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Parametric self-excitation in the dynamic cutting system

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Abstract: The problem of ensuring the stability of tool-forming motion trajectories relative to the workpiece, taking into account parametric self-excitation, is considered. The factors causing periodic changes in the parameters of the dynamic cutting system are analyzed. These factors are related to the spatial anisotropy of elasticity properties in the processed workpiece subsystem, variations in the allowance around the perimeter of the workpiece rotation, kinematic disturbances from the mechanical parts of the machine tool's drive units, and periodic processes in the cutting zone. A generalized dynamic model of the system with periodically varying parameters is presented. The influence of periodically changing parameters on the stability of the trajectories is studied. The specific features of stability loss in dynamic cutting systems are revealed. In particular, it is shown that due to parametric effects, as cutting speed increases and spindle rotation frequency rises, there is always a critical frequency at which the system loses stability.
Keywords: *Periodic parameter variation, Parametric self-excitation, Stability.*

1. Introduction

Ensuring the stability of the tool's stationary movements around the workpiece is one of the most important challenges when selecting technological parameters and design features of the tool and workpiece subsystems, which interact through the cutting process. Vibrations that arise from the loss of stability in the stationary trajectory of the tool's movements around the workpiece directly affect the geometric quality indicators of the part, productivity, production costs, and tool durability. As a result, numerous studies have focused on understanding the conditions leading to instability and the causes of self-oscillations during cutting [1-36]. In these studies, linear or nonlinear differential equations with constant parameters were used as mathematical models.

In summarizing the material presented above, the following points explain the loss of stability and the development of self-oscillations during cutting:

- The loss of stability depends on the delay in the variations of cutting force relative to variations in the cut-off layer's area. The existence of this delay between the cutting forces and the deformation shifts of the tool relative to the workpiece is proposed as an explanation for the loss of stability. This concept is discussed in the works of Kudinov A.V., Eliasberg M.E., Zharkov I.G., and others [1-3,9-11]. The effect of shear layer variations from the previous cycle is treated as a delayed argument, as examined by Y. Altintas, S.A. Tobias, H.E. Merritt, J. Tlusty, and others [4-6,20-25,27-30,32-36].
- The loss of stability and the development of self-oscillations are also explained by the dependence of cutting force on cutting speed. This mechanism of oscillation excitation is explored in the works of J. Peters, I. Grabec, M. Wiercigroch, A.I. Kashirin, L.S. Murashkin, and others [7,8,12-16,26,31].
- Another explanation for the loss of stability and the development of self-oscillations is the

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ambiguity in the dependence of cutting forces when the tool moves toward and away from the workpiece. In this case, the hysteresis characteristics define a spatial delay, which is explored as a mechanism for the formation of self-oscillations in the works of N.V. Vasilenko, V.A. Ostafiev, T.P. Putyata, A.P. Sokolovskiy, and others [17-19].

Some studies also examine the loss of system stability during cutting due to periodic changes in its parameters. This is discussed in the works of T. Insperger, G. Stepan, Z. Yao, Yu.F. Koperlev, and others [37-40]. In these works, the changing stiffness is identified as the primary cause of parametric oscillations during cutting. They often use a modified Mathieu-Hill equation. However, the main limitation of these models is their scalar representation, which does not account for the various factors contributing to the periodic changes in the system's parameters.

Practical experience shows that these concepts often contradict many well-known experimental data. For instance, according to current theories, as cutting speed increases, the system's stability reserve should also increase. Yet, practice reveals that this holds true only at relatively low spindle speeds. When the spindle speed exceeds a certain threshold, vibrations invariably increase, indicating a loss of trajectory stability. This paper, based on established concepts of the dynamic cutting system, addresses a previously under-analyzed issue of self-excitation in the system due to periodic parameter changes in the mathematical description of system dynamics.

There are numerous reasons to consider parameter variability in the dynamic cutting system. *Firstly*, the elastic properties within the machined part's subsystem are not symmetrical. *Secondly*, when machining a workpiece with periodic variations in the allowance, the parameters of the dynamic characteristics of the cutting process also exhibit periodic changes. *Thirdly*, the trajectories of the machine's actuators are always subject to periodic variations due to kinematic disturbances, which depend on the accuracy and design imperfections of the mechanical drive components. *Fourthly*, in the machining of most materials, there is a periodic change in cutting resistance associated with the periodic formation of the sliding surface in the region of primary plastic deformation. *Finally*, many technological processes inherently involve periodic changes in parameters, such as in milling operations. All of these factors lead to, at the very least, periodic changes in the total stiffness within the equations of interacting subsystems. In such systems, as known from vibration theory, parametric self-excitation is possible, depending on the frequency of the periodic changes in parameters.

