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Behaviour Analysis of the Padé Sumudu Adomian Decomposition Method Solution

Metomou Richard[1]* Weidong Zhao[1]
[1]School of Mathematics, Shandong University, China; metonourichard@yahoo.fr (M.R).

Abstract: Researchers in the past investigate the Sumudu Adomian Decomposition Method (SADM), the Laplace Adomian Decomposition Method (LADM), the Padé Sumudu Adomian Decomposition Methods (PSADM). In this paper we analyse the behaviour of the function P[L/M][.] called double Padé approximation using in the Padé Sumudu Adomian Decomposition Method (PSADM), and provide some criteriums for chosing L and M to obtain the best Padé approximation solution in the case of nonlinear Schrödinger equation and nonlinear KdV Burger's equation.
Keywords: *Adomian Decomposition Method (ADM), Padé Sumudu Adomian Decomposition Method (PSADM), Nonlinear Schrödinger equation, Nonlinear KdV Burger's equation.*
Abbreviation: *SADM-Sumudu Adomian Decomposition Method, LADM-Laplace Adomian Decomposition Method, PSADM-Padé Sumudu Adomian Decomposition Methods.*

1. Introduction

To solve optimally the problems in science engineering, many works have been proposed to study the stability for the nonlinear system [1]. Different strong scheme have been established to solve the nonlinear problems as, Adomian Decomposition Method (ADM) [2-7], Laplace transform combined with Padé approximation [8-12], Padé Sumudu Adomian Decomposition Method (PSADM) [13] homotopy perturbation method [14, 15] Modified decomposition method [16-18] have been obtained to approximate the analytical solutions. In the present paper, we analyze the behaviour of the Padé Sumudu Adomian Decomposition Methods solution (PSADM) [13] in the case of nonlinear Schrödinger equations [19] and KdV-Burgers Equations [20]. It can be seen in many literatures that the Sumudu Adomian Decomposition Methods (SADM) and Laplace Adomian Decomposition Methods (LADM) give similar results, the Sumudu transform present some advantage in calculation because have unit preserving properties (S [1] = 1). The Padé approximations have been used to control the convergence of the series solution. The function $P_{[L/M]}[.]$, called double Padé approximation [13] can also be use for Laplace Adomian decomposition method, and obtain the new solution. In this paper we analyze the behaviour of the Padé Sumudu Adomian Decomposition Methods Solution and provided some criteriums for the choice of the best PSADMs.

1.1. Padé Approximation

The [*L,M*]-order Padé approximation of the function *f* denote by $P_x^{[L,M]}$ [*f*], is the quotient of two polynomials $R_L(x)$ and $Q_M(x)$ of degrees *L* and *M*, respectively:

$$\frac{R_L(x)}{Q_M(x)}, x \in [a,b]. \qquad (1)$$

Remark 1: *The [L,M]-order Padé approximation of the function f(x) is in the form:*

$$p_{[L/M]}[f(x)] = \frac{a_o + a_1 x + ... + a_L x_L}{b_o + b_1 x + ... + b_M x^M}$$

If $L < M, \lim_{x \to \infty} P_{[L/M]}[f(x)] = 0.$

If $L < M, \lim_{x \to \infty} P_{[L/M]}[f(x)] = \infty, (-\infty or + \infty)$ according to the signe of $\frac{a^L}{b^M}$.

Definition 1: *Let f be function of two variables x and t. We defined two dimensional Padé approximation $P_{[L/M]}[f](x,t)$ of the function f as*

$$P_{[L/M]}[f](x,t) = P_x^{[L,M]}[P_t^{[L,M]}[f]](x,t), \qquad (2)$$

where $P_t^{[L,M]}$ [f] (x, t) denote the [L,M]-order Padé approximation of f(x, t) with respect to the variable t, and P_x[L,M] [f] (x, t) denote the [L,M]-order Padé approximation of f(x, t) with respect to the variable x.
If M=L, we will denote the diagonal Padé approximation of order M by $P_{[M/M]}$ [f] (x, t), and called [M,M]-order Padé approximation or M Padé approximation of f(x, t).

