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Students' commognitive conflicts in solving absolute value inequalities

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Abstract: The material of absolute value inequalities is considered more difficult to study. There are still many undergraduate students who still make mistakes in understanding and solving problems related to this topic. Moreover, some of them do not yet know how to read the symbols. Therefore, these issues need to be studied more thoroughly. This research was conducted to explore the undergraduate students' discourse on absolute value inequality tasks through commognitive analysis. A qualitative approach was applied, involving 78 participants; 11 of them were further interviewed. The analysis of the absolute value inequality tasks and the interview results revealed that there were commognitive conflicts that needed to be addressed. The findings of this research showed that commognitive conflicts arose when the undergraduate students applied the definition of absolute value and determined the solution set for the absolute value inequality problems.
Keywords: *Absolute value, Commognitive, Discourse, Meta-level learning.*

1. Introduction

Language will always be present wherever we are, even when we are silent (Sfard, 2021). Furthermore, according to Sfard (2021), attention to language in mathematics education is relatively new, as researchers have been keen on investigating mathematical activities without much consideration of the language used in those activities. Baroody (1993) stated that: (1) "Mathematics as language'; mathematics is not only a tool for thinking, finding patterns, or solving problems, but also a tool for conveying ideas precisely and clearly, and (2) "mathematics learning as social activity"; as a social activity, mathematics learning involves interactions between students and students. Additionally, communication between teachers and students is a crucial part of uncovering students' mathematical potential. Mathematics is a unique language consisting of words, tables, and illustrations, such as graphs and symbols (Tong et al., 2021).

Mathematics can be viewed as an activity of training a certain form of communication called 'discourse.' Thus, mathematics is a discourse that cannot be equated with the often-heard and criticized statement that mathematics is a language (Sfard, 2012; 2021). This is because there are several elements of mathematics that make the 'discourse' different, namely keywords, visual mediators, narratives, and routines. Based on the previous teaching experiences done by the researcher, some undergraduate students read the same symbols differently, and some of them even did not understand the symbols at all. There was also confusion in understanding mathematical terms, for instance, the students often equated the definition of derivative and differential terms. Lefrida et al. 's (2023) earlier research found that students still made errors by saying that a function having a derivative meant it also had a limit. The students also stated that a function being said to be a 'continuous' function had a limit. From these findings, the responses done by the students are incorrect. According to the theory, a function that has a derivative is the function that has equal derivatives in the left and right. The gap in the use of words by people has been a phenomenon worth studying until now, especially in mathematics classrooms (Sfard, 2021). This is also in line with Akçakoca et al. (2024), who stated that communication in the classroom will always be equivalent to the thinking process.

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Several previous studies have emphasized the relevance of the context in which learning occurs, based on Sfard's cognitive approach (Emre-Akdoğan, Güçler, & Argün, 2018; Heyd-Metzuyanim & Graven, 2019; Nachlieli & Tabach, 2019; Lefrida et al., 2023; Akçakoca et al., 2024). Commognitive theory assumes that learning is not a process where someone changes a certain cognitive structure in their mind, but rather a process of changing participation routines within a particular community (Heyd-Metzuyanim, Smith, Bill & Resnick, 2019). Consequently, discourse analysis in the context of classroom learning provides insights into how learning occurs. This is linked to Nachlieli and Heyd-Metzuyanim (2022)'s research which explored commognitive conflict in mathematics education research by focusing on students' mathematical discourse.

In this research, a commognitive perspective (Sfard, 2020) was applied to conceptualize mathematics learning as a change in a person's mathematical discourse. Incommensurable discourses usually arise when inconsistent words, visual mediators, or routines are used in communication. When communicating, incommensurable discourse can lead to commognitive conflicts between one or more interlocutors. Hence, it can be said that commognitive conflict is a situation where participants use the same words in different ways (Sfard, 2007; Ben Zvi & Sfard, 2007).

2 Literature Review

2.1. Mathematics is a Discourse

Given that communication is inherently collective, the term individual or personal discourse may seem like an oxymoron. Thought has been defined as self-communication. Indeed, personal discourse, mostly internal and silent, is largely beyond direct investigation. In fact, not all participants in a mathematical conversation group are capable of doing self-communication mathematically. Mathematics can be described as discourse about mathematical objects, such as numbers, functions, sets, and geometric shapes.

A specific form of communication must be used to ensure that the mathematical narratives produced are reliable and unambiguous. Thus, mathematical activity can be seen as a practice of a certain form of communication, namely discourse. Specifically, the elements of discourse include: 1) Keywords in mathematical discourse; they primarily express quantity and form (numbers, geometric objects), as well as relationships between them (equality, inequality, similarity, equivalence, etc.). A symbolic system consists of vocabulary, a set of basic signs called words, and syntax, a set of rules for combining vocabulary elements into meaningful (or valid) utterances. 2) Visual mediators; they are defined as visible means that support communication, such as written words, symbols, and graphs. Moreover, literacy in discourse is defined as visual media, primarily through symbolic artifacts. In addition to algebraic symbols, symbolic artifacts also include icons, such as diagrams, graphs, and other images. Visual mediation involves the entire series of written symbols, such as numbers, tables, algebraic formulas, and even lines (imaginary). 3) Written or spoken texts; the written or spoken texts "framed as descriptions of objects, relationships between objects, or processes with or by objects" are considered narratives; for example, axioms, definitions, and proofs are examples of narratives in mathematical discourse. Endorsed narratives, as potentially useful narratives, are produced with the help of language, mediators, and certain routines.

The elements that shape discourse are sets of routines with recurring forms of communicative actions. A repeated pattern of communication is called a routines. Keywords and visual mediators used regularly in narratives are also called routines. Routine mathematical discourse includes, for example, computation, problem solving and verification. It also includes new narratives that are proved (Sfard, 2008, 2020). Activities involving mathematical objects and mathematical narratives are discourse routines (Sfard, 2020). In the mathematics classroom, students' task is to individualize a discourse that historically known as mathematical discourse. The term individualizing mathematics means being able to participate competently in this discourse, not only when talking with others but also when communicating with oneself. Commognitive theory assumes that learning is not a process where someone changes a particular cognitive structure in their mind, but rather a process of changing participation routines within a particular community (Heyd-Metzuyanim et al., 2019).

2.2. Commognitive Conflict as a Stimulus for Meta-Level Learning

If learning mathematics is a change in discourse, then we can distinguish between two types of learning: object-level learning and meta-level learning. First, object-level learning, which manifests itself in the expansion of existing discourse, can be achieved through vocabulary expansion, building new routines, and generating endorsed narratives. Second, meta-level learning, which involves changes in the metarules of discourse, is usually associated with exogenous changes in discourse. These changes mean that some familiar tasks, such as defining words or identifying geometric shapes, may now be performed differently in unfamiliar ways. One of the examples of these changes can be seen when certain familiar words may change in how they are read or used. Meta-level learning is most likely to occur from direct encounters between students and new discourse. Since this new discourse is governed by different metarules from the discourse that students are previously familiar with, it requires a commognitive conflict, a situation where different discourses act according to different metarules. The idea of commognitive conflict differs from the acquisitionist notion of cognitive conflict. There are three substantial differences, as shown in Table 1 below.