2. Generalized Mathematical Model of a Cutting Dynamic System as a System with Periodic Parameter Changes

To study the conditions for self-excitation in a dynamic cutting system with parameters that periodically vary as functions of time, it is necessary to consider a generalized model of the system. Such a model should capture the main characteristics of the system without being overly complex. For this purpose, we will present the main physical assumptions adopted in the creation of the mathematical model:

- We consider that the dynamic equation of the cutting process represents a two-mass mechanical system consisting of the tool and workpiece subsystems. These subsystems interact through a dynamic link formed by the cutting process [2-3].
- Without loss of generality, we limit our consideration to the case where the stiffness of the tool subsystem is much greater than that of the workpiece subsystem, meaning the stiffness of the tool subsystem can be considered absolutely rigid.
- Elastic deformation shifts $X(t)$ of workpiece subsystem are considered in space, that is $X = \{X_1, X_2, X_3\}^T$,where , $X_1(t)$- tangential, $X_3(t)$ - radial, $X_2(t)$ - axial.

Taking into account the assumptions made the equation of dynamics of cutting process has the following form:

$$m\frac{d^2X}{dt^2} + C\frac{dX}{dt} + kX = F(f_c, t_c X, \frac{dX}{dt}) \qquad (1)$$

where $F(f_c, t_c X, \frac{dX}{dt}) = \left\{ F_1(f_c, t_c X, \frac{dX}{dt}), F_2(f_c, t_c X, \frac{dX}{dt}), F_3(f_c, t_c X, \frac{dX}{dt}) \right\}^T$ - vector - functions of cutting forces, depending on technological parameters (f_c- feed, t_c- depth of cut) and elastic deformation displacements of the workpiece subsystem at the point of contact between the tool and the workpiece.

$m = \left[m_{ij} \right], i, j = 1,2,3 \quad and \quad m_{ij} = 0, i \neq j$; $c = \left[c_{ij} \right], i, j = 1,2,3$; $k = \left[k_{ij} \right], i, j = 1,2,3$- matrixes of inertial, dissipative and elastic coefficients of workpiece subsystem. As shown in works [1-3], all these matrixes are symmetric positive-definite matrices.

To study the system (1), it is first necessary to reveal the structure of the cutting force and the periodic change of the system parameters, as well as their mathematical description.

In (1), at disclosure of regularities of formation of cutting forces we will take the following hypotheses into account [2,24,41, ...]:

+ Forces formed in cutting zone are proportional to the cross-sectional area of uncut chip, that is $F_0(t) = \sigma.S(t),$ where σ - specific force of cutting; $S(t) = a(t).h(t) = f_c(t).t_c(t)$- the current value of the cross-sectional area of uncut chip; $a(t) = \frac{t_c(t)}{\sin\varphi}$- the current value of width of the cut-off layer; $h(t) = f_c(t)\sin\varphi$- the current value of thickness of the cut-off layer; φ - the main angle on the plan; $t_c(t)$ and $f_c(t)$ - respectively the current value of cutting depth and feed which are defined by the following expressions: $f_c(t) = f_c^0 - X_3(t) + X_3(t-T)$;$t_c(t) = t_c^0 - X_2(t)$, where f_c^0, t_c^0- given values of feed and cutting depth; $X_3(t) - X_3(t-T) = f_{t-T}^{t} \upsilon_3(t)dt$, $\upsilon_3(t)$ - variable component of longitudinal feed speed, mm/s; T - period of rotation of spindle.

+ Cutting forces have invariable orientation in space: $F(t) = \chi F_0(t)$, where $\chi = \{\chi_1, \chi_2, \chi_3,\}^T$ - vector of orientation coefficients of forces in space.

+ The module of force is late at a variation of value of the area of the cut-off layer [1,9,10]: $F_0(t) = \sigma S(t-\tau)$, where τ - delay time or constant of time of a chip formation.

Thus, the cutting force can be represented in linearized form as follows:

$$F_0(t) = \chi_1\sigma\left[f_c^0(t)t_c^0(t) - f_c^0(t)X_2(t) - t_c^0(t)\left(X_3(t) - \left(X_3(t-T)\right)\right)\right] \qquad (2)$$

Now, we will consider the factors that influence the periodic variation of the system parameters and their mathematical description.