PSADM Procedure [13]

By replacing the Sumudu transform by Laplce transform and using the same procedure we will obtain the Laplace Adomian Decomposition Methods instead of Sumudu Adomian decomposition method.
We consider the PDE in the form as following:

$$L_t u(x,t) + L_x u(x,t) + R(u(x,t)) + G(u(x,t)) = F(x,t) \qquad (3)$$

with u(*x,0*)=h(*x*) the initial condition, L_x the highest order differential respect to *x*, L_t the first order differential respect to *t*, *G*(*u*(*x,y*)) is the nonlinear term, *F*(*x,t*) is the inhomogeneous term, and *R* the remaining linear terms of lower order derivative.
The procedure of PSADM for solving (3) can be write as follows.

Step 1: Take the Sumudu transform to the equation (3) and apply the differentiation property of Sumudu transform to obtain

$$S_t[u(x,t)](v) = h(x) + v.S_t[-L_x u(x,t) - R(u(x,t)) - G(u(x,t)) + F(x,t)](v). \qquad (4)$$

Step 2: Apply the inverse of the Sumudu transform to the above equation to obtain

$$u(x,t) = S_v^{-1}[h(x)](t) + S_v^{-1}[v.S_t[-L_x u(x,t) - R(u(x,t)) - G(u(x,t)) + F(x,t)](v)](t).. \qquad (5)$$

Step 3: Use Adom*ian decomposition method to decomposite the nonlinear function* G(*u*) and the *u*, respectively, as

$$G(u) = \sum_{n=0}^{\infty} A_n$$

and

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t).$$

Step 4: Write the equation in the form

$$\sum_{n=0}^{\infty} u_n(x,t) = S_v^{-1}[h(x)](t) + \sum_{n=0}^{\infty} S_v^{-1}[v[-L_x u_n(x,t) - R(u_n(x,t) + F(x,t) - A_n](v)](t).$$ which lead to the SADM recursive relations:

$$u_0(x,t) = S_v^{-1}[h(x)](t)$$

$\vdots$

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* Correspondence: metonourichard@yahoo.fr

$u_k(x,t) = S_v^{-1}\left[v\left[-L_x u_{k-1}(x,t) - R(u_{k-1}(x,t)) + F(x,t) - A_{k-1}\right](v)\right](t).$

Step 5: Deduct the SADM approximation solution $u_{SADM} = u(x,t,j)$:

$$u(x,t,j) = u_0 + u_1 + \ldots + u_j.$$

Step 6: The $[L,M]$-order PSADM solution $uPSADM = u(x,t,j,[L,M])$ is given by

$$u(x,t,j,\ [L,M]) = P[L/M]\ [u_{SADM}]\ (x,t),$$

if $L=M$, we denote M-PSADM solution by

$$u(x,t,j,M) = P_{[M/M]}\ [u_{SADM}]\ (x,t).$$

Remark 2: *In step 5 we obtain the Sumudu Adomian Decomposition Method (SADM), instead of Sumudu transform we can use other integral transform like Laplace transform to obtain in step 5 the Laplace Adomian Decomposition Method (LADM). The Sumudu Adomian Decomposition Method and Laplace Adomian Decomposition Methods give similar result. The Sumudu transform due to the unit preserving properties (S(1)=1, provided some advantages in calculation.*

For different type of Padé approximation and different order of the Padé approximation we will analyse the behaviour of the P ADM solutions.

Example 1

In the first case of the following example, we will show that for different type of Padé approximation:

$u(x,\ t, j,\ [L,M])$, for L > M,

$u(x,\ t, j,\ [L,M])$, for L = M, and

$u(x,\ t, j,\ [L,M])$, for L < M, we have different solutions and one of them is more accurate.