Table 1.
Concepts: cognitive conflict and commognitive conflict.

	Cognitive conflict	**Commognitive conflict**
Conflict happens between:	Interlocutors and the world	Incommensurable discourses
Role in learning	An alternative to eliminate misunderstanding	Practically essential for meta-level learning
How is it resolved?	By students' rational efforts	By students' acceptance and individualization of the expert interlocutor's discursive way

Source: Sfard (2008).

Commognitive conflicts also exist in mathematical discovery. The conflicts may occur within a person between two overlapping discourses that are already embedded within him/her. Sometimes, during the transition from familiar to new discourse, a mathematician may endorse conflicting narratives. One of famous cases of such inner conflict is that of George Cantor, the inventor of set theory, who, in letters to mathematician Richard Dedekind, lamented his inability to resolve the contradiction between the familiar "truth" that a part was smaller than the whole and the conclusion he reached from his new theory, which stated that a subset of an infinite set could be "as big as" the whole set (Sfard, 2008). Another example, Ioannou (2016), reported commognitive conflicts experienced by students during Group Theory exercises. As a summary, thinking, according to the participatory framework, is conceptualized as communicating with oneself, either synchronously or asynchronously. Communicating with others or with oneself is largely verbal, or aided by other symbolic systems (Sfard, 2008). According to the psychology of communication about human thought, language is no longer viewed as a window into thinking but as its determinant element. Therefore, as long as thoughts exist in language, both thought and speech cannot be separated.

3. Research Method

3.1. Type of Research

This research is an exploratory research which applies a qualitative approach. Qualitative research has four characteristics: (1) it focuses on processes, understanding, and meaning-making, (2) the researcher is the main instrument in data collection and analysis, (3) the research process occurs inductively, and (4) data presentation is in descriptive form, consisting of words and images (Merriam & Tisdell, 2016).

In this research, the subjects were selected from Mathematics Education undergraduate students for the 2024/2025 academic year. In Figure 1, the Absolute Value Inequality Task was given to 78 undergraduate students: 50 second-year students and 28 third-year students. In the data collection process, supporting instruments used to gather research data were: (a) Absolute Value Inequality Task, (b) interviews, and (c) recording devices. The data analyzed in this study included interview transcripts, documents, and field notes. Data analysis was carried out using the interactive model analysis technique proposed by Miles et al. (2014), which included data condensation, data display, and conclusion drawing and verification. The quality of qualitative research was ensured through prolonged engagement with the subjects and member-checking.

Selesaikan pertidaksamaan $|x-3|+|x+2|<11$ (Stewart, 2010)

Figure 1.
Absolute value inequality task.

After completing the task, subjects were asked to determine the criteria according to themselves. The criteria are presented in Table 2 below. Of the 70 students who completed the task, 14 said the task was difficult. The rest described it as moderate or easy.

Table 2.
Question criteria according to the subjects.

Students	Difficult	Moderate	Easy	Total
Fourth semester	8	35	7	50
Sixth semester	6	11	11	28
	14	36	18	78

4. Results and Discussion

The task in Figure 1 was assigned to second-year and third-year students. They were asked to comprehend the task. Each student was required to propose an initial idea for solving the absolute value inequality problem. Each idea they mentioned was also to be written down. The ideas were further grouped, as shown in Tables 2 and Tables 3.

Table 2.
Grouping of initial ideas (Second year students).

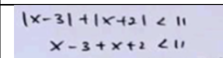
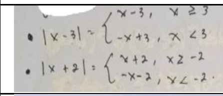
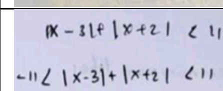
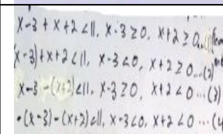
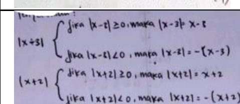
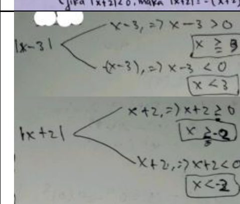
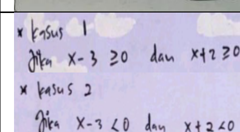
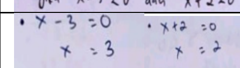
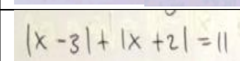
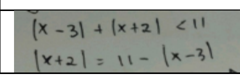
No.	Students	Understanding and initial idea	Notation	Code
1	**CHE**,KAR, SAL, NUR, TIK, SUL, SIT, KHA, YUL, YUR, FIQ, OLI	The property of absolute value is always positive		SM41
2	ANN, SAKI	Utilizing the definition of absolute value		SM42
3	NIL, FIT, VIA, SRI, FAD	Starting with $\|x\| < a$		SM43
4	CIN, FAD, **NUR**, NI, ILU SRI, ZUL, **FEL**, AZIZ, WAH	There are four possibilities, because $x-3$ and $x-2$ are within absolute value symbols		SM44
5	FIT	Removing the absolute value		SM45
6	PAT, WID, YOS	Removing the absolute value symbol		SM46
7	RAH	Consisting of two cases		SM47
8	**EKA**, KAD, LUK, NAZI, TTY, WAN, NUR	Determining the value of x		SM48
9	**EET**, MIF, ISMI, ZIAN, GUS	Replacing the < symbol with = symbol		SM49
10	RIN, AZIM, MAL	Shifting the absolute value		SM10

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Table 3.
Grouping of initial ideas (Third year students).

