected as case studies. Figure 1 illustrates the location of the selected sites on a map of Africa. Table 1 shows the geographic location of the twelve selected sites. For each selected site, hourly wind speed data collected at 10 m height for the twelve years from January 2011 to December 2023 are used. Table 2 shows the statistics; mean speed, standard deviation, kurtosis, skewness, and power density, calculated from the measured wind speed data for the selected sites.

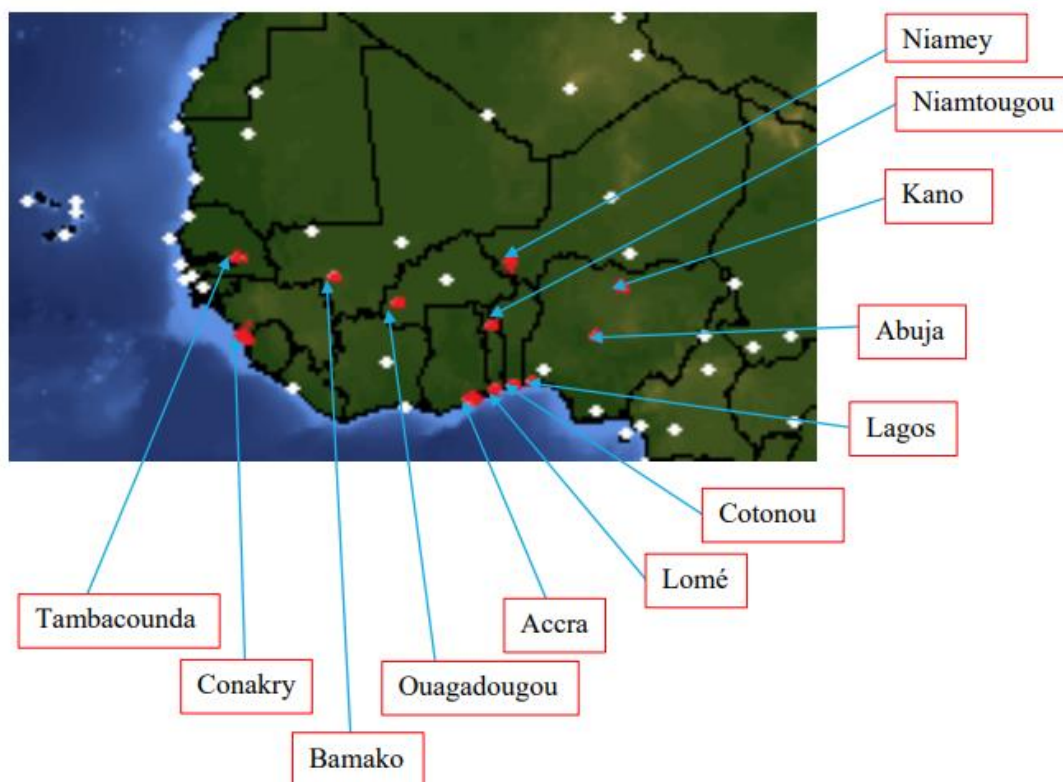


Figure 1.
Geographical location of the selected sites.

Table 1.
Graphical location of selected sites.

Sites	OACI Code	Latitude (°N)	Longitude (°E/°W)	Altitude (m)
Abuja. Nigeria	DNAA	9.25N	7.00 °E	344
Accra Kotoka. Ghana	DGAA	5.60N	0.17 °W	69
Bamako Senou. Mali	GABS	12.53N	7.95 °W	381
Conakry Gbessia. Guinée	GUCY	9.34N	13.36 °W	22
Cotonou Cadjehoun. Benin	DBBB	6.35N	2.38 °E	9
Kano Mallam Aminu. Nigeria	DNKN	12.05N	8.53 °E	481
Lagos Ikeja. Nigeria	DNMM	6.58N	3.33 °E	38
Lome Tokoin. Togo	DXXX	6.17N	1.25 °E	25
Niamey. Niger	DRRN	13.48N	2.17 °E	227
Niamtougou. Togo	DXNG	9.77N	1.10 °E	343
Ouagadougou. Burkina Faso	DFFD	12.35N	1.52 °W	306
Tambacounda. Sénégal	GOTT	13.77N	13.68 °W	50

Table 2.
Descriptive statistics of the wind data used from the selected sites.

Sites	Mean (m/s)	Std (m/s)	Skewness	Kurtosis
Abuja	2.43045	1.27036	1.25746	11.03356
Accra	4.16032	2.21591	0.08801	2.76699
Bamako	2.79606	1.72889	0.82169	5.09067
Conakry	3.35295	1.65166	0.54587	6.07934
Cotonou	4.01159	1.81438	-0.12249	2.49218
Kano	4.78558	2.32290	0.25828	4.31923
Lagos	2.71353	2.24166	1.23657	5.64726
Lome	3.52870	2.02964	0.26247	2.33358
Niamey	3.31053	1.84692	1.08897	5.66665
Niamtougou	2.61775	1.79118	0.41532	3.62772
Ouagadougou	2.99562	1.66267	0.78947	4.59290
Tambacounda	2.95754	1.64106	1.36276	8.59716

We note that Kano, Accra and Cotonou have the highest average wind speeds at 4.78558m/s, 4.16032m/s and 4.01159m/s respectively. Lome follows with an average speed of 3.52870m/s. On the other hand, the lowest wind speed is observed for Abuja. In addition, for the stations of Abuja and Tambacounda the kurtosis coefficient is significantly higher than the stations of Lome, Accra and Cotonou. It is observed that for all stations the values of the skewness coefficients are positive except for Cotonou, indicating that all distributions are skewed to the right except for Cotonou which is skewed to the left. The standard deviation for all stations is between 1 and 2.5. It should be noted, however, that Accra has the largest standard deviation, which is 2.21591.

