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Research on the volatility forecasting model of KOSPI index returns using AR(M)-GARCH(P,Q) model

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Abstract: In this study, we estimated the volatility of the KOSPI index returns and analyzed volatility trends. The data used in the study consisted of monthly observations from January 2005 to December 2022, and the KOSPI index raw data was transformed into log returns. The volatility estimation model used the AR(m)-GARCH(p,q) model, which combines the autoregressive error model and the GARCH model that explains the persistence of volatility at low orders. The goodness of fit of the model was confirmed using the Portmanteau Q-test and LM-test. Applying the autoregressive error model revealed significant autocorrelation in the log returns of the KOSPI index at lags 3 and 6. Residual analysis indicated that the residuals followed white noise, but the squared residuals exhibited heteroscedasticity. Therefore, after fitting the autoregressive error model, we applied the GARCH model and conducted residual analysis, finding both the residuals and squared residuals significant at a 5% significance level. The volatility forecasting results indicated a continuous increase in volatility. The findings of this study are expected to provide important implications for policymakers responsible for risk management in the Korean stock market.
Keywords: *AR(m)-GARCH(p,q) model, Autoregressive error model, GARCH model, Q-test and LM-test t. Return rate.*

1. Introduction

Recent uncertainties in the global economy, coupled with the proliferation of advanced financial technologies and algorithmic trading, have led to increased volatility in international financial markets. Key factors contributing to heightened market volatility include US interest rate hikes, political instability, trade wars, regulatory constraints on major financial institutions' risk investments, as well as potential shifts to a tightening monetary policy in China and fiscal crises in Europe. These global issues significantly impact various domestic industries such as financial markets, financial industries, export industries, real estate markets, and more, including the domestic financial market sectors, which involve foreign exchange, stock, bond, and short-term capital markets. Particularly, sustained US dollar strength expands financial market volatility, increasing uncertainty in both domestic and international stock indices, interest rates, exchange rates, and other factors. This acceleration of the safe asset preference by investors can result in significant declines in stock indices and bond prices. It also affects domestic real economies, thereby reducing consumer and investor confidence and hindering economic growth rates. The volatility of financial asset returns, such as derivatives, stocks, futures, exchange rates, etc., is a crucial element in financial trading, encompassing risk management, derivative valuation, and portfolio selection.

The stock market connects investors like enterprises, individuals, and institutions, through the securities issued by businesses to raise industrial capital within a country's national economy, enabling efficient distribution of capital. The expansion of stock market volatility can weaken the transmission of information regarding corporate value, resulting in misguided decision-making by enterprises,

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individuals, and government officials [1]. Factors influencing stock return volatility include governmental legislation, collective bargaining agreements, and policy decisions impacting the economy, as well as inflation, consumer spending indicators, and GDP indicators potentially influencing market performance. Furthermore, enhanced governmental regulation in specific industries and the performance of individual businesses, such as the launch of innovative products meeting consumer satisfaction, also affect volatility [2]. Volatility serves as an anticipated indicator of financial asset uncertainty and price fluctuations. The increase in stock market volatility serves as a destabilizing factor due to changes in political and economic performance, as well as corporate activities. Managing volatility is crucial for the stability and growth of the national economy, considering it as a key means of corporate financing and a representative savings method for households. Stock returns reflect the intrinsic value of companies and are therefore utilized as a significant indicator in financial markets. Therefore, this study aims to examine the volatility trends of the composite stock index using the AR(m)-GARCH(p,q) model, which combines autoregressive error and GARCH(p,q) models.

2. Prior Research

The volatility of financial assets is associated with risk premiums and plays a crucial role in the decision-making of financial participants in financial transactions. Estimating volatility in financial markets has been studied extensively over the years, with a variety of research models developed. Considering the current expansion of financial market volatility due to factors such as intensified inflation, accelerated monetary tightening, and increased risk of economic downturns, volatility estimation remains a significant issue. Various existing research models for volatility estimation include:

Engle and Rangel (2008) proposed the spline-GARCH model, which allows for long-term forecasting of volatility using financial market data from 48 countries and provides estimates for expected volatility in newly opened markets [3]. Conrad, Loch, and Rittler (2014) analyzed long-term correlations between oil and US stock returns using the DCC-MIDAS model [4].