In the second case of the following example, we will show that for diagonal Padé approximation $u(x,t,j,\ [L,M])$, for L = M, we can increase the accuracy of the method by increasing or reducing de value of M accordingly to the topologie of the solution.

Case 1: Consider the equation:

$$i\frac{\partial u}{\partial t} + \Box + 2[u]_2 u = 0 \quad (7)$$

$$U(x,\ 0) = e^{ix} \quad (8)$$

We can easily deduice the SADM solution [15]:

$$u_{SADM} = u(x,t,j) = e^{ix}\left(1 + it + \frac{1}{2!}(it)^2 + \frac{1}{3}(it)^3 + \ldots + \frac{1}{j!}(it)^j\right. \quad (9)$$

The algorithm is coded by the symbolic computation software Mathematica. We know $u(x,t) = e^{i(x+t)}$ is the exact solution for the Problem.

Figure (1a) and **Figure (1b)** show the real part and imaginary part of SADM solution u_{SADM} = u(x,t,15) in Domain D = [0,2] × [0,2].

For different values of L and M we plot different orders of the PSADM solutions to see the behaviour of the methods.

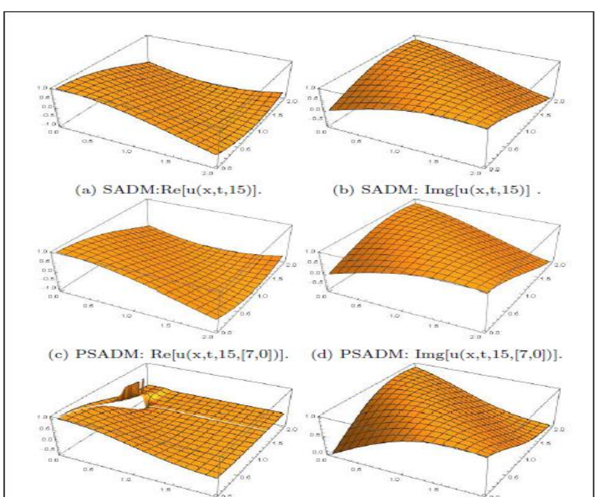
Figure (1c) and **Figure (1d)** show respectively, the real part and imaginary part of PSADM solution u_{PSADM} = $u(x,t,15,[7,0])$ in Domain D = [0,2] × [0,2].

Figure (1e) and **Figure (1f)** show respectively, the real part and imaginary part of PSADM solution u_{PSADM} = $u(x,t,15,[1,7])$ in Domain D = [0,2] × [0,2].

Figure (2a) and **Figure (2b)** show respectively, the real part and imaginary part of PSADM solution u_{PSADM} = $u(x,t,15,[7,1])$ in Domain D = [0,2] × [0,2].

Figure (2c) and **Figure (2d)** show respectively, the real part and imaginary part of PSADM solution u_{PSADM} = $u(x,t,15,[7,7])$ or in short u(x,t,15,7) in Domain D = [0,2] × [0,2].

Figure (2e) and **Figure (2f)** show the real part and imaginary part of the exact solution $u(x,t)$ in Domain D = [0,2] × [0,2].



(a) SADM:Re[u(x,t,15)]. (b) SADM: Img[u(x,t,15)] .

(c) PSADM: Re[u(x,t,15,[7,0])]. (d) PSADM: Img[u(x,t,15,[7,0])].

(e) PSADM: Re[u(x,t,15,[1,7])]. (f) PSADM: Img[u(x,t,15,[1,7])].

Figure 1.
SADM and PSADM solutions using 15 terms.