Periodic changes of parameters of the linearized dynamics equation in the vicinity of a stationary trajectory have two main reasons. The first one is periodic change of stiffness in a workpiece subsystem fixed in the chuck. In this case, as shown in experimental studies, values of stiffness of workpiece subsystem in the plane normal to its rotation axis periodically change along rotation angle of workpiece (Fig.1) for the case when the part is fixed in a three-jaw chuck.

Note that periodic changes in stiffness are due to the fact that when the workpiece is positioned opposite the chuck jaw, the stiffness is always greater than when it is positioned between the jaws. Thus, parameters of stiffness of a workpiece subsystem fixed in a spindle in the direction illustrated in Fig.3 and in the direction orthogonal to it have approximately invariable distributions, but they are shifted on a phase by an angle depending on the number of chuck jaws, for a three-jaw chuck on a angle of $\frac{\pi}{3}$. It is characteristic that the modulation level of parameters of stiffness of processed workpiece subsystem depends on geometry of jaws and on the way of fixing detail in the back rotating center. Level of modulation is defined by parameter Δ in expression $\left[c + \Delta\cos(3\Omega t)\right]$. Besides, frequency Ω is rotation frequency of spindle. Parameters of stiffness of tool subsystem in orthogonal directions are symmetric as shown in [1, 6]. Therefore, matrices of stiffness of workpiece subsystem can be expressed in the following form:

$$c = \begin{bmatrix} c_0 + \Delta\sin 3\Omega t & 0 & 0 \\ 0 & c_0 + \Delta\sin(3\Omega t + \frac{\pi}{3}) & 0 \\ 0 & 0 & c_{33} \end{bmatrix} \quad (3)$$

In general case, matrices of stiffness of workpiece subsystem can be expressed in the following form:

$$c = \begin{bmatrix} c_{11} + \Delta_{11}\sin(\omega_1 t) & c_{12} + \Delta_{12}\sin(\omega_1 t + \theta_{12}) & 0 \\ c_{21} + \Delta_{21}\sin(\omega_1 t + \theta_{21}) & c_{22} + \Delta_{22}\sin(\omega_1 t + \theta_{22}) & 0 \\ 0 & 0 & c_{33} \end{bmatrix} \quad (4)$$

where, $\omega_1 = n\Omega$, n - number of chuck jaws, Ω - rotation frequency of spindle.

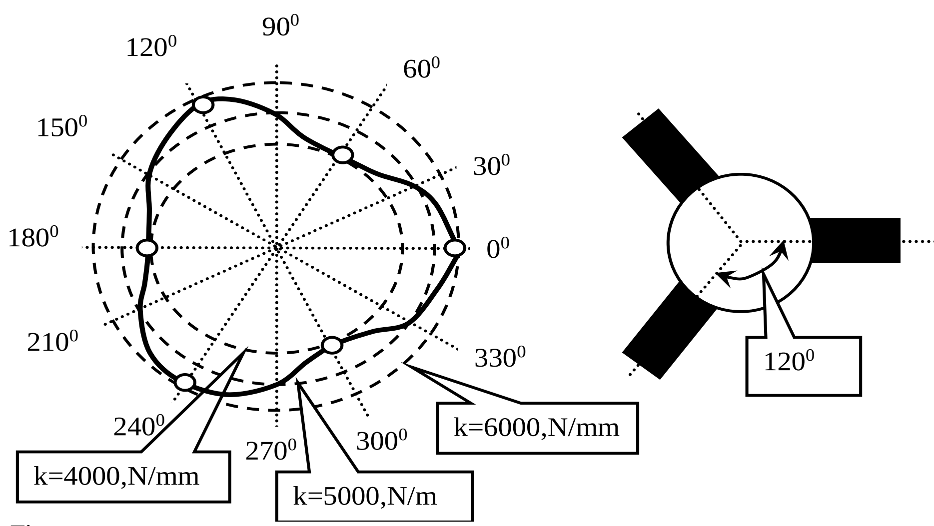

Figure 1.
An example of change of stiffness of the workpiece fixed in the three-jaw chuck at change of rotation angle of spindle.