Chauvet et al. (2012) analyzed whether the symmetric characteristics of financial market volatility are useful for predicting real economic conditions and argued that stock volatility measures and common factors improve the macroeconomic forecasting of financial indicators in the short term [5]. Conrad and Loch (2014) used the GARCH-MIDAD model to analyze the relationship between stock market risk and the macroeconomic environment and confirmed that the ability to predict long-term volatility of stock prices was improved [6]. Asgharian, Hou, and Javed (2013) demonstrated that extracting common factors from various financial and macroeconomic variables and incorporating them into the GARCH-MIDAS model improves its predictive ability [7]. Seung Hee Lee and Hee Joon Han (2016) analyzed the volatility of KOSPI index returns using a semiparametric single-index volatility model combining GARCH models explaining short-term fluctuations and single-index models explaining long-term volatility fluctuations, demonstrating that the long-term volatility of the Korean stock market is best explained when using the housing price index [8]. Young Im Lee and Jin Lee (2017) showed that the GARCH-MIDAS model effectively explains domestic stock market volatility by reflecting not only domestic macroeconomic variables but also international variables such as overseas prices, oil prices, and production [9]. Do Kyun Chun (2017) compared and analyzed the volatility of exchange rates such as USD-KRW, JPY-KRW, EUR-KRW, and GBP-KRW using stochastic volatility models and GARCH(1,1) models to explain exchange rate volatility [10]. And Conrad and Kleen (2020) argued that the mixed-frequency GARCH model has significant explanatory power for long-memory processes and types of long autocorrelation similar to volatility processes such as squared returns [11].

3. Research Model

3.1. Data Transformation

The data consists of monthly data of the composite stock index from January 2005 to December 2022.

In this study, to estimate the volatility of the composite stock index, we transformed the original data by taking the natural logarithm to use relative changes. Specifically, at time t , if the value of the composite stock index is P_t, the formula for calculating returns is given by Equation (1).

$$Z_t = log\left(\frac{P_t}{P_{t-1}}\right) \quad (1)$$

where, the logarithmic return of the composite stock index at time $[t-1,t]$ derived from Equation (1) is referred to as log-return.

3.2. AR(m)-GARCH(p,q) Model

The AR(m)-GARCH(p,q) model combines an autoregressive model of order m for the errors and a model where the variance of the errors follows a GARCH(p,q) process, and is expressed as follows.

$$\begin{aligned} Z_t &= x_t' \beta + \varepsilon_t \\ \varepsilon_t &= -\phi_1 \varepsilon_{t-1} - \cdots - \phi_m \varepsilon_{t-m} + \nu_t \\ \nu_t &= \sigma_t u_t, \ \ u_t \sim i.i.d \ \ N(0,1) \\ \sigma_t^2 &= \alpha_0 + \sum_{i=1}^{q} \alpha_i \nu_{t-i} + \cdots + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \end{aligned} \quad (2)$$

where, (Equation 2) must satisfy $\alpha_0 > 0, \alpha_i \geq 0, \beta_j \geq 0, \sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j < 1$.

An autoregressive error model utilizes local information when there is autocorrelation among error terms, allowing for more accurate estimation and prediction. The GARCH(p,q) model, on the other hand, uses the squares of all past residuals to explain variance, enabling estimation of volatility over long lags with fewer parameters.

3.3. Heteroscedasticity Test

Statistical tests such as the Portmanteau Q-test and the Lagrange multiplier test (LM-test) are employed to test for the presence of autocorrelation among squared errors. The statistic for the Portmanteau Q-test is as follows [12],

$$Q = T(T+2)\sum_{i=1}^{q} \frac{Corr(\widehat{v_t^2}, \widehat{v_{t-\iota}^2})}{(T-i)} \quad (3)$$

and the statistics for Lagrange multiplier test is as follows [13].

$$LM = \frac{T\, W'\, Z(Z'Z)^{-1}\, Z'\, W}{W'\, W} \quad (4)$$

4. Research Results

4.1. Volatility of Log Returns and Heteroskedasticity Test

It was observed that the squared data of the KOSPI index's log returns exhibit a clustering and persistence phenomenon of volatility, where once volatility increases, it tends to remain high for a period, and similarly, once it decreases, it stays low for a certain duration Figure 1.