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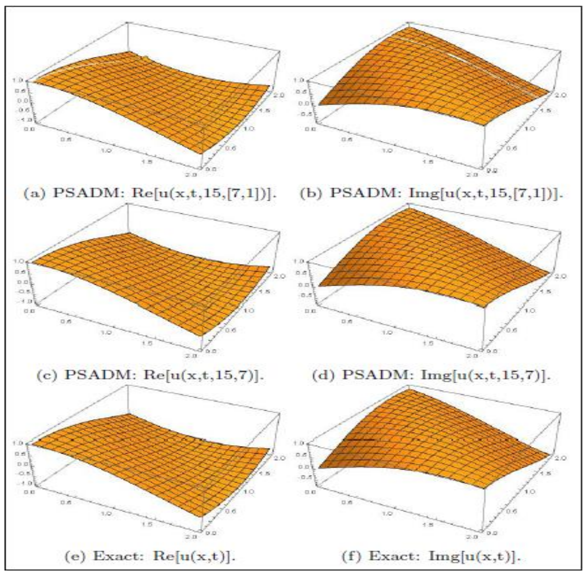

Figure 2.
(a)-(d) PSADM solutions using 15 terms, (e) and (f) exact solutions.

In domain $D = [0, 2] \times [0, 2]$ we can see that the $u(x, t, 15, [1, 7])$ and $u(x, t, 15, [7, 1])$ are not smooth compare to the other solutions. Next we plots the absolute errors.

Figure (3a) and **Figure (3b)** show respectively the absolute error for real part and imaginary part of the Sumudu Adomian Decomposition solution $u(x, t, 15)$.

Figure (3c) and **Figure (3d)** show respectively the absolute error for real part and imaginary part of the Padé Sumudu Adomian Decomposition solution $u(x, t, 15, [7, 0])$.

Figure (3e) and **Figure (3f)** show respectively the absolute error for real part and imaginary part of the Sumudu Adomian Decomposition solution $u(x, t, 15)$.

Now let see the behaviour of the SADM, PSADM solutions in domain $D = [0, 10] \times [0, 10]$.

Figure (4a) and **Figure (4b)** show the real part and imaginary part of SADM solution $u_{SADM} = \mathrm{u}(x, t, 15)$ in Domain $D = [0, 10] \times [0, 10]$.

Figure (4c) and **Figure (4d)** show respectively, the real part and imaginary part of PSADM solution $u_{PSADM} = \mathrm{u}(x, t, 15, [7, 0])$ in Domain $D = [0, 10] \times [0, 10]$.

Figure (4e) and **Figure (4f)** show respectively, the real part and imaginary part of PSADM solution $u_{PSADM} = \mathrm{u}(x, t, 15, [1, 7])$ in Domain $D = [0, 10] \times [0, 10]$.

Figure (5a) and **Figure (5b)** show respectively, the real part and imaginary part of PSADM solution $u_{PSADM} = \mathrm{u}(x, t, 15, [7, 1])$ in Domain $D = [0, 10] \times [0, 10]$.

Figure (5c) and **Figure (5d)** show respectively, the real part and imaginary part of PSADM solution $u_{PSADM} = \mathrm{u}(x, t, 15, [7, 7])$ or in short $\mathrm{u}(x, t, 15, 7)$ in Domain $D = [0, 10] \times [0, 10]$.

Figure (5e) and **Figure (5f)** show the real part and imaginary part of the exact solution $\mathrm{u}(x, t)$ in Domain $D = [0, 10] \times [0, 10]$.

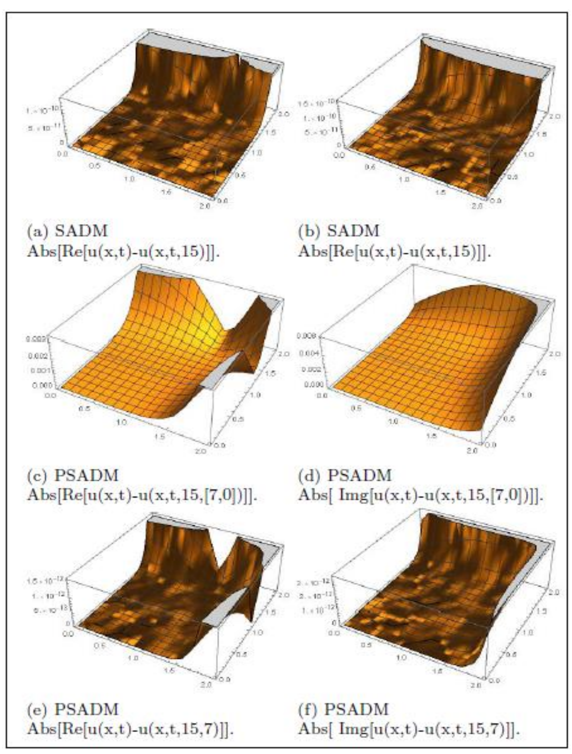

Figure 3.
Absolute errors.