No.	Students	Understanding and initial idea	Notation	Code
	Subject	Using the properties of inequalities	Jika $a>0$ dan $\lvert x\rvert < a$ maka $-a<x<a$	
1	RO, ALI, SAN, ANGG, RAF, MAR, ZAIN, INDRI	To operate on absolute value inequalities, the first step is to remove the symbol of absolute value	$-11 < x-3+x+2 < 11$	SM61
2	SAL	Expanding the inequality into four forms, namely	a). $x-3+x+2<11$; $x-3\geq 0$; $x+2\geq 0$ b). $-(x-3)+x+2<11$; $x-3<0$; $x+2\geq 0$ c). $x-3-(x+2)<11$; $x-3\geq 0$; $x+2<0$ d). $-(x-3)-(x+2)<11$; $x-3<0$; $x+2<0$	SM62
3	RIF	To simplify the inequality of absolute value functions, first multiply 1 and -1 into the absolute value functions of $\lvert x-3\rvert+\lvert x+2\rvert<11$	$\lvert x-3\rvert+\lvert x+2\rvert<11 \begin{cases} x-3+x+2<11, \text{ dikalikan } 1 \text{ (pers. 1)} \\ -x+3+(-x)-2>11, \text{ dikalikan } -1 \text{ (pers. 2)} \end{cases}$	SM63
4	SYA, FEB, DWI	Based on the properties of absolute value	$\lvert x-3\rvert+\lvert x+2\rvert<11$ $x-3+x+2<11$	SM64
5	SI	Replacing the inequality symbol with = symbol	$\lvert x-3\rvert+\lvert x+2\rvert<11$ $\lvert x-3\rvert+\lvert x+2\rvert=11$	SM65
6	RAT	Consisting of four cases	$\lvert x-3\rvert = \begin{cases} x-3 & \text{untuk } x\geq 3 \\ -x+3 & \text{untuk } x<3 \end{cases}$ $\lvert x+2\rvert = \begin{cases} x-2 & \text{untuk } x\geq 2 \\ -x+2 & \text{untuk } x<2 \end{cases}$	SM66
7	NUR, RUL, DIAN, HERI, MAG, ANI	Using the definition of absolute value	$\lvert x-3\rvert+\lvert x+2\rvert<11 \begin{cases} (x-3)+(x+2)<11, \text{ untuk } x>0 \\ -((x-3)+(x+2))<11, \text{ untuk } x<0 \end{cases}$	SM67
8	EVA	Determining the value of x There are 3 solution regions	$x-3<0$; $x+2<0$ | $x-3<0$; $x+2>0$ | $x-3>0$; $x+2>0$ -2 3	SM68
9	ARW	Squaring both sides	$\lvert x-3\rvert+\lvert x+2\rvert<11$ $\lvert x-3\rvert^2+\lvert x+2\rvert^2<11^2$	SM69

There are ten initial ideas in Table 2 and nine initial ideas in Table 3. A difference of narratives among a group of students was observed. It indicates the presence of conflict. Furthermore, the researcher revisited the data to determine if any of the ideas mentioned and written could be combined. Seven discourses were identified from the combination of the narrative of second-year and third-year students.To further explore this commognitive conflict, 11 students were selected as subjects. Interviews were conducted with the subjects, and for each identified discourse, 1 or 2 subjects were interviewed. The subjects who were interviewed are highlighted in black and bold. The following presents the findings from the differing subject discourses.

4.1. The Property of Absolute Value is Always Positive (SM41, SM64)

CHE is a second-year student at the university. The subject described the task as an easy category and mentioned the initial idea for solving the absolute value inequality problem as "The property of absolute value is always positive". The subject responses are shown in Figure 2.

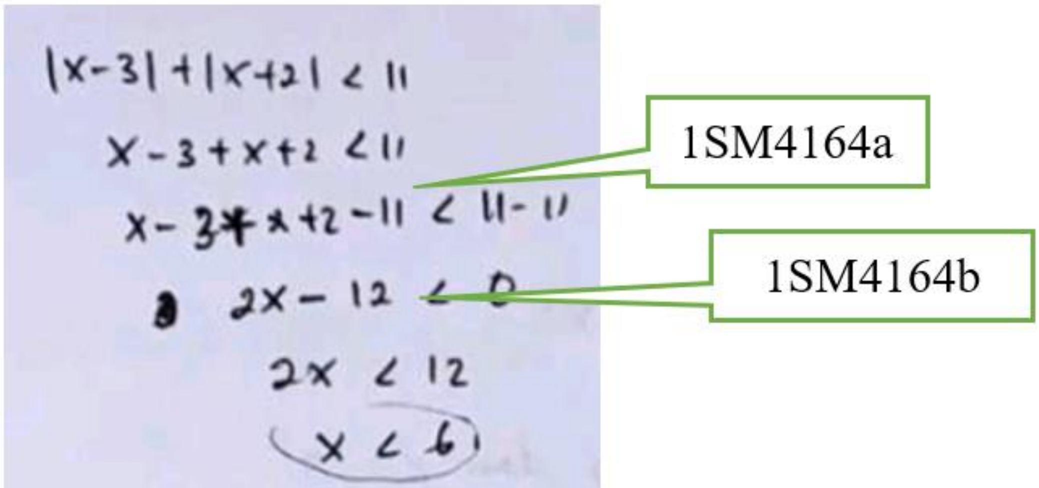

Figure 2.
Answer by subject CHE.

To gain further insight into the subject's ideas and work process, an interview was conducted, as shown in Table 4.

Table 4.
Discussion transcripts subject CHE.

Interview	Discussion
Researcher	Do you understand the problem?
CHE	Yes, ma'am. The absolute value symbol, it's always positive.
Researcher	Do you remember the definition of absolute value? (Asking 1SM416)
CHE	The absolute value of $\|x-3\|$ becomes x -3, and $\|x+2\|$ becomes $x+2$.
	Once the absolute signs are removed, the value of x can be determined.
Peneliti	Any other thoughts?
CHE	No, ma'am.

Subject CHE focused solely on substituting the value of x into the function, stating that the result would always be positive. For example, $|x-3|$ becomes $x-3$, and $|x+2|$ becomes $x+2$. According to the subject, since the absolute value is always positive, they simply removed the absolute value signs. It aligns with the findings of Gagatsis & Panaoura (2014), which suggest that students often assume that $|x+3| = x+3$ if $x > 0$. The partial understanding of the absolute value definition may be due to a tendency of the students to memorize the definitions (Papadouris et al., 2024).

4.2. Using $|x| < a$ (SM43, SM61)

Subject RO's approach to solving the absolute value inequality task is by using the property $|x| < a$. Below is the process conducted by subject RO.

