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Fractal analysis of urban growth in complex systems

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Abstract: A complex network consists of number of elements which may interact with each other to form a complex system. The fractal feature is one of the most important characteristics in complex networks. This study examines the fractal analysis of Madaba City Centre as a complex system through the period to 1953-2022. In order to reveal the fractal characteristics of the study area box-counting (Db) and information (Di) dimensions were calculated using Benoit for five satellite images for years 1953, 1981, 1992, 2000, and 2022. Results showed that Madaba road networks have fractal characteristics for the study period, and the calculated fractal dimensions are between 1 and 2. Db values for years 1953, 1981, 1992, 2000, and 2022 are 1.38, 1.51, 1.55, 1.58, and 1.65, while Di values are 1.37, 1.52, 1.57, 1.60, and 1.67 respectively. The difference between Db and Di values for the same year was small, indicating the similarity between the two methods. Increasing the complexity of the system components and dynamic activities are other indicators of its growth, and there is a significant positive relationship between the fractal dimension value and the complexity level of the city in terms of population growth and area expansion; which increase the importance of fractal theory in studying the growth of the complex system like cities.
Keywords: *Box- counting, Fractals, Information dimension, Madaba, Roads network.*

1. Introduction

Cities are defined as sets of elements connected through sets of interactions [1]. The most characteristic structure was designed based on effective linkages between land use and economic activities in terms of physical movement, that is, traffic. Traffic or movement of people and goods between origins and destinations occurs on the road network, which means that the road network is an important element in serving these spatial movements. To fulfill transportation needs and serve different activities, the road network is organized into different levels and spatial structures, increasing the level of the road network and spatial structures from a simple level to a more complicated one, which means system complexity [2]. The temporal, visual, spatial, scaling, and connectivity dimensions are key for measuring the physical complexity of any complex system. The temporal dimension describes unpredictable changes in any process over time, such as traffic jams or population growth. The visual dimension describes an individual's perception of the visual consistency of the environment, such as building façades and signage. The spatial dimension describes the diversity of land patterns and grains, similar to mixed land-use. The scaling dimension describes the similarity of a structure by applying multiple scales such as fractal urban forms and streets of all sizes. The connectivity dimension describes a city's network organization, such as intersection types and density [3]. For the scaling dimension, in particular, applying the scaling approach to the urban road network at a large scale (for example, the city scale), the city has many arterials and intersections connected to serve the traffic, whereas at a small scale (for example, the arterial scale), the arterial has many surrounding collectors and local roads branching from this arterial. The same approach can be applied to the city center, which has been developed over the years, and at the beginning of the city center formation, few roads have

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been constructed to serve the area. However, with the increasing population of that area, the demand for constructing new roads has increased, leading to a larger road network. In other words, cities are formed by repeating the same structure over the years, leading to a fractal approach that supports the self-similarity characteristics of irregular objects.

In the late 60s, the term "fractal geometry" of nature was released for the first time by Benoit Mandelbrot [4]; after that, the fundamental scaling of complex structures has been developed. From a city perspective, and based on the spatial distribution of network geometry alone, fractal geometry offers a comprehensive understanding of the origin of the sublinear exponent associated with network components [5]. Fractals are "irregular shapes whose geometry is scale-dependent. At every scale, the degree of irregularity that characterizes the geometry appears the same, which is referred to as self-similarity." [4]. another definition of a fractal clarifies that it is the value of geometric objects that can be split into smaller parts; each part has the characteristics of the entire object. Thus, the concept of self-similarity was revealed. However, by changing the scale, each part needs to maintain the type of structure only, and not the same structure as the whole object [6].