4. Weibull Distribution

The wind energy at a given location depends on the wind speed cube. Thus, the power density for time series of actual wind speed data can be calculated using Equation (1). where ρ denotes the air density, a parameter that varies with latitude and temperature, but is generally considered to be constant and averages about 1.25 kg/m³ which depends on altitude, air pressure and temperature; and v is the wind speed in m/s. S is the swept area by the wind turbine in the previous expression shows that the available power varies with the average cubic speed of the observed wind.

$$\bar{P} = \frac{1}{2} \rho S \bar{v}^3 \quad (1)$$

The latter method is based on a statistical treatment of the raw wind data and the calculation of frequencies at a given threshold of speed. The two-parameters Weibull probability density function is given by Equation (2) [21].

$$f(v) = \left(\frac{k}{c}\right) \left(\frac{v}{c}\right)^{k-1} \exp\left(-\left(\frac{v}{c}\right)^k\right) \quad (2)$$

where k is the shape parameter that indicates the wind distribution of any region, c is the scale parameter in m/s indicates how windy the location is. The cumulative function can be obtained by calculating the integral of the probability density function. The cumulative distribution function is expressed by Equation (3).

$$F(v) = \int_0^v f(V) dv = 1 - \exp\left(-\left(\frac{v}{c}\right)^k\right) \quad (3)$$

The wind density energy calculated from the density of the Weibull probability density function is estimated using the following Equation (4) [1]. The wind density energy calculated from the density of the Weibull probability density function is estimated using the following Equation (4). where $\Gamma()$ denotes gamma function.

$$P = \frac{1}{2} \rho c^3 \Gamma \left(1 + \frac{3}{k} \right) \left[\frac{W}{m^2} \right] \quad (4)$$

The estimation of the parameters of the wind speed distribution is important in terms of selecting the wind turbine to be implemented in order to obtain a good return on wind energy production and also the economic viability of the project. There are several methods in the literature to compute the parameters k and c of the Weibull distribution function. In this study, six methods such as the graphical method (GP), the empirical method of Justus (EMJ), the empirical method of Lysen (EML), the energy pattern factor method (EPF), the maximum likelihood method (ML), and the Moroccan method (MMA) are used to calculate these parameters.

The graphical method is achieved using the cumulative distribution function. In this method, the wind speed data are interpolated using least-squares regression. The observed wind speeds are divided into v_1, \dots, v_n intervals. The discrete probability of these wind speeds is respectively given by Equation (5).

$$F(v_i) = F[v < v_i] = 1 - \exp\left(-\left(\frac{v_i}{c}\right)^k\right) \quad (5)$$

By taking twice the logarithm of the equation (5) [2], [11], [22], we obtained as the Equation (6).

$$\ln\{-\ln[1 - F(v_i)]\} = k \ln(v) - k \ln(c) \quad (6)$$

Plotting $\ln(v)$ as the axis x compared to the first member of (6) as the axis of the y presents a straight line in which k is the slope of the line and the ordinate at the origin is $-k \ln(c)$ [7], [17], [23]. This determines the linear regression line of y according to x in the form given by Equation (7).

$$y_i = a \cdot x_i + b \quad (7)$$

With $a=k$, and $b = -k \ln(c)$. The Weibull parameters are then calculated according to the Equation (8).

$$\left. \begin{aligned} k &= a \\ c &= \exp\left(-\frac{b}{a}\right) = \exp\left(-\frac{b}{k}\right) \end{aligned} \right\} \quad (8)$$

Based on the empirical method introduced by Justus, the parameters k and c are calculated respectively by Equations (9) and (10) [2], [24], [25].

$$k = \left(\frac{\rho}{\bar{v}}\right)^{-1.086} \quad (9)$$

$$c = \frac{\bar{v}}{\Gamma(1+1/k)} \quad (10)$$

In the empirical method proposed by Lysen, k is calculated by Equation (9) as in the Justus method. The only difference is the calculation of c given by Equation (11) [2], [26].

$$c = \bar{v} \left(0,586 + \frac{0,433}{k} \right)^{-\frac{1}{k}} \quad (11)$$

To calculate the parameters k and c by this process, the Energy pattern factor as a parameter used for the aerodynamic design of the turbines must be defined first. The energy pattern factor is obtained by using Equation (12) [2], [3].

$$E_{pf} = \frac{\frac{1}{n} \sum_{i=1}^n v_i^3}{\frac{1}{n} \sum_{i=1}^n v_i} = \frac{\bar{v}^3}{\bar{v}^3} = \frac{\Gamma(1+3/k)}{\Gamma^3(1+1/k)} \quad (12)$$

where \bar{v}^3 is the average wind speed cube, \bar{v}^3 is the cube of the average speed. Then, the parameter k can be calculated by Equation (13).

$$k = \left(1 + \frac{3,69}{(E_{pf})^2} \right) \quad (13)$$

The parameter c is also calculated in the same way using Justus empirical method given by the Equation (10) with the obtained parameter.

The maximum likelihood method is a mathematical expression recognized as a likelihood function of wind speed data in time series format. In this method, extended numerical iterations are required to determine the k and c parameters of the Weibull distribution. Using the maximum likelihood method, these parameters are respectively calculated by Equations (14) and (15) [3], [23], [27], [28]. where v_i is the wind speed at time i in m/s and n is the number of non-zero wind speed data points.

$$k = \left[\frac{\sum_{i=1}^n v_i^k \ln(v_i)}{\sum_{i=1}^n v_i^k} - \frac{\sum_{i=1}^n \ln(v_i)}{n} \right]^{-1} \quad (14)$$

$$c = \left[\frac{\sum_{i=1}^n v_i^k}{n} \right]^{\frac{1}{k}} \quad (15)$$

This method was used in the evaluation of the wind potential in Morocco [15]. The parameters k and c are determined respectively by the Equation (16) and (10) using the obtained value of the parameter k .

$$k = 1 + (0,483 \times (\bar{v} - 2))^{0,51} \quad (16)$$

5. Evaluation Metrics

To evaluate the performance of the six methods for wind energy density estimates, different statistical approaches, including seven reliable statistical indicators are used in this study. Several statistical indicators including mean absolute percentage error (MAPE), mean absolute error (MABE), root mean square error (RMSE). In their formula, $P_{i,w}$ et $P_{i,M}$ are respectively the i -order wind power densities calculated by the Weibull function and the i -order wind power densities calculated by the measured data. In addition $P_{w,avg}$ et $P_{M,avg}$ are the averages of the values of $P_{i,w}$ et $P_{i,M}$ and n is the total number of wind speed.

The MAPE shows the average absolute percentage difference between the wind powers calculated using the Weibull function and those reached by the measured values. The MAPE is calculated by Equation (17).

$$MAPE = \frac{1}{n} \sum_{i=1}^n \left| \frac{P_{i,W} - P_{i,M}}{P_{i,M}} \right| \times 100 \quad (17)$$

The MABE represents the mean total absolute amount of polarization errors between the wind powers calculated by the Weibull function and those obtained by the measured values. The MABE is defined by Equation (18).

$$MABE = \left(\frac{1}{n} \sum_{i=1}^n |P_{i,W} - P_{i,M}| \right) \quad (18)$$

The RMSE identifies the accuracy of the model by comparing the difference between the values obtained by the Weibull function and those of the measured data. The RMSE always has a positive value and is calculated by Equation (21) [22], [29], [30]: The root mean square error is very useful for comparing several estimators. It measures the performance of the estimators based on the mean of the squares of the errors. In our study we also use it to determine the method that gives a Weibull function that better follows the frequency histograms of the different classes of measured wind speeds. The fit is better when the RMSE is low (very close to 0).