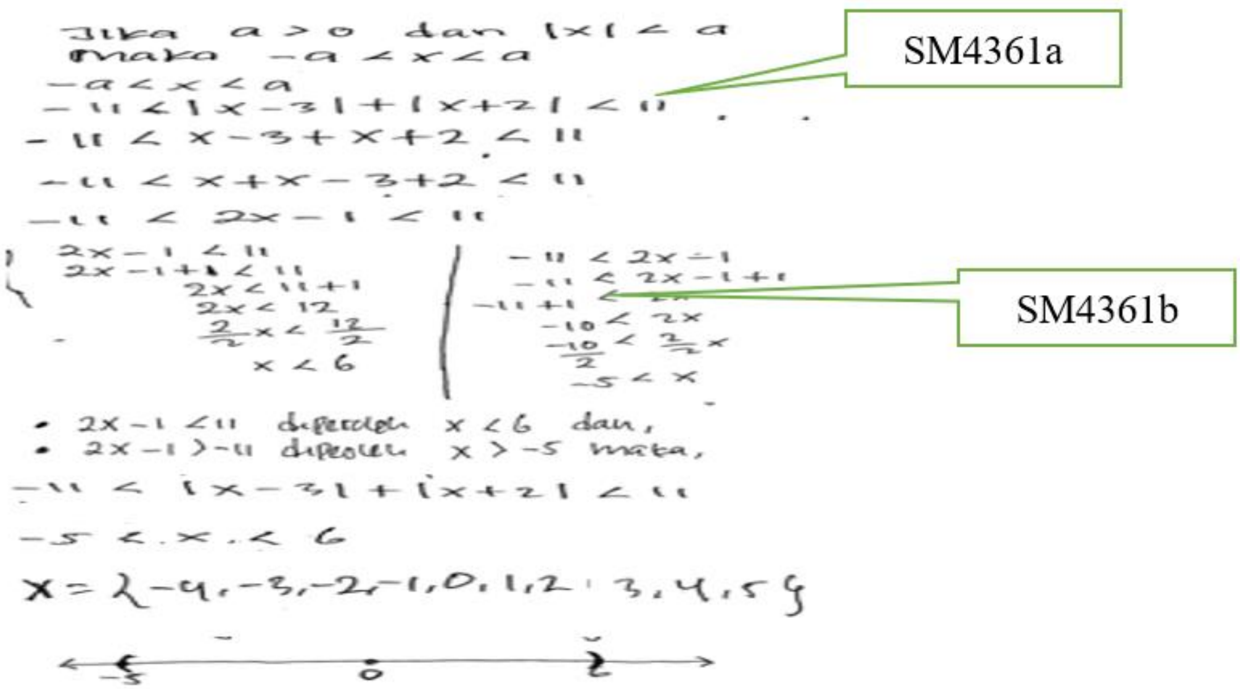

Figure 3.
Answer by subject RO.

To gather information about the subject's idea in solving the task, an interview was conducted, as shown in Table 5.

Table 5.
Discussion Transcripts Subject RO.

Interview	Discussion
RO	To solve this problem, I used the property of absolute value inequalities $|x| < a$
Researcher	How did you do it?
RO	By removing the absolute value sign, it becomes $-a < x < a$ (SM4361a)
Researcher	Why did it become [SM4361b]?
RO	$|x - 3|$ and $|x - 2|$ are positive.
	Then I performed algebraic operations by grouping like terms.
	Next, I solved for x, which is the solution to the inequality of the value that fulfills the inequality.
	After that, I wrote down the range of the solution set and the inequality in its form without absolute values.
	Finally, I drew the graph of the solution.

Subject RO used the property of absolute value inequalities $|x| < a$. However, in code (SM4361b), the absolute value signs were removed without showing the process. Based on the interview in Table 5, RO explained that absolute values are always positive. RO and CHE share the same narrative in this regard, relying on the definition of absolute value. However, neither of them paid attention to the addition sign between the two absolute values. Additionally, there is a difference in how they applied the inequality property of $-a < x < a$. CHE believed it was sufficient to solve only the right side of the inequality. On the other hand, RO worked on both sides. Nevertheless, both agreed that the absolute value is always positive.

4.3. Four Possible Solutions

The discourse from subjects who suggested the solution based on four possibilities is coded as (SM42, SM44, SM45, SM46, SM47, SM62, SM66), while two possibilities are coded as (SM47). Two students, FEL and RAT, were selected for interview. Their responses are shown in Figure 4 and Figure 5.

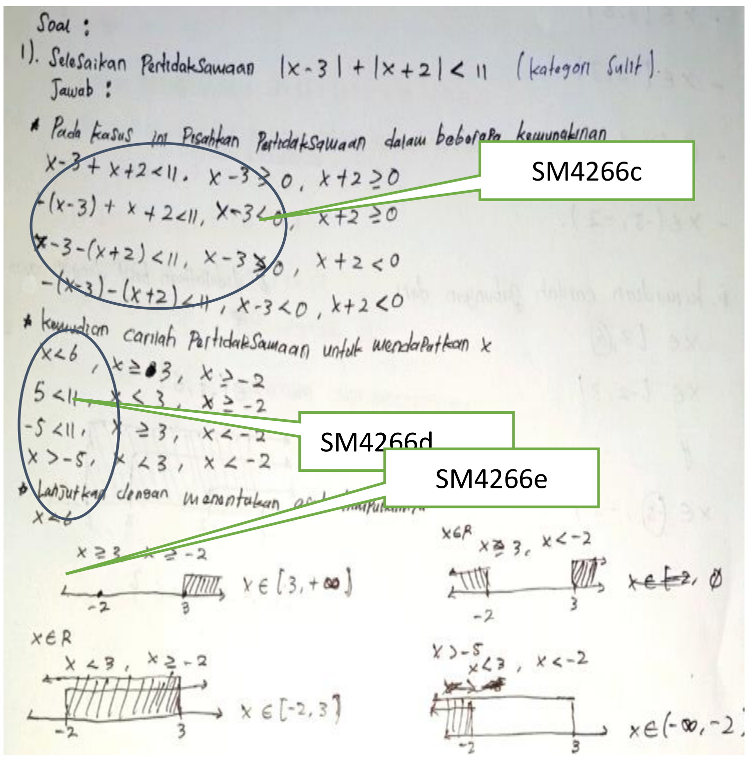

Figure 4.
Answer by subject FEL.

Table 6.
Discussion transcripts subject FEL.

Interview	Discussion
Researcher	Do you understand the problem?
FEL	Yes, ma'am.
Researcher	Which four possibilities are you referring to?
FEL	This one, ma'am (points to the marked SM4266c)
Researcher	Do you mean $-(x-3)+x+2<11$, $x-3<0$, $x+2\geq 0$
	(referring to code SM4266c)
FEL	Yes, by removing the absolute value of $x-3$, and for $x-3<0$

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	can be written as $-(x-3)$. The same applies to the other terms, ma'am.
Researcher	Why did you write the inequalities $x < 6,\ 5 < 11,\ -5 < 11, x > -5$?
FEL	To find the value of x
	$2x - 1 < 11$; $x \geq 3$; $x \geq -2$ $x < 6$
	$5 < 11$ is true for all x). Similarly, $-5 < 11$ is true for all x, so $x \epsilon R$
Researcher	Look at the graph you made. Where is the region < 6 ? (SM4266e)
FEL	It's in the region $x \in [3, \infty)$