Scaling or fractal dimension determines the roughness and self-similarity of objects by studying how their details relate to the scale at which they are observed. In addition, this process refers to fractal geometry, which has been widely used to study the complexity of nature and society in various disciplines including economics, physics, biology, and geography. Fractal dimension (D) is used to characterize the irregularity or roughness of an object. As the term fractal refers to various extremely irregular curves or shapes for which any suitably chosen part is similar in shape to a given larger or smaller part when magnified or reduced to the same size, a mathematical value should be calculated to provide a better understanding of the main concept of fractal geometry. Fractal lines have D values greater than 1 and fractal surfaces have D values greater than 2. In addition, the fractal dimension is commonly considered to be the degree of space-filling, which means increasing the density of the number of elements of any object; the D value also increases, and a curve with D very close to 1.0 behaves much like an ordinary one-dimensional line, but a curve with D very close to 2.0 has a very complicated shape, much like a two-dimensional surface [7]. The fractal dimension measures include box-counting, Hausdorff, perimeter area, information, mass, and ruler dimensions. Recently, more than one software and tool have been developed to measure and calculate different types of fractal dimensions. While some have been developed to serve a certain purpose, others can analyze objects from different fields. Benoit was used in our previous work because of its capacity to offer more than one type of fractal dimension and its friendly interface [8]. Based on previous studies [8] and [9], a comprehensive effort was made to reveal the fractal characteristics of transportation and road networks. Different fractal dimensions are defined, calculated, and compared for a certain year. The results showed that the road network of the Amman District and transportation networks in Turkey have fractal characteristics with fractal dimension values between 1 and 2. In addition, there was a strong positive correlation between the fractal dimension value and the urban characteristics of the studied networks, such as the area, population, network number of lines, and other factors. In addition to the previous findings, the research concluded that the box-counting and information dimensions have a good capacity to reveal the fractal characteristics of the road network compared with other types of fractal dimensions, and also have a strong positive correlation based on the similarity between the concepts of their methodologies.

The remainder of this paper is organized as follows. Section two introduces the literature review. Section three presents the research methodology and data sources. Section four examines the fractal properties of road networks by calculating the fractal dimensions selected over time. Finally, section five concludes the study.

2. Literature Review

[10] Calculated the Hausdorff dimension for the Road Network in both the northern and southern Jiangsu provinces in 1999 and 2009. The findings showed that the fractal dimensions of northern and

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southern Jiangsu were 1.1663 and 1.1854, respectively, in 1999, and 1.3746 and 1.3626, respectively, in 2009. [11] analyzed the change in the spatial configuration in Istanbul for the years 1975, 1980, 1985, 1990, 1995, and 2005 by calculating box-counting dimensions using the `HarFa' program. This research work found that fractal dimensions were 1.585, 1.613, 1.669, 1.671, 1.692, and 1.565 for the aforementioned years. [12] Measured the fractal growth of Baltimore over 200 years, from 1792 to 1992, by computing the box-counting dimension. This study found that the box-counting dimensions increased from 0.6641 in 1729 to 1.7211 in 1992. [13] analysed the urban settlement growth of Bodrum by calculating the box-counting dimension for 2006 and 2015. The results showed that the box-counting dimension increased from 1.43 in the year 2006 to 1.45 in the year of 2015. [14] examined several types of fractals, including geometric fractals, by calculating box-counting dimensions for Hong Kong Road networks from 1971 to 2018. Their results showed that box-counting dimensions increased from 1.527 to 1.606 during the selected period. The scope of this research is to study the urban growth of Madaba City for the years 1953, 1981, 1992, 2000, and 2022, by calculating the box-counting and information dimensions of the city's road network using Satellite Image Processing and Benoit. The reason for selecting the box-counting and information dimensions is their ability to reveal the fractal characteristics of road networks compared to other types, such as mass and ruler dimensions, as well as their popularity in the transportation engineering field as well [8].

3. Research Methodology

3.1. The Study Area

Madaba City has a population of 214,100 and a total area of 940 km^2, which means that it has 2% of Jordan's population and 1.1% of Jordan's total area[1]. As shown in Figure 1, Madaba City is located 800 m above sea level and 30 km^2 southwest of the capital of Jordan, Amman City [15].

