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Investigating different answers to one Euclidean geometry problem: a case study of grade 12 mathematics examinations in South African secondary schools

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Abstract: This article investigates the tendency for distinctly different answers to a problem in Euclidean Geometry as shown in some grade 12 examination memos for secondary schools in South Africa. The study is based on a qualitative analysis of some examples in the final grade 12 examinations where solutions to specific problems are given in distinctly different forms. Both primary and secondary data were applied to the investigation. The secondary data was the literature review of studies on the problems students come across when solving Euclidean geometry problems, and the primary data was acquired from the example of a memorandum where distinctly different answers were given to the solution of a problem. The approach to the problem-solution presentation using distinctly different answers is appreciated as it broadens the understanding of problem-solving. However, does this approach not leave the markers ignorant of other solution options, thus disadvantaging the learners and denying them a pass?.
Keywords: *Distinct different answers, Euclidean Geometry, Grade 12 examination, Problem-solving.*

1. Introduction

Euclidean geometry is a challenging subject in high schools for both teachers and students. Some people even wonder why Euclidean Geometry should be taught in high schools instead of leaving it for a few university majors. Wu [1] argues that it is unjustifiable to suddenly assert that logical reasoning in mathematics is too academically challenging for all high school students and must be the prerogative of a few college students majoring in mathematics because, broadly speaking, mathematics courses are particularly effective at training students in logical reasoning. They learn to work their way through misleading tricks until they get to the core of the problem. They also learn how to differentiate between the truth and false information that seems to be true. Many authors agree with Wu [1], including Jones [2] who states that Geometry broadly contributes to the development of the skills of visualization in students, as well as critical thinking, intuition, perception, problem-solving, speculating, deductive reasoning, logical argument, and proofs.

Some challenges were found concerning delivering Euclidean Geometry both locally and internationally.

Jones [2] states, that learners do not appreciate the need for proof, and they are unable to differentiate between the various forms of mathematical reasoning such as explanation, argument, verification, and proof. Several pedagogical problems in the teaching of Euclidean Geometry have been identified in several countries including South Africa, [3, 4, 5] South Africa, [6] Saudi Arabia, [7] Zimbabwe, [8] Malawi. In South Africa, other drawbacks were also identified, apart from pedagogical challenges. For example, a study carried out by Mthembu [9] found that some learners study mathematics against their will because their parents or the school forces them to take the subject. This greatly reduces the learners' love of mathematics and Euclidean Geometry is the section that suffers the most. The same study also found that the testing system and the prioritization of good results by the

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education department encouraged teachers to adopt teaching methods that are not ideal and hence poor results. Students from very poor schools performed better on lower-order reasoning questions but they had challenges with questions that required logic, reasoning, visualization, and critical thinking, which are typical of the study of Euclidean geometry [10]. From 2008 to 2011 Euclidean geometry in its traditional form of theorem recognition and proof construction was introduced as an optional course in the South African Grade 11 and 12 curricula [11]. This decision was made in response to a series of poor results in the Grade 12 Mathematics examinations [12]. It was also assumed that teachers did not have the necessary depth of content and pedagogical knowledge to effectively teach Euclidean geometry [13]. It was reinstated in a new Curriculum and Assessment Policy Statement (CAPS) in 2012 after numerous studies concluded that university students who had not done Euclidean geometry at high school were weaker in their mathematical skills in comparison to those who had a geometry background [12]. Several challenges concerning the teaching and learning of Euclidean Geometry have been identified and it is believed that they play a significant role in the learners' poor performance in Euclidean Geometry. Nothing significant has been noted concerning the approach to the problem solution presentation using distinctly different answers, which might also affect learners' performance.

The objective of this study is to analyse distinctly different answers to one Euclidean Geometry problem and to come up with conclusions that might help to improve the approach to teaching this topic. In most cases, different approaches to solving a problem are used, and the same answers are obtained at the end. There are situations whereby the answers are not the same depending on the question, even though the working of the problem is correct. The question is: Do examiners often take note of all the possible approaches to a solution including the ones leading to distinctly different solutions when they do the memoranda? If they do, do the markers often take note of all the possible approaches to solutions when marking or do they ignore other approaches including the ones leading to distinctly different solutions? Or do they just concentrate on the ones that they think are the obvious ones and hence disadvantage students? The authors decided to investigate the tendency for distinctly different answers to a problem in Euclidean Geometry and its impact on learners' achievement. This led to the following research question and its objective respectively:

- What are the challenges associated with the approach to the problem solution presentation using distinctly different answers in Euclidean Geometry? With the objective:
- To investigate the challenges associated with the tendency for distinctly different answers to a problem in Euclidean Geometry.