The second reason is caused by the errors of a profile of workpiece cross section caused by inaccuracy of workpiece installation in the tightening device and shift of workpiece axis and its rotation axis. Besides, periodic changes of allowance are influenced by radial beats of spindle which, as shown in work [42], also have circular trajectories in variations relative to ideal rotation axis of spindle. All these errors, which are representable in the form of a limited Fourier series, cause additional change of the current value of cutting depth, therefore, periodic changes of parameters in the equations of the interacting subsystems. Then the current cutting depth has the following form:

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$$t_c(t) = t_c^0 + \sum_{i=1}^{N} \mu_i \sin(2i-1)\omega_2 t - X_2(t) \quad (5)$$

Were, μ -coefficient of periodic component of cutting depth.

Expression (5) shows dependence of the current cutting depth on deformation shifts taking into account periodic variations of allowance on the workpiece rotation period. In particular, if absolutely round body of workpiece, axis of which is displaced about rotation axis of spindle is considered, then

$$t_c(t) = t_c^0 + \mu \sin \omega_2 t - X_2(t) \quad (6)$$

First of all, we will give a qualitative characteristic of the system (1) taking into account (2), (4), (6). It has the stationary solution $X(t) = \left\{X_1^*(t), X_2^*(t), X_3^*(t)\right\}^T$, defined from condition $\frac{dX_i}{dt} = 0$, $\frac{d^2 X_i}{dt^2} = 0$. This stationary trajectory can lose stability. To analyze stability of a stationary trajectory of movements of system (1) it is necessary to consider the equation in variations concerning this trajectory. This equation is obtained after replacing variables $X(t) = X^*(t) + x(t)$ and for small deviations we obtain the equation in variations relative to stationary trajectory:

$$\begin{cases} m_0 \frac{d^2 x_1}{dt^2} + c_{11}\frac{dx_1}{dt} + c_{12}\frac{dx_2}{dt} + c_{13}\frac{dx_3}{dt} + \left[k_{11} + \Delta_{11}\sin(\Omega_1 t + \theta_{11})\right] x_1 + \left[k_{12} + \Delta_{12}\sin(\Omega_1 t + \theta_{12})\right] x_2 \\ + \left[k_{13} + \chi_1 \sigma t_c^0 (1 + \sin(\Omega_2 t))\right] x_3 + \chi_1 \sigma f_c^0 y_2 = 0 \\ m_0 \frac{d^2 x_2}{dt^2} + c_{21}\frac{dx_1}{dt} + c_{22}\frac{dx_2}{dt} + c_{13}\frac{dx_3}{dt} + \left[k_{21} + \Delta_{12}\sin(\Omega_1 t + \theta_{21})\right] x_1 + \left[k_{21} + \Delta_{21}\sin(\Omega_1 t + \theta_{21})\right] x_2 \\ + \left[k_{23} + \chi_2 \sigma t_c^0 (1 + \mu \sin(\Omega_2 t))\right] x_3 + \chi_2 \sigma f_c^0 y_2 = 0 \\ m_0 \frac{d^2 x_3}{dt^2} + c_{31}\frac{dx_1}{dt} + c_{32}\frac{dx_2}{dt} + c_{33}\frac{dx_3}{dt} + k_{31}x_1 + k_{32}x_2 + \left[k_{33} + \chi_3 \sigma f_c^0 (1 + \mu \sin(\Omega_2 t))\right] x_3 + \chi_3 \sigma f_c^0 y_2 = 0 \\ \tau \frac{dy_2}{dt} + dy_2 = x_2 \end{cases} \quad (7)$$

where $x = \left\{x_1, x_2, x_3\right\}^T$ - vector variations (deviations) of elastic deformation shifts of system from the stationary.

It is important to emphasize that the equation in variations about stationary trajectory (7) has the periodic coefficients influencing its stability, that is, they characterize the mechanism for stability loss connected with parametrical excitement which isn't considered earlier in dynamics of machines.

In (1), the frequencies Ω_1 and Ω_2 are either equal to or multiples of the spindle rotation frequency. Therefore, Floquet theory [43, 44] can be used to analyze the stability of system (7). When studying the stability regions, the method of direct digital integration of the system of differential equations based on the fourth-order Runge-Kutta method was applied. The loss of stability was identified through the solution trajectories of the differential equations.