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (P_{i,W} - P_{i,M})^2} \quad (19)$$

6. Results and Discussion

Figures 2 to 11 show some monthly curves of k and c parameters estimated with each method and the Weibull distribution functions plotted with the obtained values. For all sites, the shape parameters follow practically the same variations for all methods. But it should be noted that for some sites there are some curves that are slightly out of phase: the MMA method for the Abuja site, the GP and MMA methods for Conakry, the MMA method for Cotonou and Kano and the ML method for Niamey. The peak for k values is reached for Cotonou with k equals to 4 for the ML method. But generally, for the k methods varies between 1 and 3. As for the variations in c curves, they are mostly in the same range for all sites. As for the variations in c curves, they are mostly in the same range for all sites. The Weibull distributions vary according to the values of the parameters k and c calculated for each method. It can be seen that for all stations the values of the parameter c for all methods are quite close to each other and only minor differences are found for the GP method and sometimes also for the ML method. Nevertheless, the values of the parameter k for the EMJ, EML and EPF methods are in the same range for all months while for the ML, GP and MMA methods the values of k are sometimes higher or lower than other methods

These differences of k and c values for each method highlight the difference in the calculated values of wind power densities with the measured data. Although the summarizes provide significant insights especially with respect to the distribution of wind power density but they cannot be used solely to determine the level of precision of the monthly power density calculation methods. Therefore, the statistical indicators are used in order to identify the level of precision of each method. The statistical indicators introduced in this section are used to assess the performance of the six estimation methods. Tables 3 to 14 provide the results of descriptive statistics and power density estimation, and Figure 12 the evaluation of the performance of the six selected methods on a monthly basis in terms of MAPE, MABE, RMSE, RMSE, respectively for Abuja, Accra, Bamako, Conakry, Cotonou, Kano, Lagos, Lome, Niamey, Niamtougou, Ouagadougou, and Tambacounda.

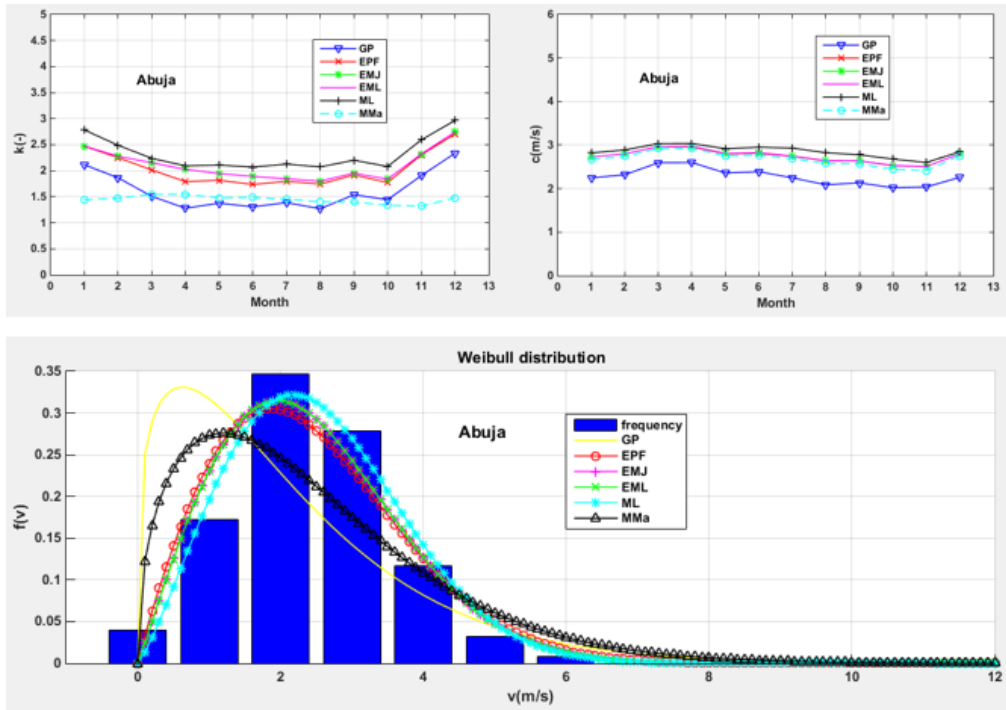


Figure 2. Monthly variation of k and c values, and Weibull distribution for Abuja.

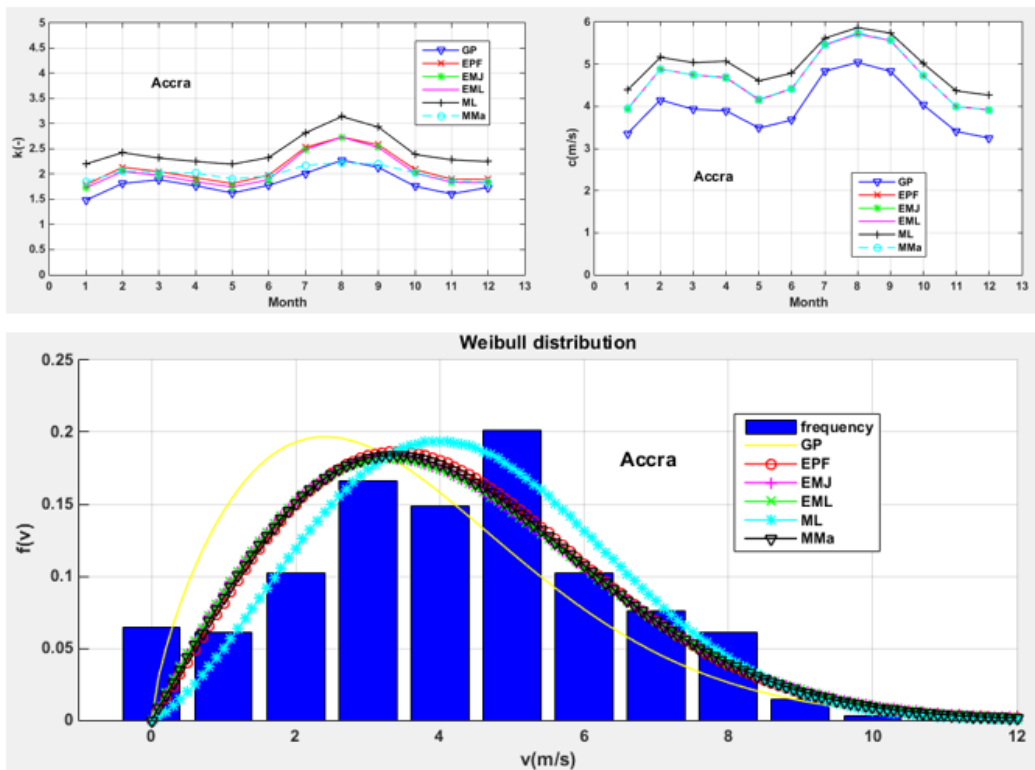


Figure 3. Monthly variation of k and c values, and Weibull distribution for Accra.