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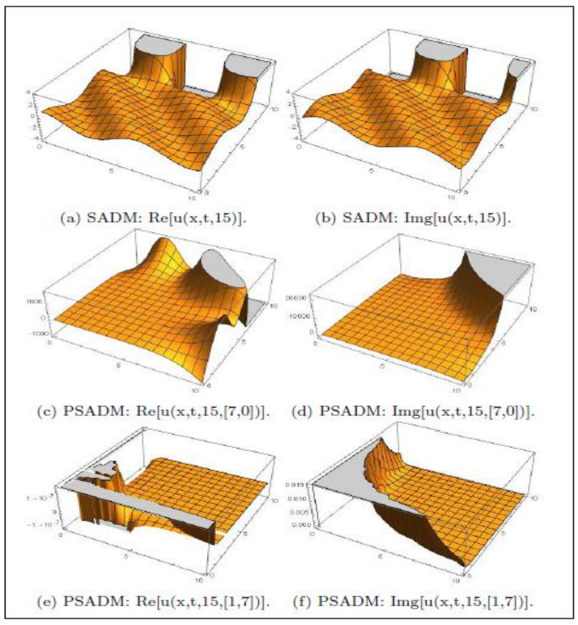

Figure 4.
SADM and PSADM solutions using 15 terms.

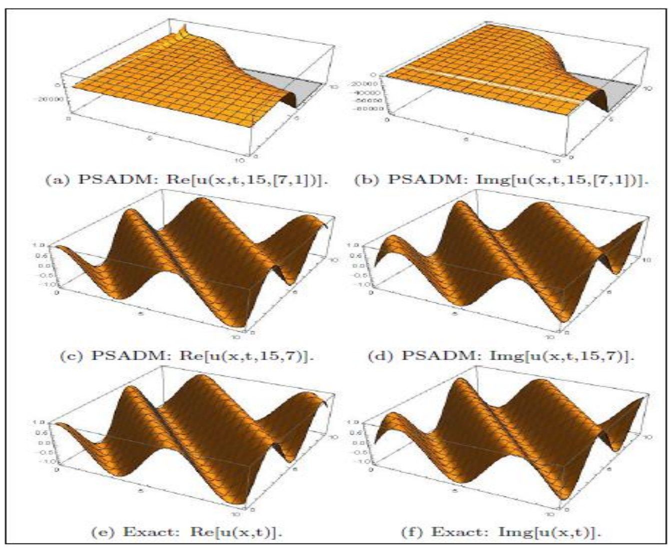

Figure 5.
(a)-(d) PSADM solutions using 15 terms, (e) and (f) exact solutions.

We can see in this example, in domain $D = [0, 2] \times [0,2]$ the SADM behave well, but in domain $D = [0, 10] \times [0, 10]$ the SADM give bad result. Only the u(x, t, 15, 7) approaching well the exact solution in the domain $D = [0, 2] \times [0,2]$ and $D = [0, 10] \times [0, 10]$. Even in domain

$D = [0, 2] \times [0, 2]$ the graph of absolute error show the [7, 7]-order PSADM (in short 7-PASDM) solution is better than SADM solution and other type of PSADM solutions. The solution can be perfom by choosing different value of M.

