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Exploring the impact of mathematics teachers' pedagogical content knowledge on student achievement: A 'H-V' perspective case study

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Abstract: The Pedagogical Content Knowledge (PCK) of teachers has attracted the attention of many educators and plays an indispensable role in their classroom teaching. The purpose of this study is to investigate the PCK of mathematics teachers with different experiences and explore the impact of PCK on student achievement. The study adopted a mixed study design, with participants consisting of two middle school mathematics teachers with different teaching experiences and their students. Interviews, questionnaire surveys, classroom observations, and statistical analysis of student grades are used as data source tools to conduct content analysis, descriptive statistics, and inference analysis on the data. The "H" perspective is a horizontal comparison of the PCK of different teachers and the achievement of students in different classes, while the "V" perspective is a vertical comparison of the achievement of students in the same class. The horizontal research results show that there is a significant difference in PCK between the two teachers, and there is also a significant difference in student performance in the class they teach. The vertical research results show that after improving the level of PCK, the achievement of students taught by teachers with short teaching experience have significantly improved, while the achievement of students taught by teachers with long teaching experience have not improved significantly. It also provides suggestions for further improving research.

Keywords: *Horizontal, Vertical, Mathematics teacher, Pedagogical content knowledge, Student achievement.*

1. Introduction

The importance of mathematics education is self-evident. In 1986, Shulman introduced the concept of Pedagogical Content Knowledge (PCK), positing that teachers must effectively integrate subject matter knowledge with pedagogical knowledge to facilitate students' ease of understanding, stimulate interest, and enhance learning capabilities.Even though education is being actively promoted, some problems are emerging, such as teachers' personal abilities and classroom teaching quality. Some teachers still adhere to the past teaching methods and fail to keep up with the times in teaching modern students,overly emphasizing the instillation of academic subject knowledge,neglecting the combination and connection between subject knowledge and teaching methods,leading to the gradual distance between subject knowledge and teachers' actual teaching practice. Some teachers experience a rigid phenomenon of applying prescribed models during the teaching process. For example, group discussions are fixed by some teachers at a certain time period, and discussions begin as soon as the time is up, resulting in some students having to stop their current thinking and join in collaborative discussions that are not beneficial to themselves. Alternatively,regardless of the actual cognitive level of students,students are blindly allowed to reason on their own,resulting in a significant waste of time and no new progress. These teachers cannot effectively apply PCK and only imitate existing models, resulting in unsatisfactory teaching results. Therefore, it is particularly important to urgently improve

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the level of PCK of teachers. Since Schulman proposed the concept of PCK, numerous scholars have subsequently conducted research on this topic, gradually delving into the development of PCK and its influencing factors, starting with the connotation and components of PCK. Currently, less research has been carried out on the relationship between teachers' PCK and student achievement. Although some studies have investigated the PCK of science teachers,which can reveal the relationship between the PCK level of science teachers and students' achievements and attitudes,this has great reference value for me to horizontally analyze the relationship between mathematics teachers' PCK and students' achievements,however,these studies were not conducted from the perspective of mathematics teachers,and did not consider whether the improvement of teacher PCK would bring new changes to students' achievements from a vertical perspective. Because teachers' PCK is constantly integrated and formed in teaching practice,and is dynamically changing. There have also been studies exploring how teachers' mathematical knowledge can help students improve their mathematical achievement how teachers accurately explain mathematical terms and concepts in the classroom,and how it affects students' mathematical achievement,however,this study only focuses on the independent contribution of mathematical subject knowledge to student achievement without mentioning other aspects of PCK, such as teaching objectives knowledge,teaching content organization knowledge,etc.,nor has it been studied from a vertical perspective. Some studies have explored the development trends and characteristics of teacher PCK at various career stages,which is of great reference value as it can help us consciously improve teacher PCK and provide a theoretical basis for longitudinal research.Therefore, exploring the impact of mathematics teachers' PCK on student achievement from both horizontal 'H' and vertical 'V' perspectives holds certain theoretical value and practical significance.

2. Research Problem

Although there are currently numerous studies on PCK, the majority of them are focused on the positioning,characteristics,development,influencing factors,and other aspects of teachers' PCK. Few studies have explored the relationship between mathematics teachers' PCK and academic achievements of middle school students from both horizontal and vertical perspectives. Based on this, the present study primarily addresses the following two problems: First, from a horizontal 'H' perspective, it compares the PCK of different mathematics teachers and the achievements of students in different classes. Second, from a vertical 'V' perspective, it compares the achievements of students within the same class. These two problems are addressed in two distinct phases.