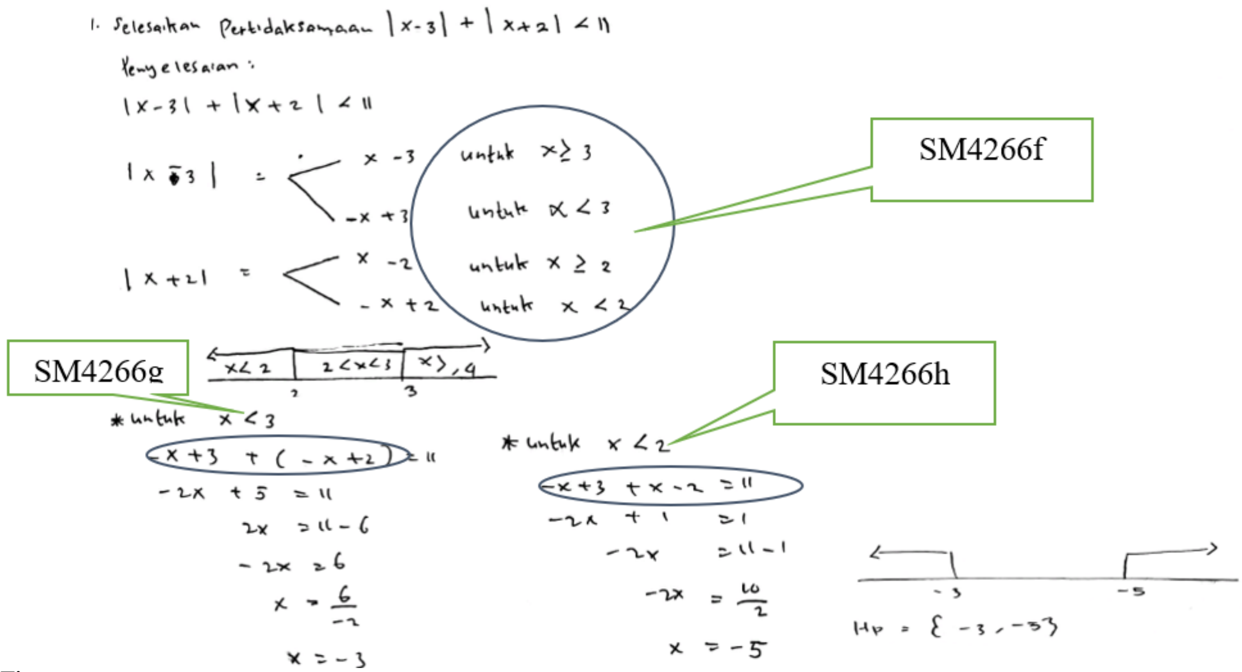

Figure 5.
Answer by subject RAT.

Table 7.
Discussion transcripts subject RAT.

Interview	Discussion
Researcher	Why did you approach the inequality with four possibilities? (SM4266f)
RAT	Based on the properties of absolute value, ma'am.
Researcher	What is that property?
RAT	$\lvert x\rvert = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$
Researcher	Can you explain this graph? (referring to SM4266g)
RAT	The graph divides the regions into three: $x < 2,\ 2 < x < 3\ and\ x \geq 4$
Peneliti	Explain your reasoning.

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RAT	Points 2 and 3 are determined by the limits of x from the four possibilities.
	$x < 2$ is from the fourth possibility.
	$2 < x < 3$ because of the boundary for x
	I'm not sure about $x \geq 4$, but I think it's because $x \geq 3$.
Researcher	Why did you test $x < 2$ and $x < 3$? (referring to SM4266h and SM4266i)
RAT	Based on points 2 and 3.
Researcher	You turned it into an equation, can you explain why?.
RAT	To find the value of x, ma'am.

According to FEL, there are four boundary intervals, which imply four possible solution sets for the inequality. In contrast, subject RAT also wrote four intervals but divided the number line into three regions to determine the solution set for the absolute value inequality. There is a noticeable difference in the narratives regarding the domain of absolute values. Both students struggled to fully understand the use of boundaries, despite having written them down. It is evident when they write the interval $x < 2$ and change the inequality to $-x + 3 + (-x + 2) = 11$. However, RAT was unable to complete the entire process of solving the task.

4.4. Multiplying 1 and -1 Into the Absolute Value Function

Subject RIF stated to solve absolute function inequalities, we first multiply the mediator in the form of numbers, namely 1 and -1 into the absolute value function $|x - 3| + |x + 2| < 11$ (SM63, SM67).

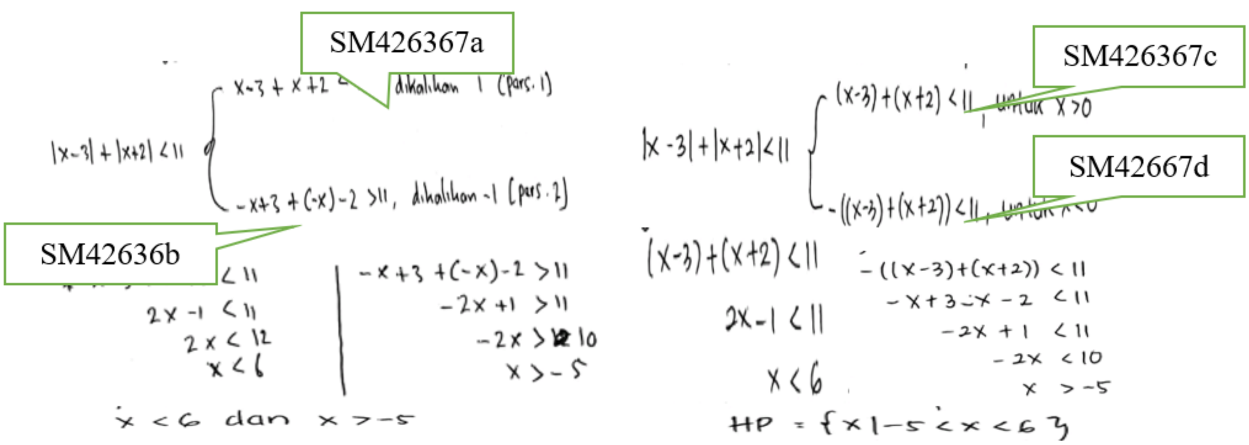

Figure 6.
Answer by subject RIF and subject NUR.

Table 8.
Discussion transcripts subject RIF and subject NUR.

Interview	**Discussion**
Researcher	What process did you need to do?
RIF	First, I multiplied the inequalities by 1 and -1
Researcher	What is the reason?
RIF	To remove the absolute sign, ma'am.
NUR	I used $x > 0$ and $x < 0$
Researcher	How did you get $x > 0$ and $x < 0$
NUR	By using the definition of absolute value, ma'am.

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Researcher	Next, what did you do?
RIF	I removed the absolute value symbol after the inequality is multiplied by 1 and -1.
Researcher	Are you sure?
RIF	Yes, ma'am.
Researcher	What are $x < 6$ and $x > -5$ for?
RIF	They are the final result, ma'am.
NUR	Solution set of inequalities, ma'am.
Researcher	How did you obtain it?
RIF	Solution set of inequalities is the combination of two inequalities.