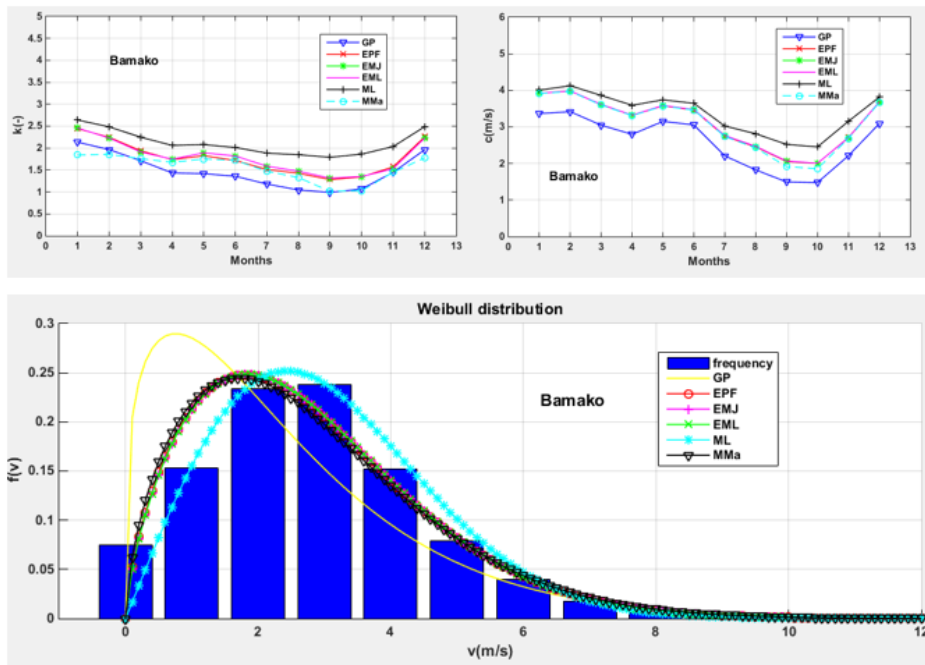


Figure 4.
Monthly variation of k and c values, and Weibull distribution for Bamako.

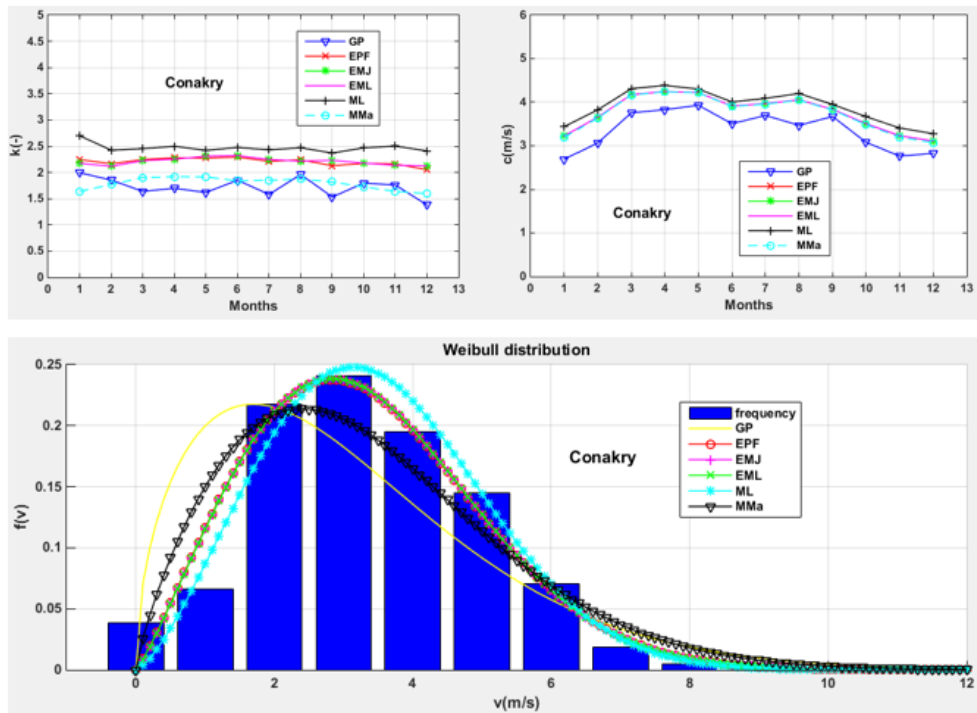


Figure 5.
Monthly variation of k and c values, and Weibull distribution for Conakry.

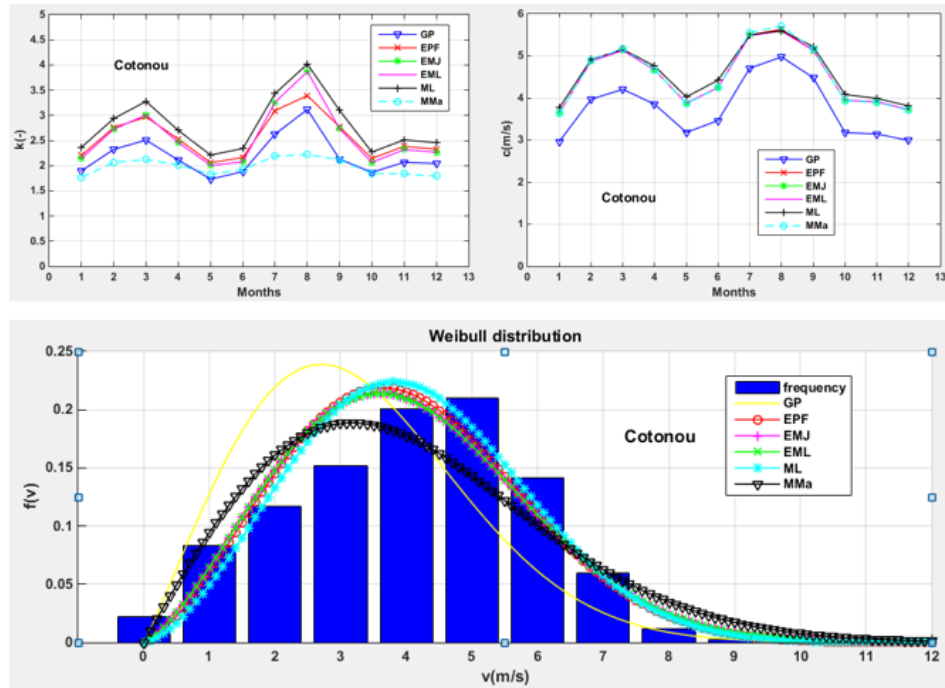


Figure 6. Monthly variation of k and c values, and Weibull distribution for Cotonou