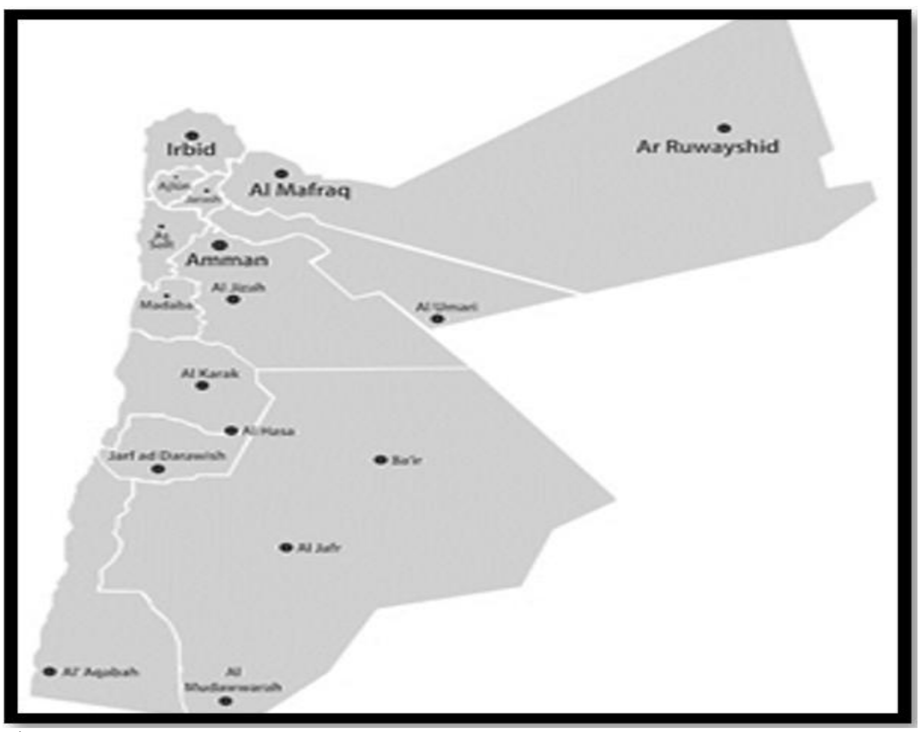

Figure 1.
Map of Jordan.

Source: Jordan in Numbers 2020", Department of Statistics, Online source: http://dosweb.dos.gov.jo/ar/products/jordan-in-figure2020/[1]

3.2. Research Materials

This research deals with the urban growth and expansion of the Madaba City Center over 69 years (1953-2022) using selected satellite images as the source of road network maps. Following the city center growth, five images were selected to cover the area within the defined time interval. The analysis period was selected based on image availability and the growth level of the study area. As shown in Figures 2(A), (B), (C), (D) one satellite image for each of 1953, 1981, and 1992 years and two satellite images for the year 2000 were collected from the Royal Jordanian Geographic Centre (RJGC). For the 2022 satellite image, the road network data were directly downloaded from OpenStreetMap according to the authors' previous work (Abid *et al.* 2021) (Abid and Tortum 2021).