3.Research Design

3.1. Research Object

A primary school in the urban area of Neijiang City, Sichuan Province, China was randomly selected, and two parallel classes were randomly selected from the ninth grade stage. There is no significant difference in the overall achievement of the students in these two classes from eighth grade to ninth grade,so the overall level variable of the students is controlled. But when choosing parallel classes, it is advisable to abandon selecting teachers with similar teaching experience and instead choose teachers with significant differences in teaching experience, which facilitates the comparison of differences in PCK. The participants are two math teachers and 30 students per class. The two teachers differ in terms of experience and professional titles. One is Junior Teacher A who has been teaching for three years,and the other is Associate Senior Teacher B who has been teaching for 12 years. I have chosen two representative new courses,the image and properties of the quadratic function $y = a(x-h)^2 + k$ and the inverse proportional function $y = k / x$ as the testing courses. Firstly,these two courses are designed for students who have just entered ninth grade for about a week in the first and second semesters,which can better control the errors caused by teacher experience. Secondly, it is because the course is one that causes a lot of students to struggle and is a very important course that better reflects the differences in teachers' PCK.Third, the selection of this course is due to my previous experience

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teaching the mathematics curriculum to ninth-grade students, which has afforded me a familiarity with the content of the textbooks. After explaining the research intention,the two teachers readily agreed.

3.2. Research Methods

The study employs a mixed-methods approach, primarily utilizing interviews, surveys, and data analysis methods to examine the impact of mathematics teachers' PCK on student achievement across two phases. In the first phase, interviews and surveys are conducted to collect views, understandings, and methods related to PCK from two teachers, analyzing and contrasting the differences between them. This is followed by a horizontal analysis of the impact of different teachers' PCK on students' achievement based on the students' stage-wise exam scores in quadratic functions. In the second phase, approximately six months later, after the teachers have consciously integrated and improved their level of PCK, students' stage-wise exam scores in inverse proportion functions are collected for a vertical analysis of the relationship between the same teacher's PCK and student achievement. It should be noted that the second phase only involves a vertical quantitative analysis of student performance, without conducting further interviews or surveys with the two teachers. This is due to the short interval between phases, where teachers' recollections of interview and survey contents could introduce subjective errors. However, observations of the two teachers' PCK are not abandoned and are mentioned throughout the research process.

3.3. Data Analysis

Quantitative data were analyzed using SPSS statistical software. Based on the characteristics of the collected data, an independent samples t-test was used for horizontal comparison, while a paired samples t-test was utilized for vertical comparison.

Qualitative data analysis employed content analysis methods to explore mathematics teachers' understanding of PCK.

4. The First Stage of Research Process and Results Analysis

Use interviews and questionnaire surveys to collect and analyze data on teachers' PCK, and collect and analyze students' stage test achievements. The details are as follows:

4.1. Conduct Interviews and Qualitative Analysis with Teachers

The interview content draws inspiration from Grossman et al.'s division of subject teaching content into six parts, as this theoretical division has been widely acknowledged and explored by many scholars. Consequently, the interview content is based on six aspects: teaching purpose knowledge, subject content knowledge,content organization knowledge,student understanding knowledge, teaching strategies and presentation methods knowledge, and effect feedback knowledge, with one question selected for each aspect. It should be noted that the interview questions will be given to two teachers in advance, so that necessary preparations can be made based on their own teaching practice, and the answers will not be too scattered or too general, resulting in inappropriate and inaccurate interview results.

4.1.1. Teaching Purpose Knowledge

It refers to the teacher's selection of knowledge and concepts with teaching value for the topic, in order to achieve a certain teaching purpose.

Interview record: How did you set the teaching objectives for each class?

Teacher A: The teaching purpose is to guide the flag direction of this lesson, and the teaching content should revolve around the teaching purpose. Firstly, we should refer to the curriculum standards and grasp the key points, difficulties, and exam points. Secondly, it is necessary to clarify the relationship between this lesson and the content of the previous lesson or chapter, grasp the context of knowledge, and thus establish appropriate teaching objectives.