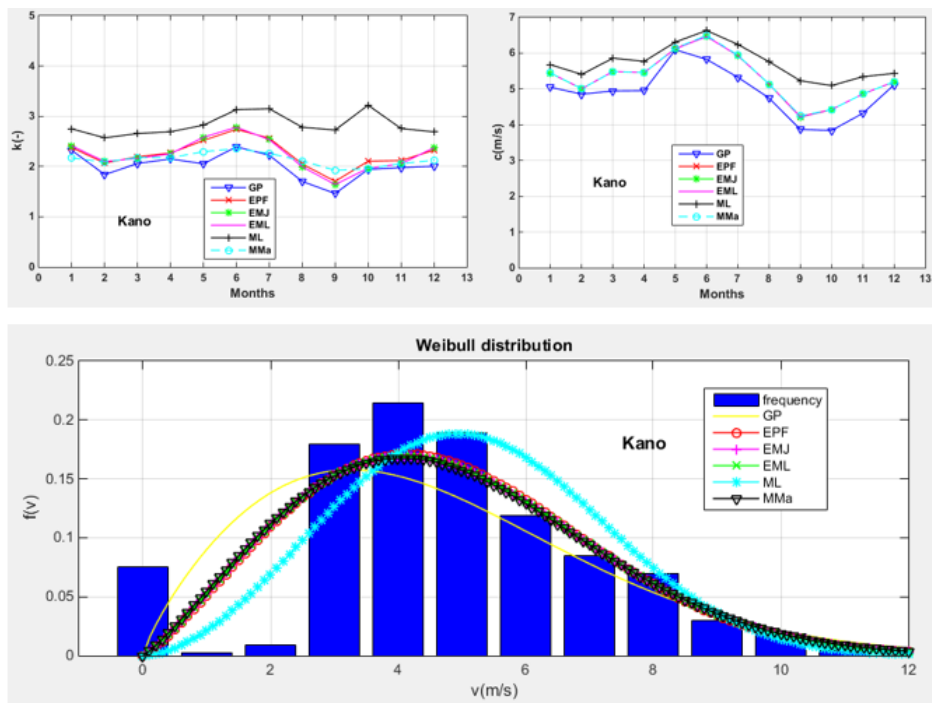


Figure 7. Monthly variation of k and c values, and Weibull distribution for Kano

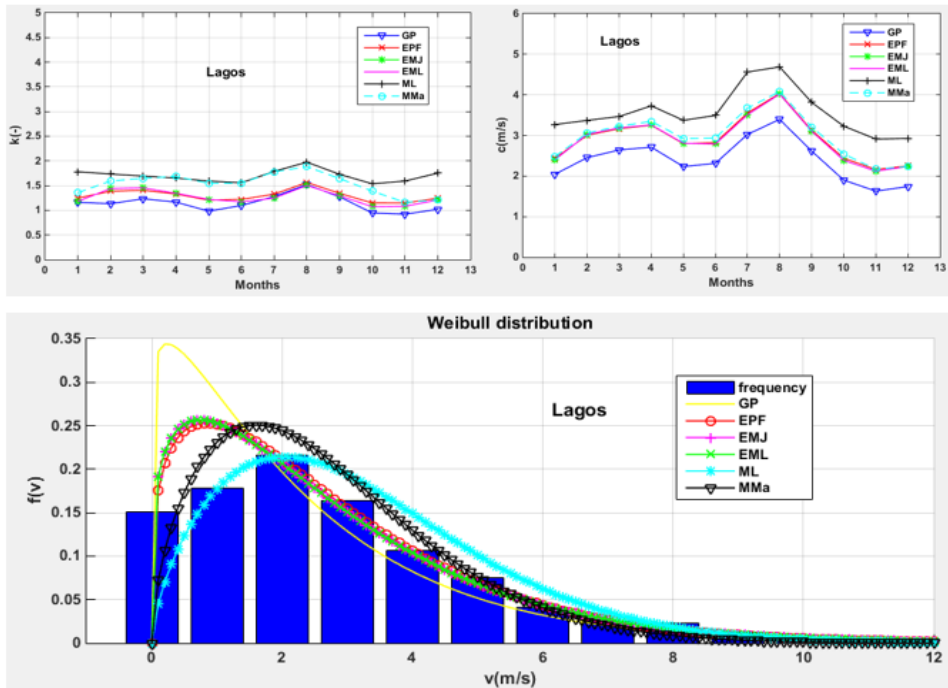


Figure 8. Monthly variation of k and c values, and Weibull distribution for Lagos

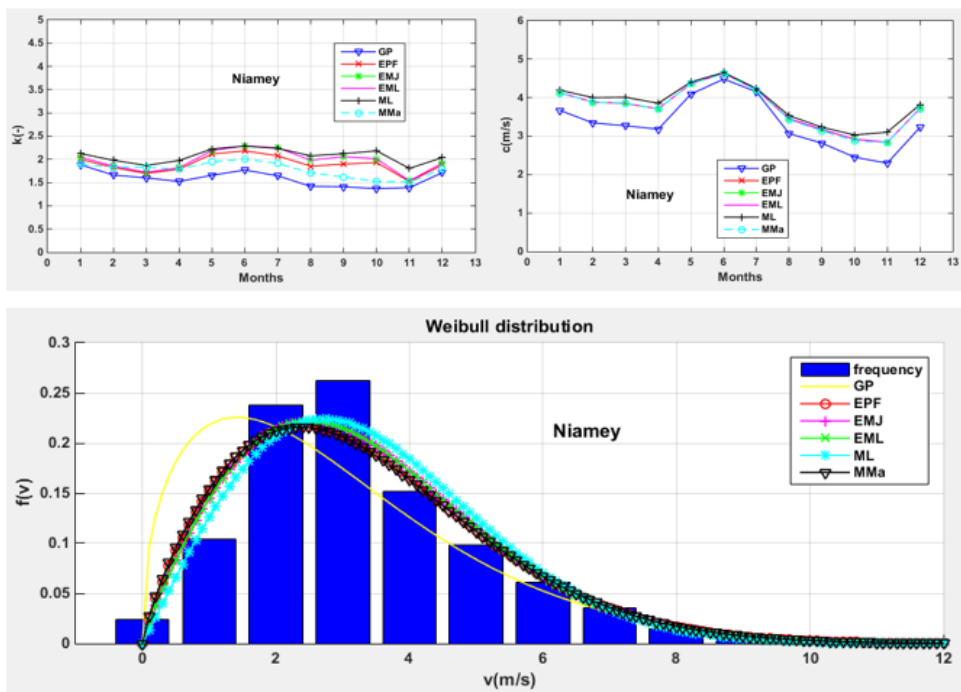


Figure 9. Monthly variation of k and c values, and Weibull distribution for Niamey