The narratives presented by RIF and NUR differ when they removed the absolute value from the inequality. However, they claimed to have used the definition of absolute value in inequalities. Subject RIF stated that to remove the absolute sign in inequalities, the left side of the inequalities was multiplied by -1 and 1. The narrative said by NUR was almost similar to RIF's. However, NUR added the interval conditions, which were $x>0$ and <0. The subjects obtained the same solution set. However, there were differences between each subject in using absolute value. Based on commognitive perspective, definitions are endorsed through narratives. This is in accordance with the research by Gagagtisis & Panaoura (2014), which found that students assume $|x + 3| = x + 3$ if $x > 0$, and $|x + 3| = - x + 3$ if $x < 0$. According to Papadouris et al. (2024), difficulties were found in understanding the definition of absolute value as a piecewise function. This definition is directly related to the challenges students face when understanding basic relationships and ultimately leads to difficulties in solving simple equations.

4.5. Determining x (SM48, SM68)

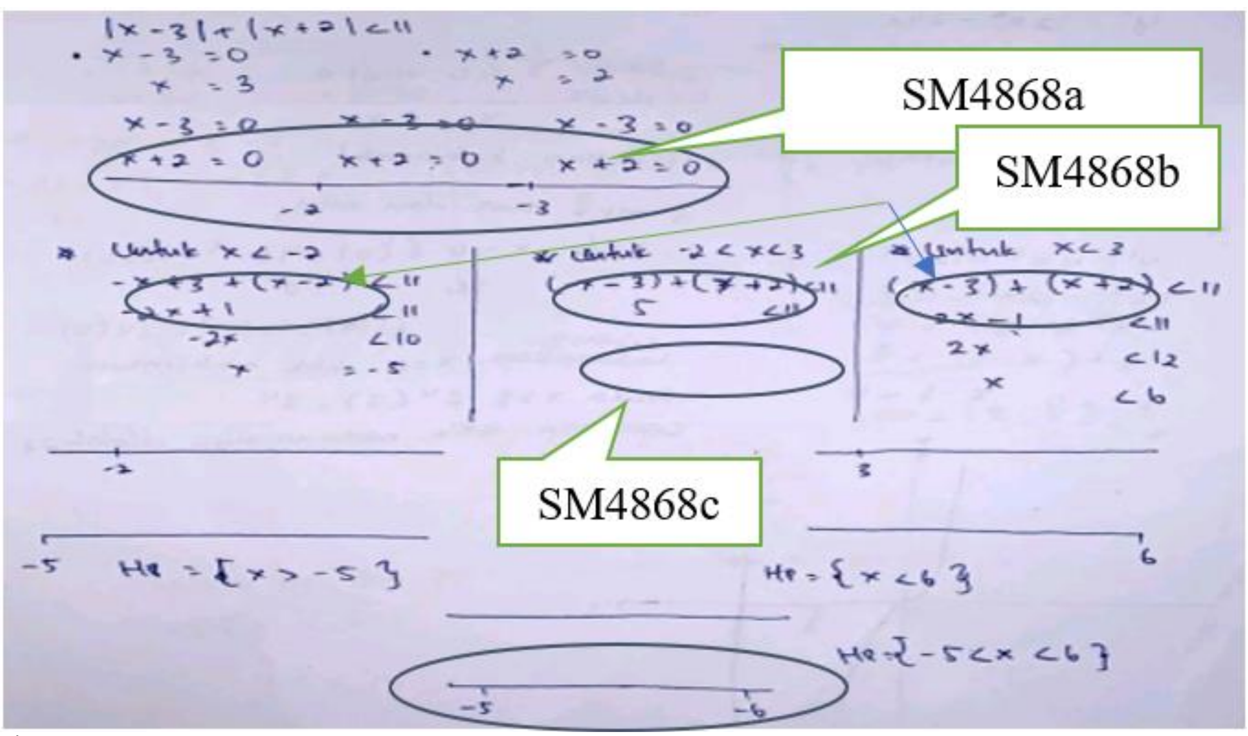

Figure 7.

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Answer by subject EKA.

Table 9.
Discussion transcripts subject EKA.

Interview	Discussion
Peneliti	You wrote the equation $x - 3 = 0$ and $x - 2 = 0$, what were they for?
EKA	I would determine the value of x, ma'am.
	Then, I divided it into three regions 3, namely $x < -2, -2 < x < 3\ \ dan\ \ x \geq 4$
Peneliti	How did you decide these regions?
EKA	I refer to the examples provided
Peneliti	You wrote $-x + 3 + (x - 2) < 11$. How did you come up with that?
EKA	It was based on equality $x < -2$.
	As were the other regions, ma'am.
Peneliti	How about $5 < 11$?
EKA	That is a true statement, ma'am, in the interval in between.
Peneliti	Hp=$\{-5 < x < 6\}$. What does it mean?
EKA	*Himpunan penyelesaian* (Solution set)
Researcher	Can you explain the process?
EKA	I took the points -5 and 6, ma'am.

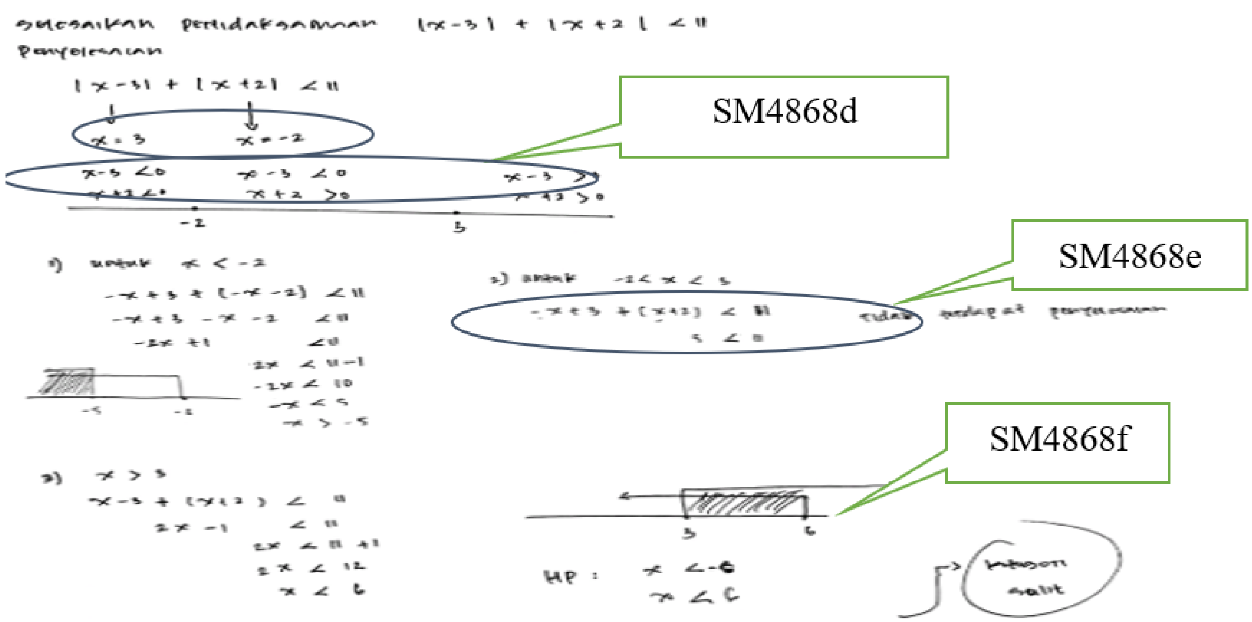

Figure 8.
Answer by subject EVA.