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Teacher B: The teaching purpose should focus on the development needs of students as the main body, that is, what basic knowledge, basic skills, practical experience, and mathematical ideas should be provided to students. We also identify the differences between students' existing knowledge and cognitive levels and curriculum standards, grasp knowledge and students from a macro perspective, adapt flexibly, and not stick to curriculum standards and key and difficult points, identify the most recent development areas of most students, and thus find the most suitable teaching objectives.】

Interview analysis: From the perspective of setting teaching objectives, Teacher A emphasizes the implementation of curriculum standards. He believes that as long as the textbook system structure is followed and gradually promoted, the curriculum objectives specified in the curriculum standards can be achieved. However, neglecting students' autonomy and actual learning situation can easily lead to a significant gap between teaching objectives and what students have learned, resulting in the goal being set but the teaching task not being completed. Teacher B has made up for the relative shortcomings of Teacher A, being able to flexibly handle the gap between the curriculum standards and students based on their actual situation. Being able to pay attention to students' subjective development and gradually enable them to truly acquire knowledge. This can also be seen from the teacher's insightful guidance and inspiration to students in the classroom records, as mentioned later.

4.1.2. Subject Content Knowledge

It refers to the overall grasp, familiarity, and understanding of specific content knowledge by teachers.

【Interview record: What is the role and status of the image and properties of quadratic functions in ninth grade?

Teacher A: The images and properties of quadratic functions are an important chapter in junior high school mathematics, laying a solid foundation for deeper learning of senior high school mathematics and other related subjects in the future. Moreover, it is a mandatory test point for the middle school entrance examination, often combined with equations and inequalities to jointly evaluate students' problem-solving ability.

Teacher B: The image and properties of quadratic functions are a very important part of junior high school mathematics, playing a role in connecting the following and the following. It is crucial to cultivate students' mathematical thinking and logical thinking from special to general, deepen their thinking and understanding of life mathematics, stimulate their interest, and help them better understand the charm of mathematics and experience the essence of mathematics.】

Interview analysis: From the perspective of importance, the views of the two teachers are consistent and both believe that they have a role in connecting the past and the future. However, Teacher A pays more attention to the guidance of exams, which is understandable, but it can easily lead to a utilitarian mindset of both teachers and students towards mathematics. Teacher B starts from interests and focuses on students' learning interests, allowing them to understand mathematics in their studies, which is more conducive to cultivating students' long-term mathematical emotions.

4.1.3. Content Organization Knowledge

It refers to how teachers arrange the logical order and link settings of teaching content.

【Interview record: Taking the graph and properties of the quadratic function $y=a\left(x-h\right)^2+k$ as an example, how would you arrange the teaching content for this class?

Teacher A: According to the conventional mode, proceed in the order of self-directed learning, classroom exploration, in class practice, and expansion and consolidation. It is worth noting that the questions or exercises set for the corresponding links should be appropriate and not too difficult or simple.

Teacher B: The arrangement of teaching content should be comprehensively considered based on students' learning level, teaching time, and course objectives. Carry out each link according to a certain

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pattern but not limited to the pattern, and adjust the teaching content according to the actual situation in the classroom in a timely manner. Introduce with real-life examples, use the knowledge from the previous section as a foundation, and combine it with animation mode to stimulate students to think independently and learn independently. Pre set exploration questions with different levels of difficulty, and based on the students' self-learning situation, propose suitable exploration questions. In short, everything is arranged according to the actual situation of the students.】

Interview analysis: Both teachers have a clear understanding of the basic teaching mode in the classroom and are able to flexibly handle difficult and easy problems. However, Teacher B is superior to Teacher A in terms of flexibility and predictability, fully considering the subjective needs of students.

4.1.4 Knowledge understand by students

It refers to the knowledge that teachers have about the internal connections and logical structures of students' understanding of concepts, principles, and other learning content.

【Interview record: Taking the graph and properties of the quadratic function y=a (x-h) 2+k as an example, what old knowledge did students have before learning this lesson? What difficulties will you encounter in learning this lesson?

Teacher A: The student has already studied functions once and has a preliminary understanding of quadratic functions. This lesson is developed based on $y=ax^2$, and there should be no major problems with learning. The possible problem encountered is that the quadratic function $y=a(x-h)^2+k$ image simultaneously shifts to the left (right) and up (down) to generate a new function image, which makes understanding more difficult.

Teacher B: Students who have just entered ninth grade are unfamiliar with quadratic functions. Although they have studied quadratic functions, the difficulty of quadratic functions is significantly higher than that of quadratic functions. This class is based on the development of $y=ax^2$, and students have a certain understanding and believe that learning will not be too difficult. The first difficulty is the relationship between the function image and the $y=ax^2$ image, the translation rules and practical applications of the image, and the second difficulty is the idea of combining numbers and shapes contained in the function and its transformation.】

Interview analysis: The viewpoints of the two teachers are basically the same, both of which can uncover students' understanding of knowledge and have a relatively consistent grasp of the difficulties of functions. But Teacher B is good at cultivating students from the perspective of mathematical thinking, standing higher and able to better cultivate students with core mathematical literacy.