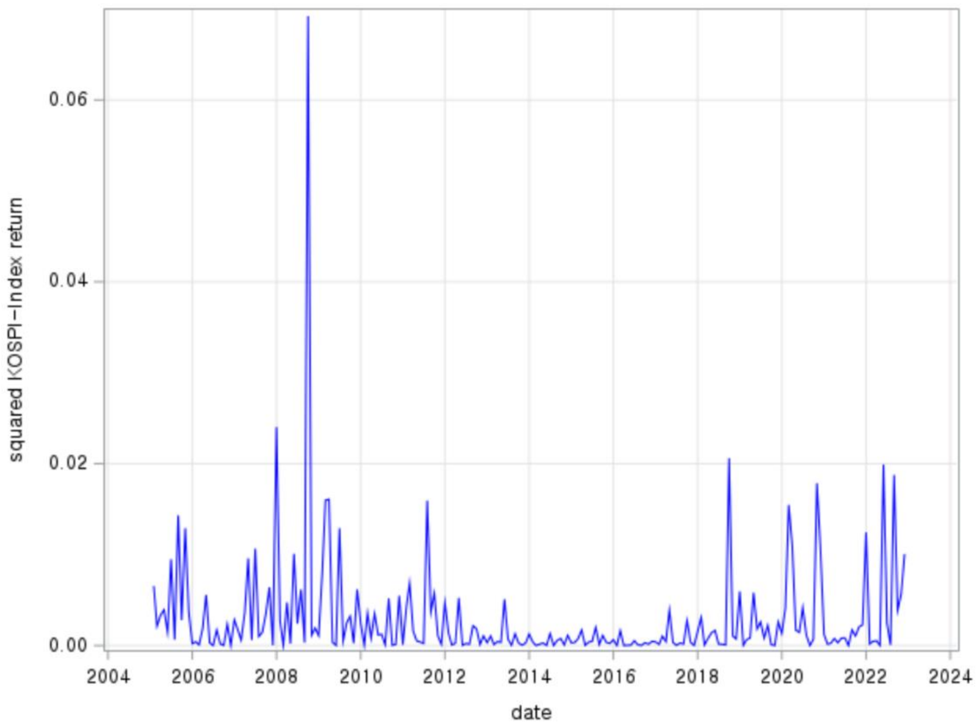

Figure 1. Volatility of squared log returns.

The Portmanteau Q-test results for the squared log returns of the KOSPI index show that the p-values of the chi-square statistics are smaller than the significance level at all lags (lag 1 to lag 30). This indicates that the squared log returns of the KOSPI index exhibit significant autocorrelation in variance, suggesting that they are not independent Table 1.

Table 1. Portmanteau Q-test of squared log returns.

To lags	Chi-square	Pr > ChiSq	Autocorrelation coefficient					
6	14.11	0.0285	0.010	0.057	0.054	0.104	0.142	0.062
12	37.90	0.0002	-0.019	0.073	0.308	-0.053	0.044	-0.008
18	45.09	0.0004	-0.018	0.072	0.077	-0.079	0.101	0.053
24	46.16	0.0042	0.004	0.023	0.020	0.027	0.043	-0.030
30	50.82	0.0102	0.014	0.088	0.032	0.031	0.084	0.044

Furthermore, heteroskedasticity testing of the KOSPI index's log return data using the Portmanteau Q-test and the Lagrange multiplier test indicates that conditional heteroskedasticity is present from lag 5 onwards in the Portmanteau Q-test and from lag 6 onwards in the Lagrange multiplier test Table 2.

Table 2. Heteroscedasticity test of log returns.

Order	Q	Pr > Q	LM	Pr > LM
1	0.3601	0.5484	1.3854	0.5796
2	1.6216	0.4445	3.0364	0.4969
3	2.7409	0.4333	4.2241	0.5218
4	5.4209	0.2468	5.1208	0.3625

5	11.8788	0.0635	8.2247	0.0937
6	20.0082	0.0028	10.2041	0.0193
7	20.0326	0.0055	16.2311	0.0290
8	22.2291	0.0045	18.5124	0.0408
9	45.1552	<.0001	21.3645	0.0001
10	45.9518	<.0001	22.0142	<.0001
11	46.5010	<.0001	22.5716	<.0001
12	46.5207	<.0001	23.8715	0.0001

4.2. Autoregressive Error Model

The AR order of the autoregressive error model was determined to be $T^{1/3}$, and significant parameters estimated by backward elimination showed ϕ_3, ϕ_6 to be significant at the 5% significance level (Table 3).