Remark 3
The real part and imaginary part of the exact solution are bounded,
and the real part and imaginary part of the SADMs are not bounded, in this case in better to use diagonal Padé approximatin to make bounded the Approximate solution.
Case 2: Consider the equation:

$$i\frac{\partial u}{\partial t}+\square u+6[u]^2u=0 \qquad (11)$$

$$u(x,o)=e^{i3x} \qquad (12)$$

We can easily deduce the SADM solution [15]:

$$u(x,t,j)=e^{i3x}\left(1-3it+\frac{(3it)^2}{(2!)}-\frac{(3it)^3}{(3!)}+\ldots\frac{(3it)^j}{(j!)}\right) \qquad (13)$$

And the [L,M] order PSADM solution:

$$u_{PSADM} = u(x, t, j, [L,M]) = P_{[L/M]}\,[u(x, t, j)]\,(x, t), \qquad (14)$$

The algorithm is coded by the symbolic computation software Mathematica.
We know $u(x, t) = e^{3i(x-t)}$ is the exact solution of the Problem.
Figure (6c) and **Figure** (6d) show respectively the absolute error for real part and imaginary part of the PSADM solution $u(x, t, 20, 7)$.
Figure (6e) and **Figure** (6f) show respectively the absolute error for real part and imaginary part of the PSADM solution $u(x, t, 20, 8)$.
Figure (6a) and **Figure** (6b) show respectively the absolute error for real part and imaginary part of the SADM solution $u(x, t, 20)$.

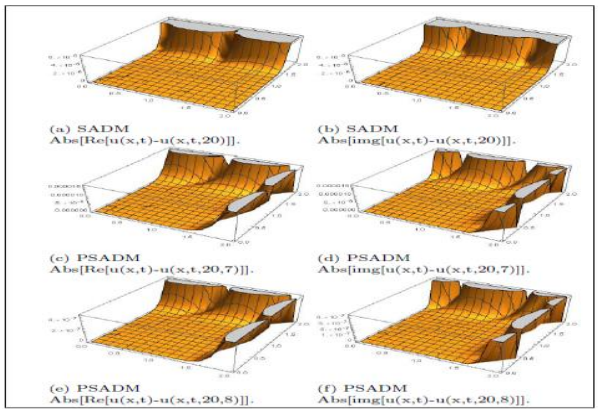

Figure 6.
Absolute error.

The graph of the absolute errors show that the SADM solution give better approximation than 7-order PSADM solution, and the 8-order PSADM solution give better approximation than SADM solution. The solution can be perform by choosing different value of M.

Example 2

Case 1: Consider the equation:

$$\begin{cases} u_t - uu_x + u_{xxx} = 0, \\ u(x,o) = -2\sec^2(x), \\ x \in \mathbb{R}, t \in \mathbb{R}_+, \end{cases} \quad (15)$$

Using the SADMs, we can deduce:

$$u_o = s_v^{-1}[u(x,o)](t), \quad (16)$$

$$u_1 = s_v^{-1}[vS_t[-\frac{\partial^3}{\partial x^3}u_o + 6A_O](v)](t), \quad (17)$$

$$\vdots \quad (18)$$

$$u_n = s_v^{-1}[vS_t[-\frac{\partial^3}{\partial x^3}u_{n-1} + 6A_{n-1}](v)](t) \quad (19)$$

The solution is given by:

$$u(x,t) = u_o + u_1 + u_2 + u_3 + u_4 + \ldots \quad (20)$$

The Sumudu Adomian Decomposition solution is given by

$$u(x,t,j) = uo + u_1 + u_2 + u_3 + u_4 + \ldots + u_j \quad (21)$$

Then the [M,N]-oder Padé Sumudu Adomian Decomposition solution is given by

$$u(x,t,j,[M,N]) = P_{[M,N]}(uSADM). \quad (22)$$

By using the symbolic computation software Mathematica.