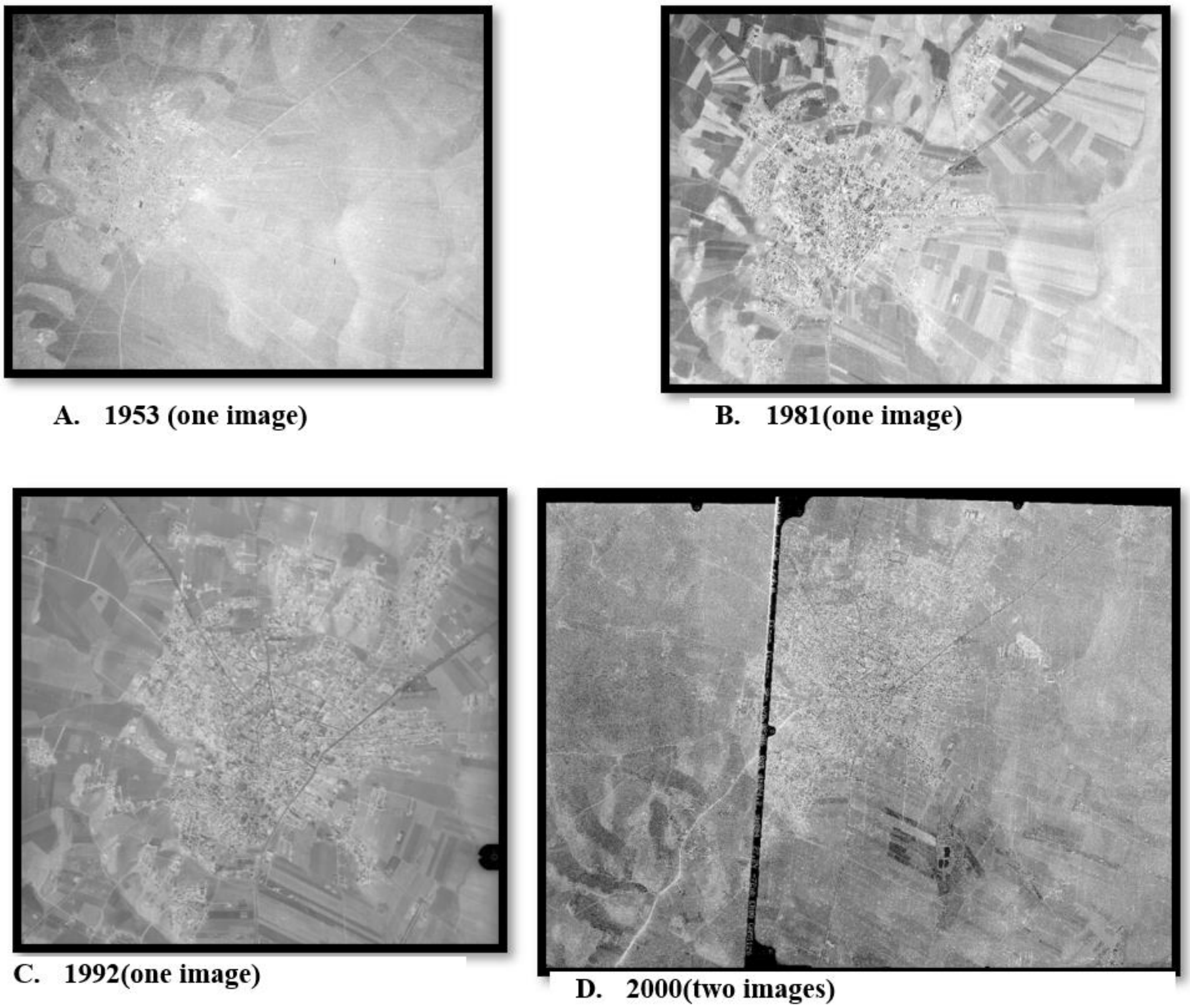

Figure 2.
Satellite images of Madaba city center from year 1953 to 2000.

3.3. Images Processing

ArcGIS was used as a digitization tool to extract road networks from the satellite images. Digitizing is a method of converting any geographic feature (natural or man-made) into spatial data (points, lines, and polygons). The satellite images were processed using ArcGIS to extract the road network layers. As shown in Figures 3 (A), (B), (C), (D), and (E), the final format of the images was

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extracted in the Bmp format, with a black and white theme, and a resolution of 500 pixels to maintain the same resolution for all images. Following the same methodology as in our previous studies [8,9], the road network for the year 2022 was downloaded from OpenStreetMap and processed using suitable tools in ArcGIS. Benoit was used to analyze the processed images and prepare them for the fractal-dimension calculations.