3. Features of Parametric Self-Excitation in a Dynamic Cutting System

In a real dynamic cutting system, there is always a change in parameters due to various reasons. Typically, there are several frequency components of parameter variation, which are multiples of the spindle rotation frequency. The variations in forces lag behind the changes in elastic deformation displacements, and this lag depends on the technological conditions, the geometry of the tool, and the physical and mechanical properties of the processed material. Therefore, even without parametric effects, the system may lose stability. Periodic parameter changes typically contribute further to this loss of stability.

The elastic properties of the workpiece subsystem, without the cutting process, are linear, and the stiffness matrices are symmetric and positively defined. Therefore, in the modeling space of elasticity, based on the rotation matrix, it is possible to define the axes of collinear and orthogonal directions, which rotate as the spindle turns under a given anisotropy of the elastic properties. This condition can cause parametric self-excitation of the system, which complements the self-excitation conditions due to the influence of circulatory forces formed by the dynamic interaction of the machining process.

For further analysis, some typical examples of parametric self-excitation systems are given. For this purpose, we will consider various parametric self-excitation diagrams on a plane: $\eta = \omega / \omega_0 - \mu$, where ω_0 is the natural frequency of the main oscillatory circuit of the system, *Hz*; ω is the parametric excitation frequency, *Hz*; μ is the parametric excitation level ($\mu \leq 1$).

To more clearly clarify the influence of various system features on the loss of stability, we first consider the scalar analogue of system (7). For this case, we analyze the influence of two-frequency parametric excitation for the system:

$$\begin{cases} m_0 \dfrac{d^2x}{dt} + c\dfrac{dx}{dt} + k(1+\mu_1 \sin(\omega t) + \mu_2 \sin(3\omega t))x = -k_c y \\ \tau \dfrac{dy}{dt} + y = x \end{cases} \tag{8}$$

where m,h,c - respectively, the mass, damping coefficient and stiffness of the subsystem; k_c - stiffness of cutting process, N/mm; τ- chip formation time constant, s.

Let us consider the change in the parametric excitation region of the system on the plane of parameters (μ_1, η) with variation of μ_2 from 0 to 0.8 and for given values of the parameters of the dynamic system $k_c = 5000\,N/mm$, $\tau = 0.1*10^{-3}s$; values of system parameters: $m = 2.25\,N.s^2/mm$, $c = 0.1\,N.s/mm$, $k = 10000\,N/mm$. The corresponding parametric excitation diagrams of the system are shown in Figure 2.

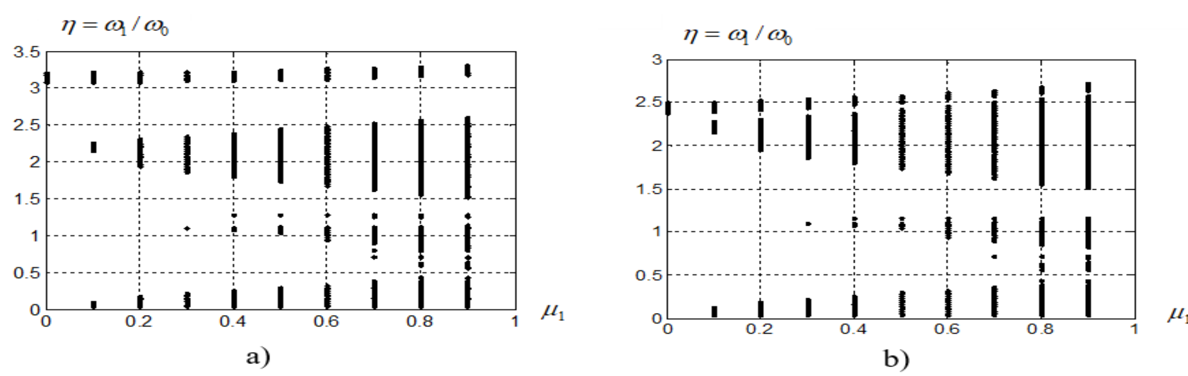

Figure 2.

An example of changing the region of parametric excitation of the system on the plane of parameters (μ_1, η) for the cases $\mu_2 = 0$ (a) and $\mu_2 = 0.8$ (b).

The given example is calculated for a system having a natural frequency of the conservative system $\omega_0 = 150\,s^{-1}$. If we consider the spindle rotation frequency equal to $3000\,rpm$, i.e $50\,Hz$, then the second diagram is already in the region of parametric self-excitation of the system. However, if the scalar case is considered, then parametric self-excitation of the system is observed at high spindle

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rotation frequencies, characteristic only for high-speed cutting. The situation changes fundamentally if we analyze the vector case of spatial oscillatory displacements.