The **figure (7a)**, **Figure (7b)**, **Figure (7c)** and **Figure (7d)** show respectively the curve of SADM solution $u(x, t, 3)$, PSADM solution $u_{PSADM} = u(x, t, 3, [2,2])$, $u_{PSADM} = u(x, t, 3, [1, 2])$ and the exact solution u = u(x, t), in domain $D = [-0.5, 0.5] \times [0, 0.1]$. The exact solution is given by $u(x, t) = -2\text{sech}^2(x-4t)$

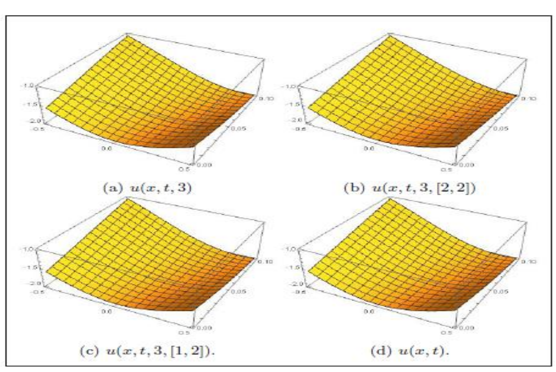

Figure 7.
(a) SADM solution, (b) and (c) PSADM solutions using 3 terms, (d) exact solutions.

Figure (8a), **Figure (8b)**, **Figure (8c)** and **Figure (8d)** show respectively the curve of SADM solution $u(x, t, 3)$, PSADM solution $u_{PSADM} = u(x, t, 3, [1, 2])$ and the exact solution $u = u(x, t)$, in domain $D = [-1, 1] \times [0, 1]$

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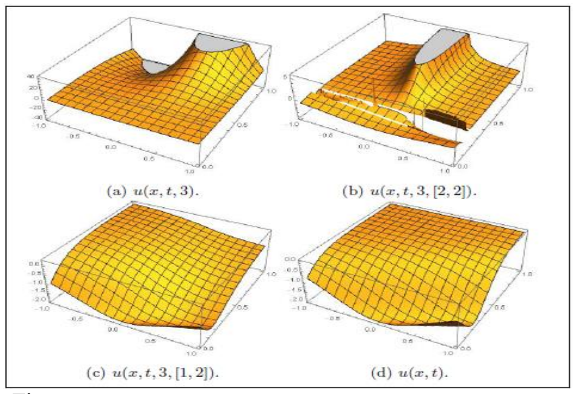

Figure 8.
(a) SADM solution, (b) and (c) PSADM solutions using 3 terms, (d) exact solutions.

We can see the SADM, [2, 2]- order PSADM and [1, 2]-order PSADM solutions behave well in domain D = [-0.5, 0.5] × [0, 0.1], but only [1, 2]-order PSADM solution give better result in domain D = [-1, 1] × [0, 1]. The [2, 2]-order PSADM solution in this case in not better than [1, 2]-order PSADM solution. Then the diagonal Padé approximation are not accurate in this case. It is recommended in case to use [M,N]-order Padé approximation with M ≠ N.

Case 2: Consider the equation:

$$\begin{cases} u_t + qu^2u_x - \beta u_{xxx} = 0, \\ u(x,o) = h(x), \\ x \in \mathbb{R}_+, t \in \mathbb{R}_+, \end{cases} \tag{23}$$

We know for q = 6, β = -1, and subject to the initial condition

$$u(x,o) = \frac{2ke^{kx}}{1+e^{2kx}}$$

the exact solution is given by:

$$u(x,t) = k\,\mathrm{sech}[k(x-k^2t)]$$

For k = 0,5, the **figure (9a)**, **Figure (9b)**, **Figure (9c)**, **Figure (9d)**, and **Figure (9e)** show respectively the curve of the SADM solution $uSADM = u(x,\ t,\ 4)$, PSADM solutions $u(x,\ t,\ 4,\ 2)$, $u(x,\ t,\ 4,\ [0,\ 2])$, $u(x,\ t,\ 4,\ [2,0])$ and the exact solution u_{exact} in domain D = [0,1] × [0,2].