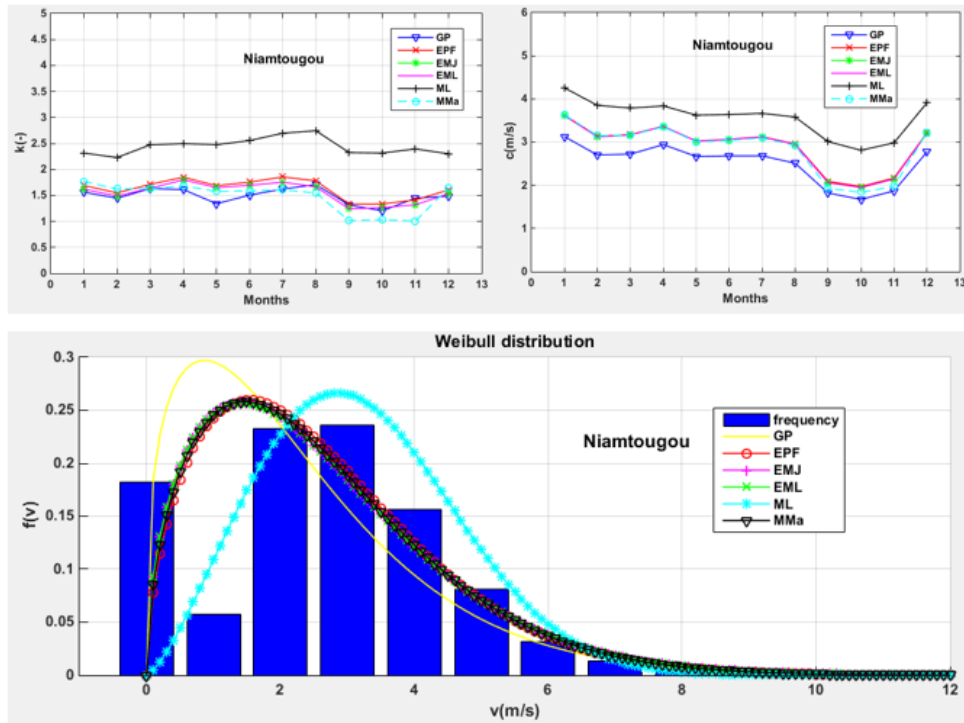


Figure 10.
Monthly variation of k and c values, and Weibull distribution for Niamtougou

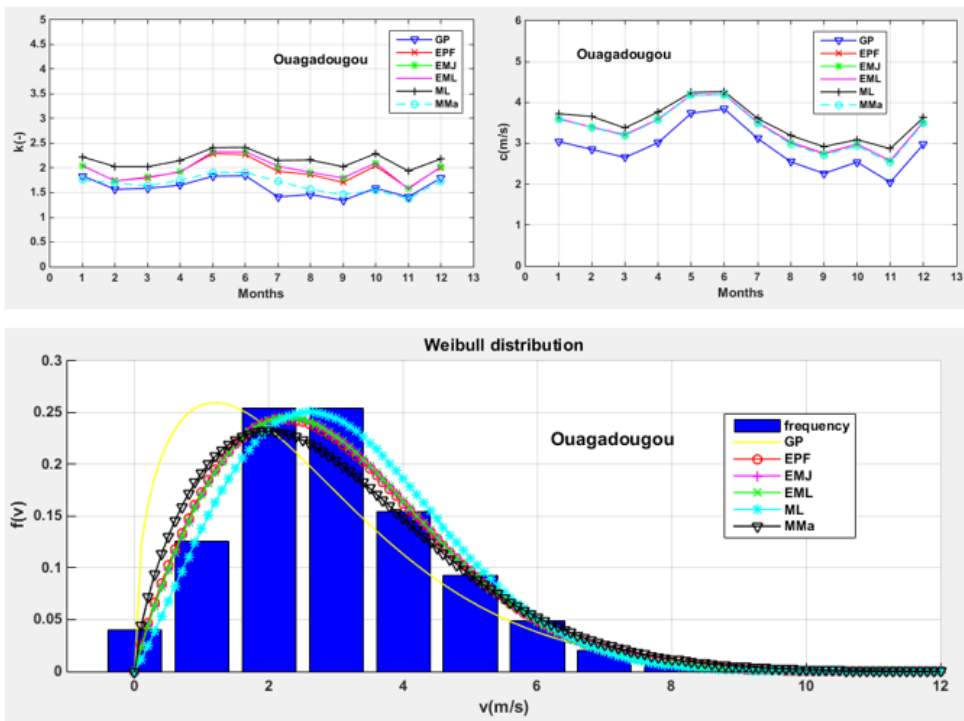


Figure 11.
Monthly variation of k and c values, and Weibull distribution for Ouagadougou

Table 1.
Descriptive statistics and power density for Abuja site.

Results	Real values	GP	EPF	EMJ	EML	ML	MMa
Vit.moy.	2.43776	2.0605	2.4378	2.4378	2.439	2.5368	2.4378
Ec.typ.	1.24492	1.367	1.2832	1.233	1.2337	1.1775	1.7101
Dens.puiss.	17.4943	15.229	17.462	16.612	16.638	17.177	25.541
Kurtosis	8.91043	6.9212	7.4299	9.2685	9.2434	10.451	3.1199
Skewness	1.04677	1.6215	0.9168	1.0794	1.0747	0.9273	0.4831

Table 2.
Descriptive statistics and power density for Accra site.

Results	Real values	GP	EPF	EMJ	EML	ML	MMa
Vit.moy.	4.15723	3.55301	4.15723	4.15723	4.15930	4.43428	4.15723
Ec.typ.	2.14720	2.01581	2.06641	2.12624	2.12736	1.92788	2.15189
Dens.puiss	82.12856	60.10977	82.17615	84.62223	84.74185	87.43053	86.26740
Kurtosis	2.92631	4.65209	3.39907	3.04094	3.03396	4.57299	3.03982
Skewness	0.08977	1.21591	0.10664	0.09400	0.09078	-0.41216	0.12566

Table 3.
Descriptive statistics and power density for Bamako site.

Results	Real values	GP	EPF	EMJ	EML	ML	MMa
Vit.moy	2.79765	2.37492	2.79765	2.79765	2.79934	3.01251	2.79765
Ec.typ.	1.62835	1.65579	1.63012	1.60993	1.61095	1.49114	1.83500
Dens.puiss	31.39636	25.65127	31.29463	30.79267	30.84730	31.90702	36.68506
Kurtosis	6.00753	6.71940	5.81369	6.30352	6.28229	8.14727	3.92648
Skewness	0.91573	1.66482	0.89034	0.95025	0.94505	0.66773	0.66526

Table 4.
Descriptive statistics and power density for Conakry site.