Let us give an example of the conditions of parametric self-excitation of the system for the case when the deformation displacements of the tool occur in the plane. The linearized equation in variations for this case has the form:

$$\begin{cases} m_0 \dfrac{d^2 x_1}{dt^2} + c_{11}\dfrac{dx_1}{dt} + c_{12}\dfrac{dx_2}{dt} + k_{11}\left[1+\mu\sin(\omega t+\theta_{11})\right]x_1 + k_{12}\left[1+\mu\sin(\omega t+\theta_{12})\right]x_2 = -\chi_2 k_c y_1 \\ m_0 \dfrac{d^2 x_2}{dt^2} + c_{21}\dfrac{dx_1}{dt} + c_{22}\dfrac{dx_2}{dt} + k_{21}\left[1+\mu\sin(\omega t+\theta_{21})\right]x_1 + k_{22}\left[1+\mu\sin(\omega t+\theta_{22})\right]x_2 = -\chi_2 k_c y_2 \\ \tau_1 \dfrac{dy_1}{dt} + y_1 = x_2; \tau_2 \dfrac{dy_2}{dt} + y_2 = x_2 \end{cases} \quad (9)$$

where $k_c = \sigma f_c^0$.

In system (9) it is additionally taken into account that the delay of forces in two orthogonal directions differs. Let us pay attention to the fact that in system (9) there is a rotation of the orientation angle of the stiffness ellipse. This fact causes additional conditions for parametric self-excitation of the cutting system.

Let's look at a specific example (Fig. 3). Matrixes of coefficients of workpiece subsystems are given in Table 1. Parameters of a dynamic characteristics of cutting process: $k_c = 5000\,N/mm$; vector of force orientation coefficients $\chi = \{\chi_1, \chi_2\}^T = \{0.5, 0.8\}^T; \tau_1 = 2\cdot 10^{-4}\,s; \tau_1 = 10^{-4}\,s$. The frequency of the first form of oscillation corresponding to workpiece subsystem is equal $\omega_0 = 150\,s^{-1}$. The diagram is constructed in relation to the first natural frequency of the system without cutting. The orientation angle of the stiffness ellipse.

Table 1.

$m, Ns^2/mm$	$c, Ns/mm$	$k, N/mm$
$\begin{bmatrix} 2.25 & 0 \\ 0 & 2.25 \end{bmatrix}$	$\begin{bmatrix} 1.0 & 0.5 \\ 0.5 & 1.0 \end{bmatrix}$	$\begin{bmatrix} 10000 & 5000 \\ 5000 & 10000 \end{bmatrix}$

Here, the conditions of exchange of force flows between the subsystems considering oscillatory displacements in two orthogonal directions in the plane normal to the spindle rotation axis are of great importance in the self-excitation mechanism. The data presented show that in the system under consideration, parametric self-excitation is observed already at spindle rotation frequencies equal to (400–500) *rpm*. A more scrupulous analysis shows that the following factors influence the self-excitation conditions in this case:

1) It is known [2] that due to the reaction from the processing process, the total elasticity matrix becomes asymmetric. Therefore, without parametric effects $(\mu = 0)$ circulatory forces are naturally formed in the system. Depending on the direction of action of the circulatory forces (forces orthogonal to the direction of deformation displacements) and the direction of rotation of the stiffness ellipse, conditions can be formed that promote self-excitation or are aimed at stabilizing the equilibrium.

2) The system under consideration has two resonant frequencies that are separated in the frequency domain by antiresonance. When these frequencies approach each other, vibration modes of the beat type are formed in the system with a frequency equal to the difference in resonant frequencies. As a result, the spindle rotation frequency at which the parametric self-excitation effect is observed can be significantly reduced.