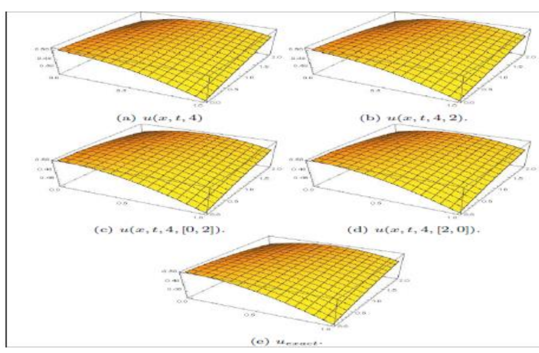

Figure 9.
(a) SADM solution, (b) and (d) PSADM solutions using 4 terms, (e) exact solutions.

For k = 0.5, the **figures** (10a), (10b), (10c), and (10d) show respectively the absolute error curve for the SADM solution u_{SADM} = $u(x,t,4)$, PSADM solutions $u(x,t,4,2)$, $u(x,t,4,[0,2])$, and $u(x,t,4,[2,0])$ in domain D = [0,1] × [0,2]
The SADM and [2, 2]-order PSADM (or in short 2-PSADM) provide same results. The [0, 2]-order PSADM solution in this case providing better error than [2, 2]-order PSADM and [2, 0]-order PSADM solutions. The diagonal Padé approximations are not recommended in this case.

Remark 4
The following conditions can help to choose the best PSADM solution.
Condition (*)
If

$$\lim_{x\to\infty} u_{exact} = 0,$$

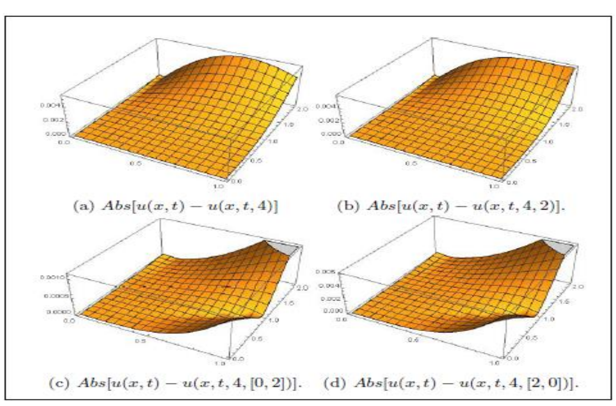

Figure 10.
Absolute errors.

We comput the new solution by

$u(x, t, j, [L,M]) = P_{[L/M]}\ [u_{SADM}]\ (x, t)$, with $L < M$

then

$$\lim_{x\to\infty} u(x,t,j,[L,M]) = 0,$$

Condition (**)

If

$$\lim_{x\to\infty} u_{exact} = \infty,$$

We comput the new solution by

$u(x, t, j, [L,M]) = P_{[L/M]}\ [u_{SADM}]\ (x, t)$, with $L > M$

then

$$\lim_{x\to\infty} u(x,t,j,[L,M]) = \infty,$$

Condition (***)

If we are not in the case mentioning in conditions (*) and (**), we comput the new solution by

$u(x, t, j, [M,M]) = u(x, t, j, M) = P_{[M/M]}\ [u_{SADM}]\ (x, t)$, with $L = M$

2. Conclusion

In this work, we show the behaviour et of the function $P_{[L/M]}[.](x, t)$ using to obtain the Padé Sumudu Adomian Decomposition Methods solution for nonlinear partial differential equations such as the Schrödinger equations, and the KdV-Burger's equations. The proposed function provide us a suitable way for controlling the convergences of series solutions with high accuracy by using different order of Padé approximation and different type of the Padé approximation according to the topology of the exact solution $u(x, t)$ and the topology of the SADM solution $u(x, t, j)$. When the exact solutions are unknown, we have some mathematical approach to obtain more information about the topology of the exact solution. This approach can be generalized to investigate more complicated nonlinear partial differential equations that can only be solved by numerically.

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