4.1.5. Knowledge of Teaching Strategies

It refers to the series of teaching activities and their effects that teachers need to study how to present teaching content to students, design by teachers, and demonstrate their teaching behavior.

【Interview record: How do you usually improve your teaching ability and classroom effectiveness?

Teacher A: My teaching career is only three years, and there are still many areas that need to be learned and improved. I will actively learn from my predecessors, carefully summarize and reflect, strengthen the management of classroom content and teaching language, fully understand the learning situation, take students as the main body, let more students participate in discussions and cooperation, and continuously improve teaching level.

Teacher B: Firstly, teachers must adhere to a lifelong learning attitude, grasp the forefront of education, and constantly update their teaching concepts. Secondly, learn more from young people about educational technology and keep up with the pace of the times. Last but not least, take students as the main body, inspire more, encourage more, and coax more, cultivate students' interests, and make them act actively and think actively.】

Interview analysis: Both teachers answered based on their own situation, and on the surface, there was no distinction between good and bad. They all mentioned the need for continuous learning and

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believed that the classroom should be student-centered. These are all basic ideas that an excellent teacher must possess. Teacher B also mentioned cultivating students' interest, which is the most effective way for students to experience mathematics.

4.1.6. The Knowledge of Effectiveness Feedback

It refers to the teacher's evaluation of students' learning situation, including their answers, ideas, etc. Timely adjust and optimize teaching progress, difficulty, etc. from student feedback.

【Interview record: How do you determine how much students have mastered the content taught in class?

Teacher A: Based on the students' reactions to the teacher's questions in class, whether they can accurately answer the exercises set by the teacher, and observe the students' attentiveness in class. Generally speaking, practice can best reflect the true mastery of the content, as students' language organization ability can affect the effectiveness of answering questions.

Teacher B: There will always be students in a classroom who react quickly. If these students answer questions, it obviously cannot represent the overall student. If the teacher arranges the teaching progress according to the responsive students at this time, it will make many students unable to keep up with the teacher's pace. So teachers should ask more questions to middle school students, regularly arrange tests, and communicate with college students more frequently to grasp the actual learning situation.】

Interview analysis: Both teachers believe that classroom questioning, exercise tests, etc. have good testing effects. But the teacher emphasized the questioning of middle school students, which shows that Teacher B is aware that students who respond quickly may mislead the teacher and cannot reflect the effectiveness of most students' listening. This is a manifestation of a better understanding of students.

Overall, based on the analysis of six aspects of PCK, Teacher B's comprehensive mathematics PCK is superior to Teacher A.

4.2. Conduct A Questionnaire Survey on Teachers and Quantitative Analysis

The questionnaire still draws inspiration from Grossman et al.'s division of subject teaching content into six parts, namely teaching purpose knowledge, student understanding knowledge, content organization knowledge, teaching strategies and presentation methods knowledge, subject content knowledge, and effect feedback knowledge. Each section has 3 questions, totaling 18 questions. Each question has 5 options, which are very inconsistent, not very consistent, generally consistent, quite consistent, and very consistent. The corresponding score is 1-5 points, and the higher the score, the higher the teacher's level of subject teaching. The scores of teacher A and B are 58 and 74 respectively.

Table 1.
Basic descriptive statistical results of the achievements of two teachers

Group statistics					
	Teacher	**N**	**Mean**	**Std. deviation**	**Std. error mean**
Achievement	A	18	3.2222	0.73208	0.17255
	B	18	4.1111	0.67640	0.15943

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Table 2.
Independent sample t-test results.

Independent samples test										
		Levene's test for equality of variances		t-test for equality of means						
		F	Sig.	t	df	Sig. (2-tailed)	Mean difference	Std. error difference	95% confidence interval of the difference	
									Lower	Upper
Achievement	Equal variances assumed	0.104	0.749	-3.784	34	0.001	-0.88889	0.23493	-1.36633	-0.41145
	Equal variances not assumed			-3.784	33.789	0.001	-0.88889	0.23493	-1.36644	-0.41134

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Result analysis: The independent sample t-test results of the average scores of the two teachers showed that the homogeneity of variance test was not significant ($p>0.05$), indicating that the variance of the two groups' grades was homogeneous. There is a significant difference in scores between the two groups ($t=-3.784$, $df=34$, $p=0.001<0.05$), which means that Teacher B has significantly better mathematical PCK than Teacher A.