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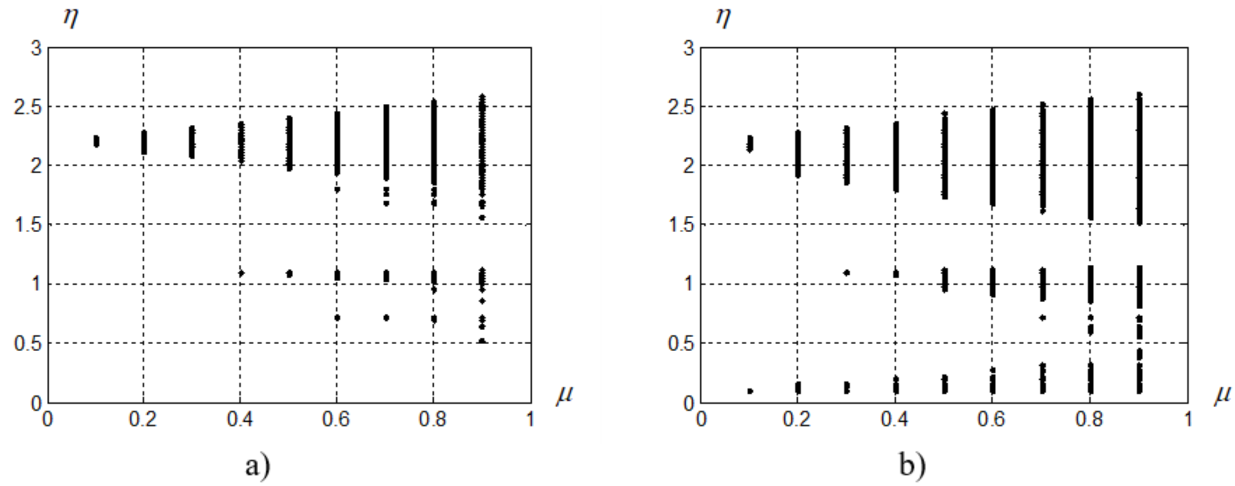

Figure 3.
Change in the parametric excitation region in the plane (η, μ) with variation of the orientation angle of the stiffness ellipse: $a-\alpha=0^0$; $b-\alpha=45^0$.

Now let us consider a more general case, when the periodic change in the stiffness of the workpiece subsystem and the change in the machining allowance are taken into account simultaneously. In this case, the main properties of the system can be considered based on the study of the loss of stability of the system. In addition, in this system the frequencies ω_1 and ω_2 are not equal or are not multiples of each other.

$$\begin{cases} m_0\dfrac{d^2x_1}{dt^2}+c_{11}\dfrac{dx_1}{dt}+c_{12}\dfrac{dx_2}{dt}+c_{13}\dfrac{dx_3}{dt}+k_{11}\left[1+\mu_1\sin(\omega_1t+\theta_{11})\right]x_1+k_{12}\left[1+\mu_1\sin(\omega_1t+\theta_{12})\right]x_2 \\ +\left[k_{13}+\chi_1\sigma t_c^0(1+\mu_2\sin(\Omega_2t))\right]x_3=-\chi_1\sigma k_c y_2 \\ m_0\dfrac{d^2x_2}{dt^2}+c_{21}\dfrac{dx_1}{dt}+c_{22}\dfrac{dx_2}{dt}+c_{13}\dfrac{dx_3}{dt}+k_{21}\left[1+\mu_1\sin(\omega_1t+\theta_{12})\right]x_1+k_{12}\left[1+\mu_1\sin(\omega_1t+\theta_{22})\right]x_2 \\ +\left[k_{23}+\chi_2\sigma t_c^0(1+\mu_2\sin(\Omega_2t))\right]x_3=-\chi_2\sigma k_c y_2 \\ m_0\dfrac{d^2x_3}{dt^2}+c_{31}\dfrac{dx_1}{dt}+c_{32}\dfrac{dx_2}{dt}+c_{33}\dfrac{dx_3}{dt}+k_{31}x_1+k_{32}x_2+\left[k_{33}+\chi_3\sigma t_c^0(1+\mu_2\sin(\Omega_2t))\right]x_3=-\chi_3\sigma k_c y_2 \\ \tau\dfrac{dy_2}{dt}+dy_2=x_2 \end{cases} \tag{10}$$

where $k_c=\sigma f_c^0$.

As before, let's look at an example. Initial data: matrix of inertial coefficients, matrix of dissipative coefficients, stiffness matrix is given in Table 2. Parameters of a dynamic characteristics of cutting process: $k_c=5000\,N/mm$; vector of force orientation coefficients $\chi=\{\chi_1,\chi_2,\chi_2\}^T=\{0.5,0.7,0.51\}^T$; $\tau=2\cdot10^{-4}\,s$.