Results	Real values	GP	EPF	EMJ	EML	ML	MMa
Vit.moy	3.33148	3.00024	3.33148	3.33148	3.33280	3.46414	3.33148
Ec.typ.	1.60201	1.81798	1.59259	1.58902	1.58965	1.50113	1.91915
Dens.puiss	40.69449	40.50231	40.56237	40.46758	40.51499	41.68727	49.60185
Kurtosis	6.44260	4.09952	6.33274	6.65489	6.64252	8.07227	3.04739
Skewness	0.48592	0.88131	0.47693	0.49775	0.49461	0.25320	0.27832

Table 5.
Descriptive statistics and power density for Cotonou site.

Results	Real values	GP	EPF	EMJ	EML	ML	MMa
Vit.moy	4.02330	3.33855	4.02330	4.02330	4.02384	4.10702	4.02330
Ec.typ.	1.69862	1.60903	1.68343	1.69004	1.69037	1.59787	2.11385
Dens.puiss	63.79816	42.86103	65.06774	65.18791	65.20482	65.20710	80.72832
Kurtosis	2.69366	4.49615	2.78383	2.74248	2.74035	3.46772	1.21240
Skewness	-0.14677	1.41403	-0.13552	-0.14623	-0.14725	-0.35331	-0.05093

Table 6.
Descriptive statistics and power density for Kano site.

Results	Real values	GP	EPF	EMJ	EML	ML	MMa
Vit.moy	4.71277	4.36121	4.71277	4.71277	4.71446	5.10566	4.71277
Ec.typ.	2.25113	2.27993	2.21547	2.23338	2.23425	1.96098	2.30906
Dens.puiss	112.7769	101.1049	112.9734	113.7094	113.8243	120.7997	118.0493
Kurtosis	4.51041	4.58891	4.78343	4.65114	4.64290	8.46210	4.44586
Skewness	0.28197	0.77250	0.28125	0.28838	0.28574	-0.54907	0.26923

Table 7.
Descriptive statistics and power density for Lagos site.

Results	Real values	GP	EPF	EMJ	EML	ML	MMa
Vit.moy	2.70026	2.28902	2.70026	2.70026	2.70148	3.18704	2.70026
Ec.typ.	2.16501	2.00671	2.09101	2.14185	2.14275	1.92858	1.78841
Dens.puiss	45.03188	33.96930	43.48667	45.64691	45.71185	47.88559	32.88635
Kurtosis	6.37692	9.48775	7.22131	6.64091	6.62908	8.87452	13.44755
Skewness	1.27921	2.26594	1.40586	1.31888	1.31577	0.81327	2.23905

Table 8.
Descriptive statistics and power density for Lome site.

Results	Real values	GP	EPF	EMJ	EML	ML	MMa
Vit.moy	3.53826	2.91092	3.53826	3.53826	3.54031	3.70109	3.53826
Ec.typ.	1.94333	1.72320	1.87766	1.92214	1.92333	1.82855	1.97795
Dens.puiss	55.29799	35.38807	55.31737	56.78762	56.88031	58.08273	59.19682
Kurtosis	2.40566	5.58399	2.75858	2.51307	2.50547	2.99925	2.37624
Skewness	0.26244	1.84950	0.29571	0.27278	0.26891	0.01376	0.28701

Table 9.
Descriptive statistics and power density for Niamey site.

Results	Real values	GP	EPF	EMJ	EML	ML	MMa
Vit.moy	3.32004	2.99810	3.32004	3.32004	3.32192	3.40313	3.32004
Ec.typ.	1.77855	1.92446	1.81939	1.75988	1.76090	1.72087	1.91881
Dens.puiss	47.52339	44.26524	47.32130	45.62783	45.70436	46.48500	50.15246
Kurtosis	6.39370	5.23938	5.58891	6.65319	6.63403	6.92268	4.28204
Skewness	1.12474	1.43759	1.03090	1.15983	1.15461	1.06955	0.86939

Table 10.
Descriptive statistics and power density for Niamtougou site.

Results	Real values	GP	EPF	EMJ	EML	ML	MMa
Vit.moy	2.60855	2.27461	2.60855	2.60855	2.61052	3.17567	2.60855
Ec.typ.	1.73066	1.55230	1.63536	1.70829	1.70956	1.39119	1.80945
Dens.puiss	27.76409	21.10048	27.68213	29.44720	29.51493	32.85143	31.83122
Kurtosis	3.77115	6.61094	4.70233	3.96792	3.95486	9.74914	3.23382
Skewness	0.35281	1.35784	0.41839	0.36637	0.36213	-1.38592	0.30581

Table 11.
Descriptive statistics and power density for Ouagadougou site.

Results	Real values	GP	EPF	EMJ	EML	ML	MMA
Vit.moy	2.99765	2.58985	2.99765	2.99765	2.99936	3.12567	2.99765
Ec.typ.	1.60551	1.64402	1.61133	1.58862	1.58954	1.51770	1.83538
Dens.puiss	34.01567	27.77934	33.88799	33.38351	33.43866	34.33200	39.77047
Kurtosis	4.98264	5.57020	4.77967	5.19495	5.17930	5.92498	2.82876
Skewness	0.80675	1.55254	0.78654	0.83299	0.82820	0.66273	0.53418

Table 12.
Descriptive statistics and power density for Tambacounda site.

Results	Real values	GP	EPF	EMJ	EML	ML	MMA
Vit.moy	2.97820	2.69982	2.97820	2.97820	2.97987	3.05704	2.97820
Ec.typ.	1.57577	1.83930	1.64332	1.55950	1.56039	1.51973	1.83429
Dens.puiss	34.72223	35.61662	34.56636	32.65425	32.70756	33.35933	39.48413
Kurtosis	10.16318	5.60248	8.01953	10.61158	10.57942	11.59459	5.24357
Skewness	1.41329	1.28658	1.20300	1.45946	1.45355	1.41555	0.87873

It is important to note that each statistical parameter offers different perspectives that are useful for comparing methods. Thus, the combination of all these statistical indicators offers a possibility to compare the differences between the wind power calculated by the measured data and that of the Weibull distribution function with much more reliability. The results show that the accuracy of the computed wind power density values changes with estimates methods. It is clear that for all stations when the four methods EPF, EMJ, EML and ML are used to compute the Weibull parameters, the values of wind power density computed by the Weibull distribution function are in favorable agreement with the value of wind power density computed by measured data. This conclusion is drawn by the low values of the MAPE, MABE, RMSE. On the other hand, it can be seen that the lowest agreement indices are reached when the MMA and GP methods are applied for the calculation of k and c parameters. Abuja, Accra, Bamako, Conakry, Cotonou, Kano, Lagos, Lome, Niamey, Niamtougou and Tambacounda present the best results in terms of wind energy density calculation when the EPF method is used to calculate the k and c parameters. After the EPF method, for the stations Accra, Conakry, Cotonou, Kano, Lagos, Lome, Niamtougou, the most accurate results are obtained using the EMJ method. For Abuja, Niamey, Ouagadougou and Tambacounda stations, the ML method is the most accurate after the EPF method. As for the Bamako station, the EML method. However, the EPF method gives the best accuracy for all sites. The reason why the most appropriate methods come after the EPF method are different between sites with the wind characteristics variation. It is also important to note that the performances of the EMJ and EML methods are very close to each other based on all statistical indicators with a slight difference but the highest precision is often obtained by the EMJ process. In each table, the most accurate method for each station is indicated in bold. With respect to the weakest methods, the GP and MMA methods show relatively high differences from the other selected methods. In the rankings they occupy the last places so that their use leads to much higher errors than the other methods.