4.3. Collection and Quantitative Analysis of Students' First Stage Exam Achievement

Conduct a horizontal comparative analysis of the first stage achievement of classes A and B. After the completion of all courses in Chapter 1, Quadratic Function Images and Properties, the Mathematics Teaching and Research Group organized a final exam for this chapter, with a maximum score of 100 points and a 90 minute answer time. This article collects and organizes the scores of students from Class A and Class B in this exam, and uses SPSS to analyze the data of student scores.

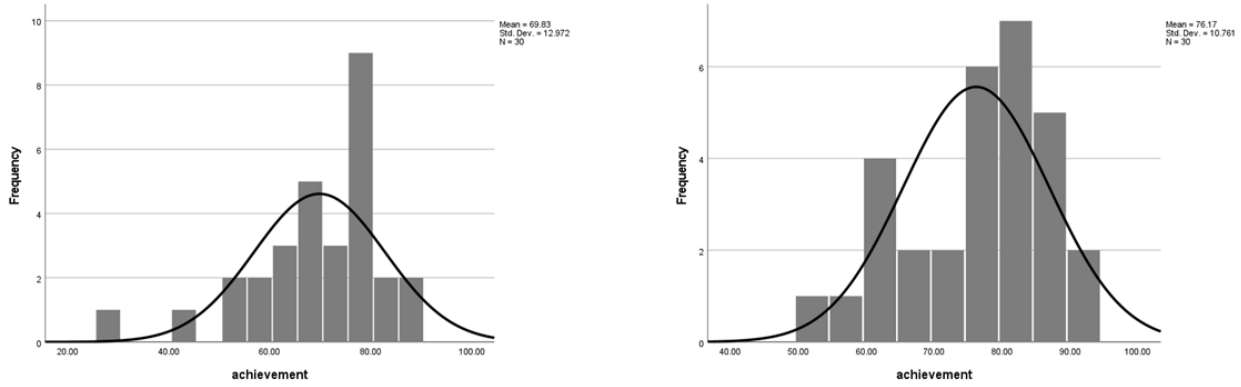

Figure 1.
Histogram of the frequency of achievements in two classes.

Table 3.
Basic descriptive statistical results of two classes' achievements.

Group statistics					
	Class	**N**	**Mean**	**Std. deviation**	**Std. error mean**
achievement	A	30	69.8333	12.97234	2.36841
	B	30	76.1667	10.76099	1.96468

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Table 4.
Independent sample t-test results.

Independent samples test										
		Levene's test for equality of variances		t-test for equality of means						
									95% confidence interval of the difference	
		F	Sig.	t	df	Sig. (2-tailed)	Mean difference	Std. error difference	Lower	Upper
achievement	Equal variances assumed	0.108	0.744	-2.058	58	0.044	-6.33333	3.07723	-12.49308	-0.17359
	Equal variances not assumed			-2.058	56.086	.044	-6.33333	3.07723	-12.49756	-.16911

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Result analysis: The independent sample t-test results of the average test scores of students in classes A and B showed that the homogeneity of variance test was not significant ($p=0.74>0.05$), indicating that the variances of the two groups' scores were homogeneous. There is a significant difference in grades between the two classes ($t=-2.058$, $df=58$, $p=0.044<0.05$), indicating that students in Class B have significantly better stage test scores than those in Class A.

Therefore, from the perspective of horizontal comparative analysis, Teacher B's subject teaching level is higher, and the students' achievements in the class he teaches are correspondingly better. The scientific and judicious application of PCK by teachers proves advantageous in enhancing the mathematical achievement of students in the class.

5. The Second Stage of Research Process and Results Analysis

After approximately 6 months, gather student scores on the second stage exam and conduct a vertical comparison analysis with the results of the first stage exam. It should be noted that Teacher A is aware that there is still a significant gap in his knowledge of mathematics teaching. Therefore, in the next six months, Teacher A will start from multiple aspects, such as reading relevant books, seeking advice from excellent teachers, observing excellent teachers' classroom teaching, etc., to improve his mathematics teaching level, and continuously reflect and summarize his teaching behavior and methods in the classroom. In the second stage, although no interviews or questionnaires were conducted with the two teachers, during the past 6 months, we often had phone discussions with Teacher A and irregularly observed Teacher A's teaching classroom. We found that Teacher A's teaching level, concepts, methods, and other aspects had indeed improved significantly. Due to Teacher B's extensive experience and limited communication, we have seldom questioned or suggested Teacher B's mathematical teaching knowledge. However, Teacher B continues to engage in ongoing learning and refinement his mathematical PCK.