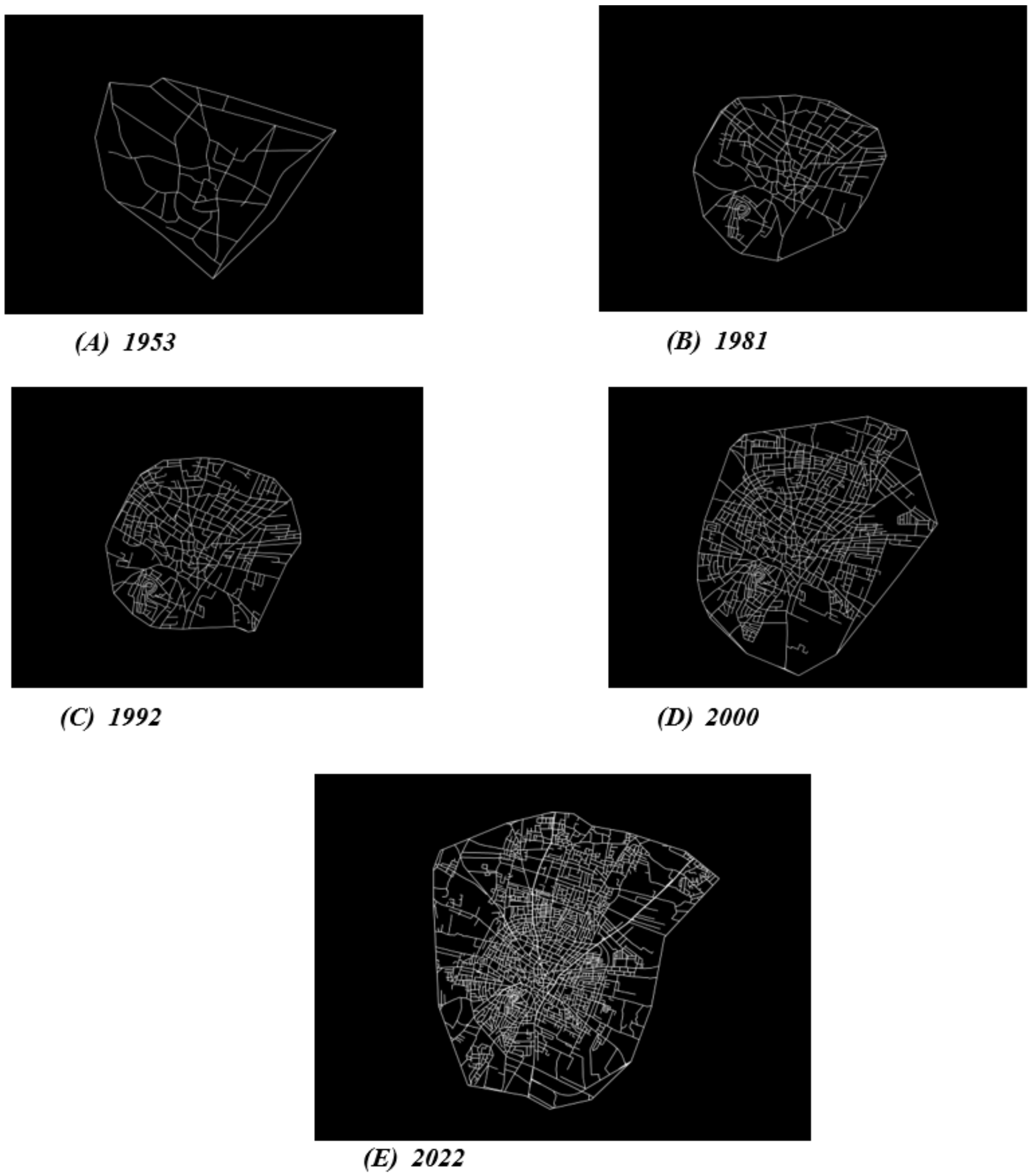

Figure 3.
Road networks of Madaba city center from year 1953to 2022.