Table 3. Estimation of parameters for the AR (3,6) model.

AR (3,6) model parameter estimates				
Variable	**Estimate**	**S. E**	**t -value**	**Pr > |t|**
Intercept	0.0004024	0.000136	2.01	0.0224
AR3	-0.1423	0.0680	-2.21	0.0366
AR6	0.1484	0.0695	2.13	0.0339

Therefore, the autoregressive error model estimated using Table 3 can be represented as follows.

$$\begin{aligned} ln\, Z_t &= 0.0004024 + \varepsilon_t \\ \varepsilon_t &= 0.1423\, \varepsilon_{t-3} - 0.1484 \varepsilon_{t-6} \end{aligned} \quad (5)$$

4.3. Conditional Heteroskedasticity Model

After fitting the autoregressive error model in Section 3.2, the Portmanteau Q-test results for the residuals and squared residuals at all lags indicate that while the residuals show no significant autocorrelation at the 5% significance level, the squared residuals exhibit significant autocorrelation (Table 4).

Table 4. Portmanteau Q-test for residuals and squared residuals.

AR (3,6) model residual analysis								
Residual								
To lags	**Chi-square**	**Pr > ChiSq**	**Autocorrelation coefficient**					
6	1.38	0.9668	-0.031	0.025	0.001	0.060	-0.030	-0.010
12	5.28	0.9478	0.099	-0.043	0.006	-0.045	0.017	-0.057
18	13.82	0.7407	-0.054	-0.129	0.042	-0.082	-0.072	-0.056
24	17.31	0.8352	0.032	0.045	-0.027	0.072	-0.040	0.061
30	26.44	0.6523	-0.120	-0.137	0.026	0.018	0.014	-0.051
Residual squared								
To lags	**Chi-square**	**Pr > ChiSq**	**Autocorrelation coefficient**					
6	13.34	0.0379	0.013	0.051	0.110	0.106	0.123	0.137
12	36.27	0.0003	-0.006	0.079	0.298	-0.065	0.042	0.002
18	41.92	0.0011	-0.014	0.067	0.037	0.089	0.085	0.053

24	42.59	0.0111	0.004	0.006	-0.006	0.016	0.025	0.042
30	45.61	0.0338	0.010	0.078	0.025	0.014	0.066	0.027

In the AR (3,6) model, the fact that the residual square does not follow white noise means that heteroscedasticity exists. When heteroscedasticity exists, the volatility estimation model mainly used in reality is the low-order GARCH (1,1). Therefore, in this study, the AR (3,6)-GARCH (1,1) model was applied to estimate the model, and as a result of the estimation, all parameters were found to be significant at the significance level of 5% (Table 5).

Table 5. Parameter estimation of AR (3,6)-GARCH (1,1) model.

AR (3,6)-GARCH (1,1) Model Parameter Estimates				
Variable	**Estimate**	**S. E**	**t -Value**	**Pr > |t|**
Intercept	0.000513	0.000197	2.19	0.0201
AR3	-0.1123	0.0324	-5.09	<.0001
AR6	-0.0632	0.0212	-2.13	0.0216
ARCH0	0.0000496	0.0000333	1.49	0.1368
ARCH1	0.0812	0.0483	2.30	0.0216
GARCH1	0.8523	0.0477	18.49	<.0001

4.4. Diagnosis of AR (3,6)-GARCH (1,1) Model

After fitting the AR (3,6)-GARCH (1,1) model, the Portmanteau Q-tests for residuals and squared residuals showed no significant autocorrelation at a 5% significance level across all lags Table 6.

Table 6. Portmanteau Q-tests for residuals and squared residuals.