Let us consider the influence of the frequency ratio $k_\omega = \dfrac{\omega_1}{\omega_2}$ on the parametric excitation region at $\mu_2 = 0.5$. For clarity, especially in the low-frequency region, the illustrations are provided with graphs presented in different scales. In the low-frequency region they are designated (a), (b) (Fig. 4).

Table 2.

$m, Ns^2/mm$	$c, Ns/mm$	$k, N/mm$
$\begin{bmatrix} 2.25 & 0 & 0 \\ 0 & 2.25 & 0 \\ 0 & 0 & 2.25 \end{bmatrix}$	$\begin{bmatrix} 1.0 & 0.5 & 0.3 \\ 0.5 & 1.0 & 0.3 \\ 0.3 & 0.3 & 1.5 \end{bmatrix}$	$\begin{bmatrix} 10000 & 5000 & 2000 \\ 5000 & 10000 & 2000 \\ 2000 & 2000 & 20000 \end{bmatrix}$

For us, the most significant diagrams are in the low-frequency range, as these correspond to traditional technological modes. High-frequency ranges pertain to high-speed cutting, with spindle rotation frequencies exceeding ten thousand revolutions per minute. When analyzing parametric excitation, it is necessary to take into account that, in this case, parameter variations due to the superposition of two non-multiple frequencies occur with beats, the frequency of which is equal to the difference between the considered frequencies of the periodic parameter changes. This is why, in the low-frequency range, depending on the frequency ratio, a self-excitation effect is observed.

An increase in the reduction of the lower spindle frequency, at which parametric self-excitation of the system is observed, is also noted as the frequency of parametric excitation approaches. In defining the dynamic cutting system, additional degrees of freedom and conditions that cause periodic parameter variations are introduced, expanding the scope of parametric self-excitation.

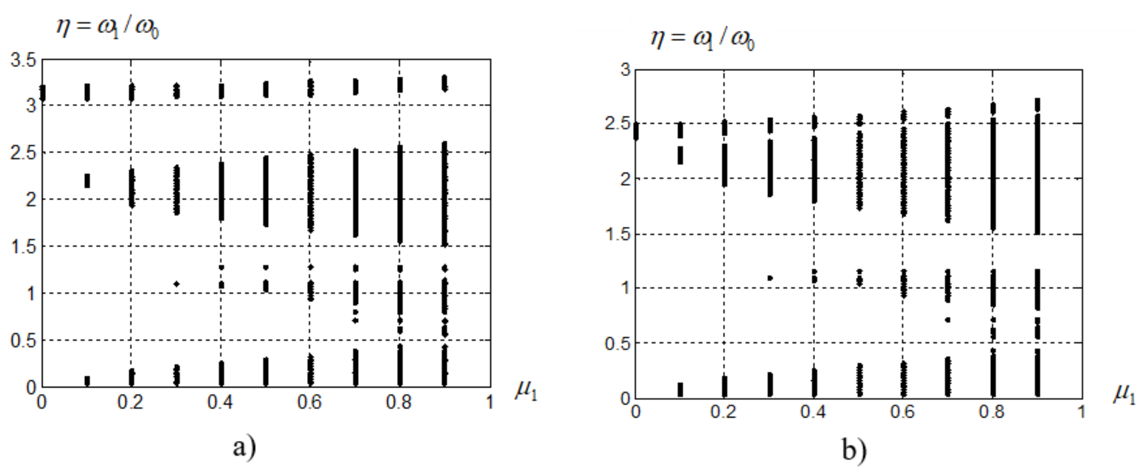

Figure 4.

Change in the parametric excitation region in the plane with variation of the frequency ratio $k_\omega = \dfrac{\omega_1}{\omega_2}$: a) $k_\omega = 0.2$, b) $k_\omega = 0.9$.

4. Conclusion

Parametric effects in a dynamic cutting system play a significant role in the stability of the trajectories of form-shaping motions. According to traditional views on stability improvement, an increase in cutting speed, associated with an increase in the spindle's rotational cycle, is expected to expand the stability region. However, in practice, this does not correspond with experimental data. With even a slight increase in spindle rotation frequency, starting from a certain critical value, the system again loses stability. This phenomenon is attributed to parametric self-excitation. Therefore, depending on the state of the machine, which causes kinematic disturbances in the trajectories and its design factors, there is a limited frequency range of spindle rotation. Within this range, as the cutting speed increases, the stability of the cutting process is first lost in the low-frequency region due to lagging arguments. In the high-frequency region, stability is again compromised due to parametric self-excitation of the system.

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