3.4. Calculation of Fractal Dimensions

Benoit offers five different types of fractal-dimension calculations; each type has its own characteristics, area, and limitations. The spatial analysis of road network box-counting methods has

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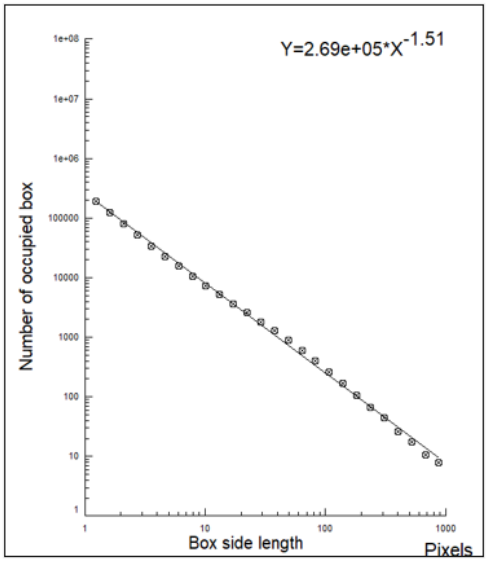
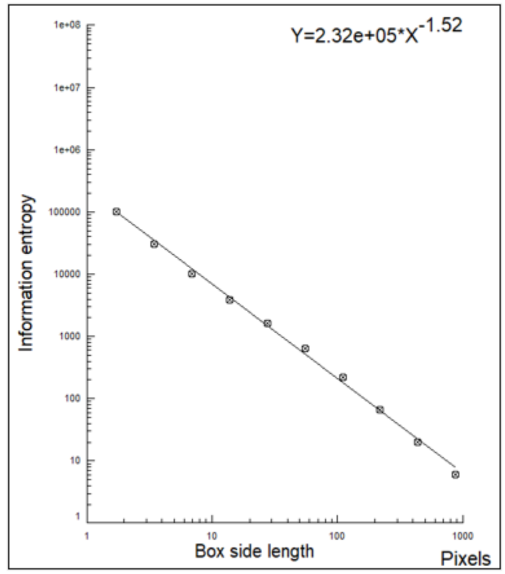
been widely used. Although the Information dimension has a concept close to the box, the counting concept has limited application until now. In the box-counting method, the minimum number of boxes required to cover the entire network is determined [16]. The linear relationship between the logarithm of the box-side length (d) and the logarithm of the number of occupied boxes N(d) is plotted in Figure 4 (A) and the slope of the linear relation is the box-counting dimension (D_b). In the information method, boxes with more points or objects are counted more often than boxes with fewer points or objects. In Figure 4 (B), the linear relationship between the logarithm of the box-side length (d) and the logarithm of information entropy I(d) is plotted, and the slope of this linear relation is the information dimension (D_i).



(A) Box- Counting dimension



(B) Information dimension

Figure 4.
Box- counting and Information dimensions for the year 1981.

4. Results and Discussion

Table 1 summarizes the box-counting and information dimensions, standard division (SD) of the results, and estimated population and network area.

The results show that the D_b and D_i values are between 1 and 2, which makes the urban road network of Madaba city center fractal throughout the year. D_b values were determined by plotting the logarithm of N(d) on the vertical axis versus the logarithm of (d) on the horizontal axis to form a straight line with a negative slope that equals $-D_b$. D_b increased from 1953 to 2022, with values ranging from 1.38427 to1.64794. D_i values were determined by plotting the logarithm of I(d) on the vertical axis versus the logarithm of (d) on the horizontal axis to form a straight line with a negative slope that equals $-D_i$. D_i increased from 1953 to 2022, with values ranging from 1.37027 to 1.67301. These results match the findings of other studies on road networks, which were mentioned in the Introduction section [10, 11, 12, 13, 14]. These findings approve and support the assumption in the particular research [9], which was carried out by the author, and analyzed the fractal geometry of Turkey's urban transportation networks, which assumed "small cities would become larger and denser with time. The Db values of these growing cities' D_b values will likely to increase in line with the findings of the data analysis, which show that large cities have larger fractal dimensions than small cities". Comparing D_b and D_i for the same year, the results show that the differences between D_b and D_i values for 1953, 1981, 1992, 2000, and 2022 are 1.4%, 0.7%, 2.0%, 2.2%, and 2.5%, respectively. This finding indicates a similarity between the two methods. In line with previous findings, the population and urban area values of Madaba City Center increased over the years, indicating physical and socioeconomic urban growth and expansion of the area. From 1953 to 2022, the population, area, D_b value, and D_i value increase from 8545 inhabitants to 147670 inhabitants, 1.00 km^2 to 34.15 km^2, 1.38427 to 1.64794, and 1.37027 to 1.67301, respectively.