Table 10.
Discussion transcripts subject EVA.

Interview	Discussion
Peneliti	You wrote $x - 3 = 0$ and $x - 2 = 0$, what were they for?
EVA	To determine x, to create a number line, ma'am.
Peneliti	Why did you create a number line?

EVA	To determine the solution region.
	For example, in the region $x < 2$, it is determined that $-x + 3 + (-x - 2) < 11$
Peneliti.	Alright. Next, how did you get the solution set?
EVA	From the solutions 1), 2) and 3)
Peneliti	What did you do?
EVA	It's the combination of $x > -5$ and $x < 6$
Peneliti	Okay

Subject EKA and EVA created three intervals to determine the solution set of the absolute value inequality. Both subjects wrote the equations x-3=0 and x-2=0 and used them to create points on the intervals. EKA mentioned that the interval in the middle produced a true statement because it showed 5<11. Subject EVA stated that five minus three is a true statement. In the process of obtaining the solution set, EKA said it can be seen from points -5 and 6. Meanwhile, subject EVA stated that it is the union of x>-5 and x<6. EKA and EVA differed in their interpretations.

4.6. Replacing the Less Than Sign to an Equal Sign (SM49, SM65), Moving Absolute Value (SM410)

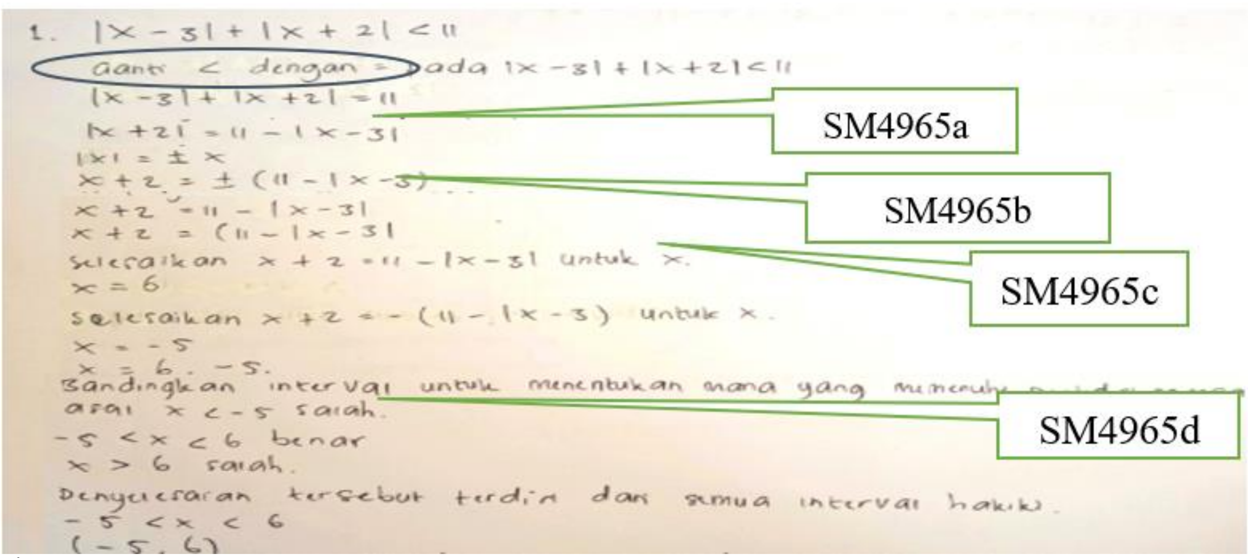

Figure 9.
Answer by subject EET.

Table 11.
Discussion transcripts subject EET.

Interview	**Discussion**
Researcher	Why did you change < symbol with = symbol?
EET	To create an equation, ma'am, to make it easier to determine x
Researcher	How is the process?
EET	Subtract $\|x - 3\|$ from both sides of the equation.
Researcher	Is there still an absolute value?
EET	To eliminate the absolute value, I used the property $\|x\| = \pm x$
	The algebraic process gives $x = 6$ and $x = -5$
Researcher	How did you obtain the interval (-5,6)?

EET	The value $x = 6$ and $x = -5$ were used to determine the interval of the inequalities that fulfill the condition.
Researcher	Why did it change back to an inequality?
EET	It is based on the question, ma'am.

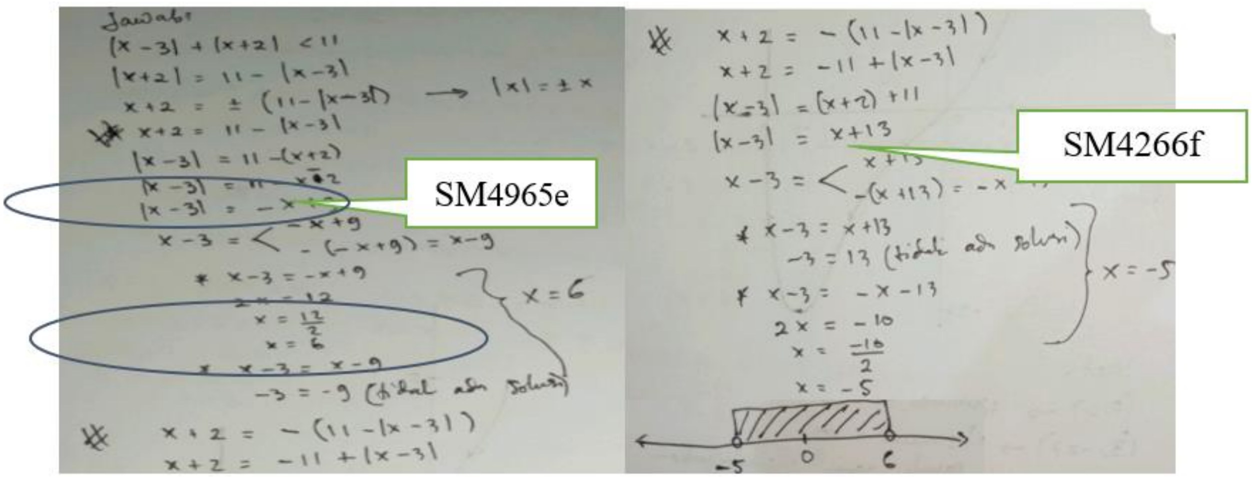

Figure 10.
Answer by subject MAL.