AR (3,6)-GARCH (1,1) Model Residual Analysis								
Residual								
To lags	**Chi-square**	**Pr > chisq**	**Autocorrelation coefficient**					
6	0.38	0.9990	-0.017	0.017	-0.008	0.003	0.033	-0.003
12	4.35	0.9763	0.088	-0.022	0.010	0.026	-0.006	-0.091
18	12.65	0.8122	-0.075	-0.105	0.044	-0.068	-0.081	-0.075
24	17.50	0.8265	-0.043	0.076	-0.051	0.020	-0.037	0.090
30	24.15	0.7651	-0.091	-0.133	-0.016	0.011	0.002	-0.027
Residual squared								
To Lags	**Chi-Square**	**Pr > ChiSq**	**Autocorrelation Coefficient**					
6	1.96	0.9230	-0.072	-0.042	0.032	-0.016	0.027	0.003
12	13.29	0.3484	-0.040	0.087	0.166	-0.112	-0.023	-0.018
18	24.50	0.1394	-0.069	0.013	-0.019	-0.080	0.189	0.020
24	26.57	0.3247	-0.009	-0.034	-0.032	0.017	-0.036	-0.069
30	28.25	0.5573	0.051	0.053	-0.036	-0.007	0.003	-0.007

Using Table 5, the final estimated form of the AR (3,6)-GARCH (1,1) model is represented as follows.

$$lnZ_t = 0.000513 + \varepsilon_t$$
$$\varepsilon_t = 0.1123\varepsilon_{t-3} + 0.0632\varepsilon_{t-6} \quad (6)$$

$$\sigma_t^2 = 0.0000496 + 0.0812\nu_{t-1} + 0.8523\sigma_{t-1}^2$$

4.5. Forecasting Volatility Using AR (3,6)-GARCH (1,1) Model

The one-step ahead forecast of volatility and multi-step ahead forecasts of volatility for the forecasting period based on the AR (3,6)-GARCH (1,1) model are shown in (Fig. 2). It is observed that volatility increases gradually from January 2023 to December 2023.

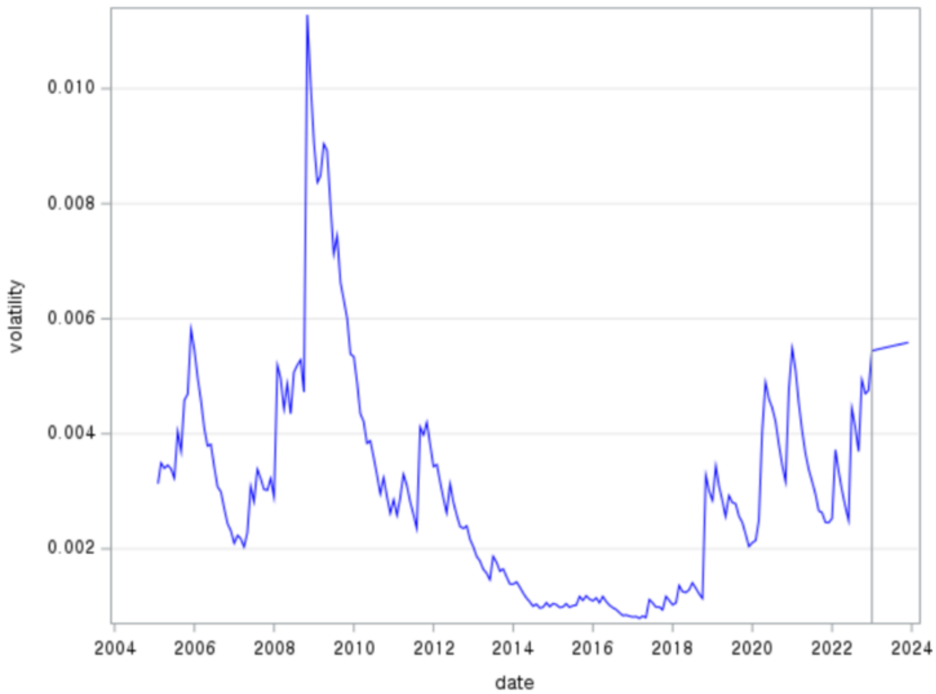

Figure 2. Volatility Forecasting by AR (3,6)-GARCH (1,1) Model.