After the completion of all courses on the inverse proportion function images and properties in the first chapter of the ninth grade second volume, the mathematics teaching and research group organized a final exam for this chapter, with a maximum score of 100 points and a 90 minute answer time. This article collects and organizes the scores of students from Class A and Class B in this exam, and uses SPSS to analyze the data of student scores.

5.1. Horizontal Comparative Analysis of Grades in Class A and Class B in the Second Stage

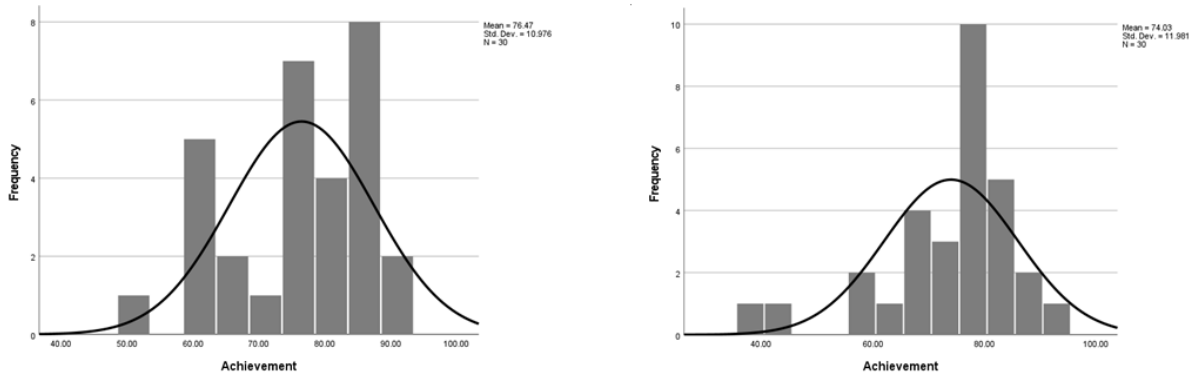

Figure 2.
Histogram of the frequency of achievements in two classes

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Table 5.
Basic descriptive statistical results of two classes' achievements.

Group statistics					
	Class	**N**	**Mean**	**Std. deviation**	**Std. error mean**
Achievement	A	30	74.0333	11.98126	2.18747
	B	30	76.4667	10.97563	2.00387

Table 6.
Independent sample t-test results.

Independent samples test										
		Levene's test for equality of variances		**t-test for equality of means**						
									95% confidence interval of the difference	
		F	**Sig.**	**t**	**df**	**Sig. (2-tailed)**	**Mean difference**	**Std. error difference**	**Lower**	**Upper**
Achievement	Equal variances assumed	0.002	0.964	-0.820	58	0.415	-2.43333	2.96656	-8.37156	3.50489
	Equal variances not assumed			-0.820	57.560	0.415	-2.43333	2.96656	-8.37252	3.50586

Result analysis: From the independent sample t-test of the average scores of students in the two classes in the second stage, it was found that assuming equal variance, $p=0.96>0.05$, the homogeneity test of variance was not significant, and there was no significant difference in the scores of students in the two classes ($t=0.82$, $df=58$, $p=0.415>0.05$). Although the average score of Class B is 2.4 points higher than that of Class A, there is no significant difference in the grades between the two classes. From this, it can be inferred that Teacher A's level of PCK has significantly improved.The rapid growth of achievements among Class A students was what caused it.

5.2. Vertical Comparative Analysis of the Achievements of Class A Students in Two Stages

Table 7.
Basic descriptive statistical results of students' achievements in two stages of Class A.

Paired samples statistics					
		Mean	**N**	**Std. deviation**	**Std. error mean**
Pair 1	Stage 1	69.8333	30	12.97234	2.36841
	Stage 2	74.0333	30	11.98126	2.18747

Table 8.
Paired samples correlation.

Paired samples correlations				
		N	**Correlation**	**Sig.**
Pair 1	Stage 1 & Stage 2	30	0.990	0.000

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Table 9.
Paired sample t-test results.

Paired samples test									
		Paired differences					t	df	Sig. (2-tailed)
		Mean	Std. deviation	Std. error mean	95% confidence interval of the difference				
					Lower	Upper			
Pair 1	Stage 1 – Stage 2	-4.20000	2.02399	.36953	-4.95577	-3.44423	-11.366	29	0.000

Result analysis: From the average score of Class A students, it can be seen that the second stage is 4.2 points higher than the first stage. Comparing the paired sample t-test results of two stages, the results showed that $t=-11.366$, $df=29$, $p<0.05$, indicating that Class A students' grades in the second stage were significantly higher than those in the first stage. The results can explain the significant improvement in students' achievements after Teacher A consciously improved their PCK.This also indicates that Teacher A initially possessed insufficient levels of PCK, suggesting a significant potential for improvement.