The standard deviation data were analyzed by calculating the coefficient of variation (Cv), which is the ratio of the standard deviation to the mean, showing the extent of variability to the mean of the population. If the Cv value ≥ 1, this indicates a relatively high variation, whereas if the Cv value is < 1, this indicates a relatively low variation. The maximum Cv was calculated to be 0.058, whereas the minimum Cv was 0.021 for the case study data, which means that there was a significantly low variation in the D_b and D_i values from the mean.

Table 1.
Fractal dimensions, standard division values, and the estimated populations and areas.

	D_b		D_i			
Year	**D_b Value**	**SD**	**D_i value**	**SD**	**Area[2] (KM^2)**	**Population[3] (Inhabitants)**
1953	1.38427	0.087834	1.37027	0.046525	1.00	8545
1981	1.51101	0.080735	1.51815	0.041668	4.46	28236
1992	1.55036	0.089102	1.57014	0.04892	5.76	55749
2000	1.57962	0.07873	1.60178	0.046721	13.56	70338
2022	1.64794	0.066091	1.67301	0.032115	34.15	147670

5. Conclusions

Scaling, or fractal dimension, is an important measure for revealing the fractal geometry of complex systems, such as cities. This study aims to examine the urban growth of Madaba City Center by calculating the box-counting dimension D_b and the Information dimension for the period 1953-2022. Satellite images from 1953, 1981, 1992, 2000, and 2022 were processed using ArcGIS software. BENOIT was used to compute D_b and D_i using the processed maps in the bitmap file format. The fractal dimension was calculated by analyzing black and white images with a resolution of 500 pixels. From the urban growth of Madaba City's Road network over the years, this research work can conclude the following.

(1) Madaba City's road networks have fractal characteristics for 1953, 1981, 1992, 2000, and 2022, as the calculated fractal dimensions vary between 1 and 2.
(2) The fractal dimension explains the complexity level of the system, and with time, the city expands, which is reflected in an increase in the system components (roads); for example, the fractal dimension in 1953 was less than that in 1981, and its value increased with time.
(3) The difference between the box-counting and information dimensions for the same year is small, which refers to the similarity between the two methods.
(4) Increasing the complexity of the system components and dynamic activities are other indicators of its growth, and there is a significant positive relationship between the fractal dimension value and the complexity level of the city in terms of population growth and area expansion.
(5) The findings of this research succeeded in filling the gap in the author's previous research, as they approve of increasing the fractal dimension value with time for the same case study.
(6) This research, in line with previous works as mentioned in the analysis and discussion section, argues that the complexity of cities can be explained by the fractal growth of the city's physical

[2] Source: Attribute table in ArcGIS.
Source: Department of Statistics (DOS) for the year of 2020 (estimated numbers).[3]

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patterns, but does not explore the fractal approach for dynamic characteristics such as traffic volume, capacity, and journey time, which makes this approach a viable idea for any future work.

Ethical Compliance: All procedures performed in this study involving human participants were in accordance with the ethical standards of the institutional and/or national research committee and with the 1964 Helsinki Declaration and its later amendments or comparable ethical standards.

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