5. Conclusion

There are various models for predicting the volatility of financial assets' returns. In this study, we used the AR(m)-GARCH(p,q) model to forecast monthly volatility of KOSPI index returns in order to understand the trend in volatility. Analysis using data from 2005 to 2022 revealed the following:

The squared log returns of the KOSPI index exhibited clustering and persistence in volatility, confirmed by Portmanteau Q-tests showing significant autocorrelation from lag 5 onwards, and Lagrange multiplier tests indicating heteroscedasticity from lag 6 onwards. To estimate KOSPI index volatility, we applied the autoregressive conditional heteroskedasticity model after employing backward elimination, revealing significant parameters at the 5% significance level for lags 3 and 6. Testing the independence of residuals using the AR (3,6) model showed that residuals followed white noise, although squared residuals exhibited autocorrelation. Adding the GARCH (1,1) model to the AR (3,6) model resulted in all parameters being significant, and both residuals and squared residuals following white noise. Therefore, the final model for KOSPI index return volatility was determined as AR(3,6)-GARCH (1,1), allowing for forecasts of volatility during the fitting period and the forecasting period.

Given the current uncertainty in the global economy leading to increased stock market volatility, this study emphasizes the importance of risk management practices, strategic diversification in investments, and adjustments in macroeconomic and financial regulatory policies for policymakers and financial market participants. For more refined predictions of KOSPI index return volatility, future research could explore models such as EGARCH, QGARCH, TGARCH, PGARCH, as well as mixed-frequency GARCH models, SVM models, and AI-GARCH(1,1) models. These remain as future research topics.

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References

[1] Karolyi, Andrew G., Why Stock Return Volatility Really Matters, Strategic Investor Relations, March (2001).
[2] https://www.fidelity.co.kr/insight-and-learning/learn-about-investing/what-is-volatility/
market-volatility
[3] Engle, R. and Rangel, J., The spline-GARCH model for low-frequency volatility and its global macroeconomic causes, *Review of Financial Studies*, 21 (2008), 1187-1222. 10.2139/ssrn.939447
[4] Conrad, C., Loch K. and Rittler, D., On the macroeconomic determinants of long-term volatilities and correlations in US stock and crude oil markets, *Journal of Empirical Finance*, 29 (2014), 26-40. 10.1016/j.jempfin.2014.03.009
[5] Chauvet, M., Zeynep, S. and Emre. Y., What Does Financial Volatility Tell Us about Macroeconomic Fluctuations?, FEDS Working Paper, (2012).
https://doi.org/10.1016/j.jedc.2015.01.002
[6] Conrad, C. and Loch, K., Anticipating Long-Term Stock Market Volatility, *Journal of Applied Econometrics*, 30 (2015), 1090-1114. https://doi.org/10.1002/jae.2404
[7] Asgharian, H., Hou, A. J. and Javed, F., The importance of the macroeconomic variables in forecasting stock return variance: A GARCH MIDAS approach, *Journal of Forecasting*, 32 (2013), 600-612. http://doi.org/10.1002/for.2256
[8] Seung Hee Lee, Hee Joon Han, Adopting Economic and Financial Variables to Explain Stock Market Volatility in Korea, *The Korean Journal of Economic Studies*, 64 (2016), 67-95. UCI: I410-ECN-0102-2017-320-000177235
[9] Young Im Lee, Jin Lee, Forecasting Stock Market Volatility Using GARCH-MIDAS Model, *Korea Deposit Insurance Corporation*, 18 (2017), 141-162.
http://dx.doi.org/10.2139/ssrn.3001724
[10] Do Kyun Chun, Volatility Estimation with Stochastic Volatility Model: Application in Korean Exchange Market, *Korea Advanced Institute of Science and Technology*, Theses Master. (2017).
[11] Conrad, C. and Kleen, O., Two are better than one: volatility forecasting using multiplicative component GARCH-MIDAS models, *Journal of Applied Econometrics*, 35 (2020), 19-45. https://doi.org/10.1002/jae.2742
[12] McLeod, A. I. and Li, W. K., Diagnostic Checking ARMA Time Series Models using Squared_Residual Autocorrelations, *Journal of Time Series Analysis*, 4 (1983), 269-273.
https://doi.org/10.1111/j.1467-9892.1983.tb00373.x
[13] Eagle, R. F. Autoregressive Conditional Heteroskedasticity with Estomation of Variance of United Kingdom Inflation, *Econometrica*, 50 (1982), 987-1007. https://doi.org/10.2307/1912773