5.3. Vertical Comparative Analysis of the Achievements of Class B Students in Two Stages

Table 10.
Basic descriptive statistical results of students' achievements in two stages of Class A.

Paired samples statistics					
		Mean	N	Std. deviation	Std. error mean
Pair 1	Stage 1	76.1667	30	10.76099	1.96468
	Stage 2	76.4667	30	10.97563	2.00387

Table 11.
Paired samples correlation.

Paired Samples Correlations				
		N	Correlation	Sig.
Pair 1	Stage 1 & Stage 2	30	.997	.000

Table 12.
Paired sample t-test results.

Paired samples test									
		Paired differences					t	df	Sig. (2-tailed)
		Mean	Std. deviation	Std. error mean	95% confidence interval of the difference				
					Lower	Upper			
Pair 1	Stage 1 – Stage 2	-0.30000	0.91539	0.16713	-0.64181	0.04181	-1.795	29	0.083

Result analysis: From the average score of Class B students, it can be seen that the second stage is 0.3 points higher than the first stage. Comparing the paired sample t-test results of two stages, the results showed that $t=-1.795$, $df=29$, $p=0.083>0.05$, indicating that there was no significant difference between the grades of Class B students in the second stage and the first stage. This may be due to the fact that Teacher B already has good teaching experience, making it difficult to significantly improve the level of PCK in a short period of time and there is no positive impact on students' achievement. It

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may also be due to reaching a stage teaching bottleneck, resulting in no significant improvement in student achievement.

6. Conclusion and Recommendations

This study investigates the impact of mathematics teachers' PCK on student achievement from both horizontal and vertical "H-V" perspectives, involving two mathematics teachers with differing years of teaching experience. The results indicate a clear positive impact of mathematics PCK on student achievement, a conclusion that is validated among teachers with shorter teaching experience, while no significant correlation was found between the PCK of more experienced teachers and student achievement. This finding may be incidental, given the investigation's focus on individual cases and the small sample size, thus lacking broad generalizability. However, as a research discovery, it undoubtedly provides new insights, especially the innovative value of dual comparisons—horizontal and vertical—filling a gap in previous research and offering a theoretical foundation for subsequent larger-scale studies. Future research should closely observe the classroom dynamics between teachers and students, capturing the application of teachers' PCK during instruction. It should also design reasonable surveys to comprehensively understand students' perceptions of their teachers' PCK.

Based on the research findings, a conscious effort by teachers to enhance their PCK can facilitate improved student achievement. Therefore, the following recommendations are proposed for the enhancement of mathematics teachers' PCK: firstly, actively utilize curriculum standards to improve the level of teaching objectives, subject knowledge, and content organization knowledge. Regardless of teaching experience, curriculum standards should be organically integrated with actual teaching content and objectives to construct classroom teaching that is most suitable for students.The second recommendation is to create life situations related to the curriculum to enhance the level of teaching strategy knowledge. Foster a lively classroom atmosphere and use appropriate language arts to stimulate student interest.The third recommendation is to fully understand the learning situation to improve the level of knowledge in understanding students. Do not deviate from the actual cognition and level of students, and maximize classroom effectiveness. The fourth recommendation is to continuously summarize and reflect to improve the level of knowledge in feedback effectiveness. Observe excellent classrooms and seek advice from outstanding teachers to learn from their strengths and weaknesses, enhance and optimize one's teaching ability, and shape the most suitable and efficient teaching model for oneself.

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Appendix 1

Investigation on the current situation of mathematics teachers' PCK.

Dear teacher:

Hello! Thank you very much for taking part in this survey. This is a survey on the basic status quo of Pedagogical Content Knowledge(PCK for short) of mathematics teachers. In order to obtain real data, please read the questions carefully and answer the questions in the questionnaire according to your actual situation. There are 18 questions in the questionnaire. Please choose one of A, B, C, D, and E according to the frequency of events in each question. Conform to, very conform to, the degree increases in turn, and the corresponding score is 1-5 points. thank you for your support!