Many high school learners regard Euclidean Geometry as one of the most difficult topics in mathematics. As a result, most of them do not do well in the final examinations. A lot of research has been done involving Euclidean Geometry in many countries. Many factors both internal and external leading to poor performance in the topic have been found. In South Africa, Makhubele [14] found a lot of misapplication of concepts caused by:

i) Learning concepts without understanding.
ii) Challenges with understanding the features and properties of shapes.
iii) Difficulties in dealing with proof questions.

The above was also echoed by Abosede [15] who found that learners had noticeable difficulties in understanding properties of parallel lines, congruency, and proofs of parallelograms which eventually led to problems in calculating the required angles. Ndlovu & Mji [16] identified the following barriers that students face in solving Euclidean Geometry problems: lack of basic geometrical knowledge and vocabulary, use of an inappropriate frame of reference (or geometrical knowledge), inability to make logical deductions, and inability to organise information in a logical chain of arguments leading from givens to conclusions.

The layout of the paper is as follows: the first part focuses on the background of the problem to highlight the challenges of teaching and learning Euclidean Geometry in high schools. Research questions are also given. The second part is the literature review. The third part explains the research instrumentation and proceedings. The fourth section analyses data gives and discusses examples of distinctly different solutions found in one Euclidean Geometry question which markers might not take note of.

2. Methodology

An interpretive paradigm was followed. A single case study was used because the cases were only grade 12 mathematics examination memoranda which were selected and analysed qualitatively.

The memoranda were selected randomly from 2012 to 2022 grade 12 Mathematics past examinations papers. Only memoranda from 2012 to 2022 we chosen because before that Euclidean Geometry was optional in South African high Schools. An example from one previous memorandum was given where the authors were able to find alternative and distinct solutions to the same problem. Not many costs were involved since memoranda were selected from past examination papers online. The authors are aware that the result cannot be generalized since not all past examination memoranda were analyzed.

The analysis was done to check whether all the options that were supposed to be given for every Euclidean Geometry question appeared in the memorandum. Literature was also carefully reviewed based on the problems students come across with when solving Euclidean geometry problems. In some memoranda, not all options were given. Figure 1 below is an example where a grade 12 final examination question with different options for a solution and a distinctly different solution was identified but only one option was shown in the memorandum. More options are then given.

3. Results and Discussion

For the below chosen Euclidean Geometry item, the examiner gave one answer, yet it had varied approaches and at least 2 distinct solutions depending on the approach used to answer the question. The examiner just used one approach and got one answer. There could have been a situation whereby some students used a different approach from what was shown on the memorandum and got penalised by markers which disadvantaged them. A distinct different answer was also found as the solution which was not presented in the memorandum. On the other hand, assuming that all the options were there on the memorandum, were the markers going to take note of all of them or they were still going to concentrate on only one option that they thought was the most appropriate and forget about the rest and hence disadvantage the students? The memorandum showed the below option.

3.1. Example of the Memorandum

This memorandum was taken from the Education Department in South Africa. (5.44.1 Mathematics paper 2 memorandum 2021). The memorandum is based on the question, where learners had to determine, with reasons, the size of a particular angle in terms of a given variable. The figure below was provided to help learners determine the angle, and from the diagram, the angle is M_1 which was to be determined in terms x.