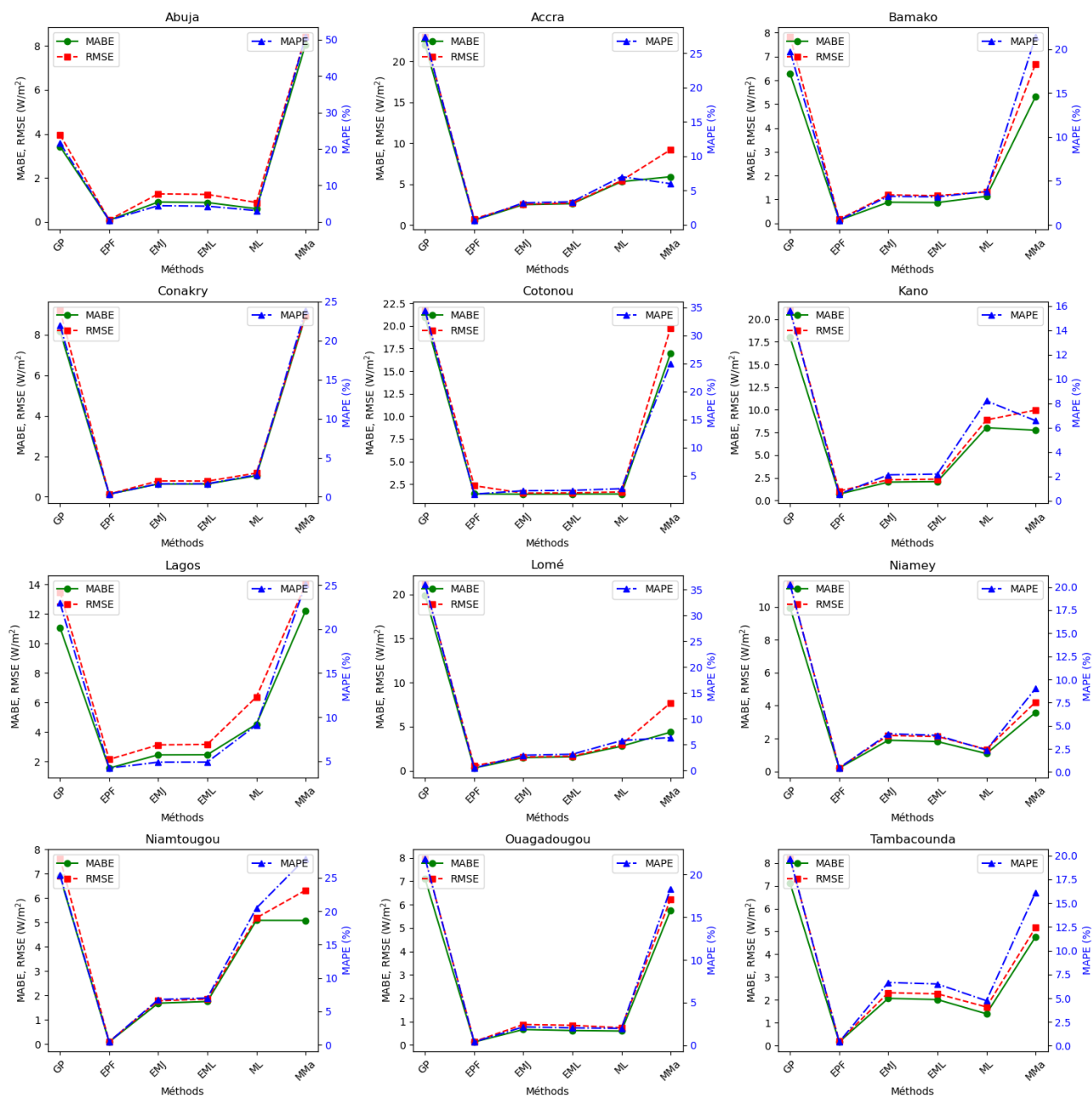


Figure 12. Metrics for methods evaluation.

7. Conclusion

Knowledge of wind power density is therefore of vital importance in assessing the potential of wind energy and in determining the suitability of the site for wind energy development. The two parameters Weibull distribution function has been widely used in various wind energy applications because of its simplicity, adaptability and accuracy. In this work, the performance of six numerical methods namely GP, EPF, EMJ, EML, ML and MMa were evaluated to determine the k and c parameters of the Weibull distribution function for the calculation of wind energy density at twelve stations distributed in the

West African sub-region. To achieve this goal, the wind power density derived from the Weibull function is compared to the power wind density calculated using measured wind data. Evaluations were performed on monthly basis in order to provide more complete analysis. The results indicated that by using different estimation methods to determine the k and c parameters, the accuracy of calculating wind power density values using the Weibull function changes. Based on the analysis of the results of, it is proven that the EPF, EMJ, EML and ML methods provide highly desirable better performance while the GP and MMA methods showed poor performance for all stations. Furthermore, the analysis of the results shows that the most appropriate parameter estimation methods are not the same for all stations examined due to the wind characteristics. At all sites, the EPF method was recognized as the most appropriate of the methods except for the Lagos site where some indicators put the EMJ method in first place. The methods that occupy second place after EPF vary according to the sites: the EMJ method for Accra, Conakry, Cotonou, Kano, Lome, Niamtougou; EML for Bamako, ML for Abuja, Niamey, Ouagadougou and Tambacounda. The parameters estimated can be used with excellent performance to represent the monthly wind speed distribution and determine the different statistical properties of the power density. Nevertheless, it should be mentioned that since each station benefits from specific wind power characteristics, the results obtained in this study with respect to the efficiency of the estimation methods of the Weibull distribution function parameters can only be extended to regions with identical wind power characteristics.

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Conceptualization, Agbassou Guenoukpati; methodology, Agbassou Guenoukpati, Pierre Akuété Agbessi and Komi Amoussou; validation, Adekunlé Akim Salami and Agbassou Guenoukpati; formal analysis, Adekunlé Akim Salami and Agbassou Guenoukpati; writing-original draft preparation, Agbassou Guenoukpati; writing-review and editing, Pierre Akuété Agbessi and Komi Amoussou; supervision, Adekunlé Akim Salami. All authors have read and agreed to the published version of the manuscript.

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