1. Teaching target knowledge

(1) The target requirements of the new curriculum standards under the background of the senior high school entrance examination are very clear

A Strongly Disagree B Slightly Agree C Moderately Agree D Quite Agree E Strongly Agree

(2)The teaching objectives of the function content of the teaching section are very clear

A Strongly Disagree B Slightly Agree C Moderately Agree D Quite Agree E Strongly Agree

(3)Be very clear about the exploratory goals of the functions taught

A Strongly Disagree B Slightly Agree C Moderately Agree D Quite Agree E Strongly Agree

2. Knowledge understand by students

(1)Understand the current cognitive level and basic ability of each student very well

A Strongly Disagree B Slightly Agree C Moderately Agree D Quite Agree E Strongly Agree

(2)Be able to accurately predict the confusion and difficulties that students may encounter in the learning process

A Strongly Disagree B Slightly Agree C Moderately Agree D Quite Agree E Strongly Agree

(3)Be able to accurately capture the changes in students' thinking and willpower in the classroom and make corresponding adjustments

A Strongly Disagree B Slightly Agree C Moderately Agree D Quite Agree E Strongly Agree

3. Content organization knowledge

(1)Able to accurately draw the logical structure diagram of the knowledge in this chapter

A Strongly Disagree B Slightly Agree C Moderately Agree D Quite Agree E Strongly Agree

(2)Be able to scientifically arrange the sequence of teaching content and accurately grasp the proportion of each link
A Strongly Disagree B Slightly Agree C Moderately Agree D Quite Agree E Strongly Agree
(3)Able to adjust the content in time according to the actual situation of classroom teaching
A Strongly Disagree B Slightly Agree C Moderately Agree D Quite Agree E Strongly Agree
4. Teaching strategy knowledge
(1)Be able to formulate appropriate teaching strategies to enable students to learn actively and actively
A Strongly Disagree B Slightly Agree C Moderately Agree D Quite Agree E Strongly Agree
(2)Pay attention to the use of heuristic teaching and be good at encouraging students to think
A Strongly Disagree B Slightly Agree C Moderately Agree D Quite Agree E Strongly Agree
(3)Able to set up effective teaching strategies corresponding to different course types
A Strongly Disagree B Slightly Agree C Moderately Agree D Quite Agree E Strongly Agree
5. Subject content knowledge
(1)Familiar with the mathematical thinking methods corresponding to the mathematics content taught
A Strongly Disagree B Slightly Agree C Moderately Agree D Quite Agree E Strongly Agree
(2)Able to use other resources to reasonably replace the content of textbooks according to teaching needs
A Strongly Disagree B Slightly Agree C Moderately Agree D Quite Agree E Strongly Agree
(3)Able to explain mathematically the general laws and procedures of mathematics
A Strongly Disagree B Slightly Agree C Moderately Agree D Quite Agree E Strongly Agree
6. Effect feedback knowledge
(1)Able to use a variety of methods, such as classroom observation, questions, tests, etc. to evaluate students' knowledge mastery
(2)Able to adjust teaching methods and teaching content in a timely manner according to students' feedback, and improve teaching efficiency
(3)Be able to clearly point out the strengths and weaknesses of students through the feedback of students, and make reasonable suggestions

Appendix 2.
Scores of the PCK status survey of teachers A and B.

Question number	A teacher score	B teacher score	Question number	A teacher score	B teacher score
1	3	4	10	3	4
2	4	4	11	3	4
3	4	4	12	3	3
4	3	3	13	2	4
5	3	4	14	4	5
6	3	4	15	3	4
7	4	5	16	3	5
8	2	3	17	3	5
9	5	5	18	3	4

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Appendix 3.
The test achievements of students in the two classes of the first stage.

Student serial number	Class A achievement	Class B achievement	Student serial number	Class A achievement	Class B achievement
1	28	52	16	73	79
2	41	58	17	74	82
3	54	60	18	76	82
4	55	60	19	77	82
5	60	61	20	77	83
6	60	62	21	77	83
7	65	66	22	78	84
8	65	67	23	78	84
9	65	71	24	79	85
10	68	74	25	79	86
11	68	75	26	79	86
12	69	76	27	81	87
13	69	76	28	85	88
14	69	77	29	86	90
15	72	78	30	88	91

Appendix 4.
The test achievements of students in the two classes of the second stage.

Student serial number	Class A achievement	Class B achievement	Student serial number	Class A achievement	Class B achievement
1	38	51	16	77	78
2	45	59	17	78	83
3	59	61	18	78	83
4	59	59	19	79	84
5	64	60	20	79	82
6	66	63	21	80	83
7	68	66	22	80	85
8	68	68	23	82	86
9	69	72	24	81	84
10	72	74	25	84	87
11	73	74	26	85	87
12	75	76	27	85	87
13	77	77	28	86	88
14	77	77	29	88	91
15	77	78	30	92	91

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