Table 12.
Discussion transcripts subject MAL

Interview	**Discussion**
Researcher	Why did you change < symbol to = symbol?
MAL	To make it easier to determine x.
	Then, I used the property $|x| = \pm x$.
Researcher	There is still an absolute value on the right side (SM4266)
MAL	I will work on it step by step, ma'am.
Researcher	This result is in an equation. Why did it become an interval?
MAL	Try it out.

Subject EET stated that to simplify his work, he changed the less than sign to an equal sign. Next, EET used the definition $|x| = \pm x$ to eliminate the absolute value. The two subjects did not simultaneously eliminate the absolute value, as seen in codes (SM4965e) and (SM4965f). There were commognitive conflicts of the subjects in the endorsed narratives when applying the definition $|x| = \pm x$. This aligns with the findings of Gagatsis & Panaoura (2014), where students assumed that $| \ |x| = \pm x$ implies that for every x, |x| has two values.

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4.7. Are Squared (SM69)

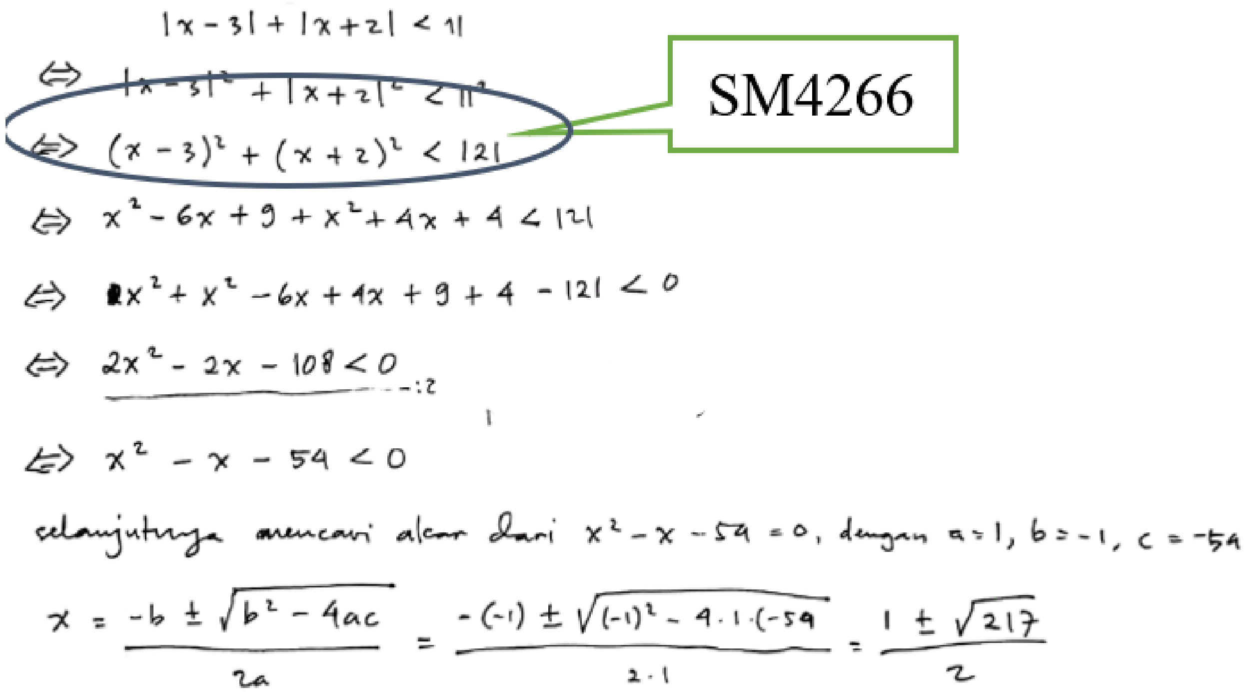

Figure 11.
Answer by subject ARW.

Table 13.
Discussion transcripts subject ARW.

Interview	**Discussion**
Researcher	What is your idea?
ARW	Absolute value inequalities can be solved by squaring both sides.
Researcher	Why must it be squared?
ARW	It uses the property $|x|^2 = x^2$
Researcher	Explain this (SM4266)
ARW	Separate the plus sign.
	I'm not able to do it yet, ma'am.

Subject ARW used endorsed narratives that also represented the property of absolute value, namely $|x|^2 = x^2$. However, based on interview results, they only knew the property without deeper understanding. What they did was merely replace the absolute value sign with a square sign, followed by the process of squaring the function.

Based on findings from 78 students working on absolute value inequality assignments, there were 10 different ideas identified as commognitive conflicts. To explore further, in-depth interviews were conducted with 11 students selected as research subjects. This study found commognitive conflicts among subjects regarding the use of visual mediators and endorsed narratives. Thus, understanding the concept of absolute value is considered fundamental. In secondary and higher education, the concept of absolute value is regarded as one of the most fundamental concepts because it encompasses many mathematical concepts (Baştürk, 2023).

When subjects used the property of absolute value inequalities |x|<a, they did not pay attention to the visual mediator represented by the plus sign in the problem. Subjects did not differentiate between absolute values bounded by addition, for example in the inequalities $|x-3|+|x+2|<11$ and $|x+2|<11$. According to Papadouris et al. (2023), students' misinterpretations of the signs regarding the concept of absolute value led many to write $|-x|=x$. Similarly, subjects' interpretations of the domain of absolute values remained varied when defining the domain of $|x-3|$ and $|x+2|$. Subjects assumed $|x-3|=x-3$ and $|x-2|=x-2$ are always positive, without further consideration of the properties of absolute values. This narrative became ingrained in students' minds, indicating a need for clarification. Thus, absolute values became important with the introduction of negative numbers, which also presents challenges for them. This means we must change the entrenched perceptions about the concepts of numbers they have developed since elementary school (Almeida & Bruno, 2014; Papadouris et al., 2024). A piecewise function is a function defined by more than one formula. Each formula has its own domain of definition, and the domain of the piecewise function is the union of all the smaller domains from each formula. The definition of absolute value as a piecewise function is another source of difficulty (Tall & Vinner, 1981), especially when students tend to memorize that definition.

5. Conclusion

The study was conducted on even semester students who have studied absolute value inequality. A total of 78 students who worked on this task, there were 10 main categories of commognitive conflicts. The cause of the emergence of commognitive conflicts is because students do not understand the absolute value inequality. Students only know the definition or theorem of absolute value but cannot use it in solving problems. In addition, they are confused about the intervals that exist in the absolute value interval. This provides an opportunity for researchers to explore other commognitive conflicts in this material.

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