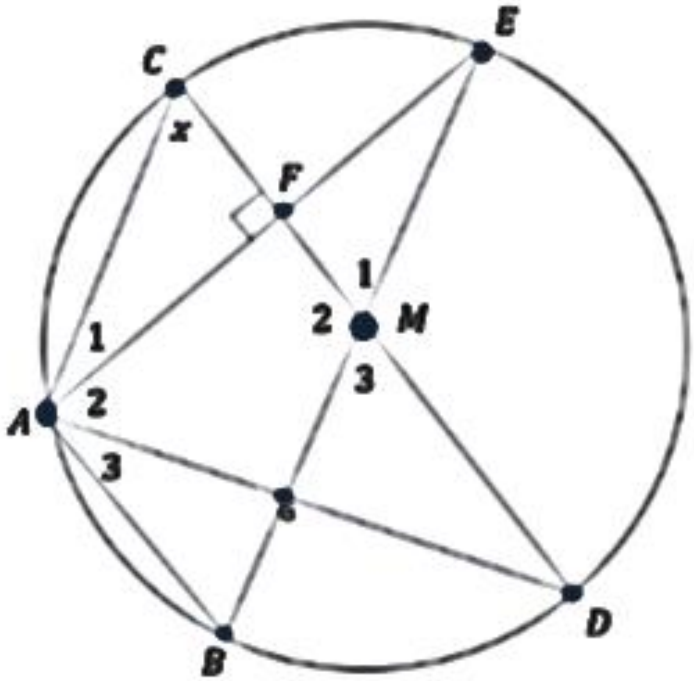

Figure 1.
The Geometry of the Problem (Mathematics paper 2 memorandum 2021).

This is an extract of the solution route to the first different answer to the problem, the actual problem is: Determine, giving reasons, the size of $\widehat{M_1}, in\ terms\ of\ x.$ (Mathematics paper 2 Memorandum 2021).

Memorandum solution (Mathematics paper 2 memorandum 2021):

$\widehat{A_1} = 90° - x \quad [sum\ of\ \angle s\ in \Delta]$

$\therefore \widehat{M_1} = 180° - 2x\ [\angle at\ centre = 2 \times at\ circumf.]$

The other routes to the possible options not in the memorandum provided by the authors are as follows:

Suppose G is the intersection point of AD and BE.

$C\hat{A}D = 90°(\angle\ in\ semi-circle) = A\hat{G}E\ (MB\ from\ centre \perp chord).$

$\therefore AC \parallel BE \therefore \widehat{M_1} = x\ (Alternating \angle s)$

Or

Comparing $\Delta s\ ACD\ and\ ACF$: $C\hat{A}D = 90° = C\hat{F}A\ and\ x\ is\ common \therefore \widehat{A_1} = \hat{D}$(Remaining $\angle s$)

In $\Delta GMD\ \widehat{M_3} = 180° - (90° + \hat{D}) = 180° - (90° + \widehat{A_1})$

$= 90° - \widehat{A_1} = x.$ But $\widehat{M_3} = \widehat{M_1}(vert.opp.\angle s) \therefore \widehat{M_1} = x.$

Or

$C\hat{A}D = 90°(\angle\ in\ semi-circle) = A\hat{G}E\ (MB\ from\ centre \perp chord).$

$\therefore AC \parallel BE$. This implies $\widehat{M_1} = \widehat{M_3}$(vertically opposite) $= x\ (corresponding\ \angle s)$

Different options which include a distinct different solution which did not appear in the memorandum were given. This strengthens the argument of the authors that giving only one solution where there is a possibility of distinct different solutions shows that learners who gave different correct solution with different approach may be disadvantaged. It could be that the examiner knew the other options and just thought that the option used in the memorandum was the obvious and best for the learners or it may be that the examiner did not think about other options. Learners who wrote grade 12 examinations in that particular year might have been disadvantaged in this question if they used one of the above options which were not in the memorandum. Normally during marking, markers have a lot of scripts to mark so they do not have time to work out other options during marking time. Whatever is not in the memorandum they do not consider it. This could be one of the factors leading to underachievement at grade 12.

4. Conclusion

In this article, the authors investigated different answers to one Euclidean Geometry problem where an example of a memorandum in one grade 12 mathematics examination was considered. Alternative approaches to solutions not given in the memorandum, including the one for a distinctly different solution were given to show that some learners might have been disadvantaged during marking. The approach to the problem solution presentation using distinctly different answers is appreciated to broaden the understanding of problem-solving but the question is: Is this approach not leading the markers to be ignorant of the other solution options and disadvantage the learners passing?

5. Recommendations

The authors recommend that the grade 12 examiners are supposed to make sure that they include all the possible options when making the memoranda so that during marking learners are not disadvantaged by markers. Another study on markers' perceptions of problem solution representation using distinct different solutions is needed.

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