

Geometric programming technique: Sizing the geometric parameters and performance indices of a single sided linear induction motor

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Abstract: This paper presents the use of Geometric Programming (GP) technique for the optimal geometric sizing design of a three phase single sided linear induction motor (SLIM). Two objective functions were proposed to enhance the determination of the best sizing parameters, while, nine constraints were imposed on the objective function using the GP tool in the MATLAB/Octave environment. Some quantities were chosen as the independent design variables: stator length $[L]_s$, stator width W_s , stator yoke depth h_y , stator slot width w_s , and stator tooth width w_t ; these variables are on the primary of the SLIM. The variables considered on the secondary of the SLIM are the width of back iron W_b and the depth of the back iron T_b . The results suggested that stator losses were 203.4 W, while weight of stator was 6.44 kg. Furthermore, comparison was made with existing SLIM design, and results suggest that GP is a viable technique. Also comparison with the literature showed that the findings of this work are within specified limits.

Keywords: Geometric, Induction, Motor, Performance, Programming, Technique.

1. Introduction

Geometric Programming technique is one of the numerous techniques of optimization methods employed in determining the best or most effective result utilizing a quantitative measurement system. Cost minimization is the consideration of this work. [1-2] presented geometric programming as the formulation of engineering problems with specific kinds of non-linear optimization problems with flexible variables. Authors in [3] stated that geometric programming is a non-linear optimization technique that is robust, very flexible and can be used to approximate linear optimization problems.

A single sided linear induction motor (SLIM) is an electrical motor composed of coil assembly as primary and reaction plate as secondary and where the coil assembly is made up of steel laminators and phase winding. Authors in [4-5] opined that the single-sided linear induction motor (SLIM) is by far the most widely used in linear induction motor and that SLIM has some advantages over rotary induction motors, which include simple construction, direct electromagnetic thrust propulsion, precise linear positioning, separate cooling, all electro-mechanical controlled systems used for an induction motors can be adopted for a SLIM without any bigger changes, safety and reliability, economical and cheap maintenance. Presently, single sided linear induction motor is gaining attention in the transport sector and elevator systems. This is attributed to the call for clean energy, simplicity of design and low construction cost [6]. Also, because of low operating costs and extremely high reliability, SLIM-propelled systems have become an ever more frequent part of the public transport offering [7]; thus, several efforts have been put in place to improve its efficiency and speed. In [8-9], a novel structure of single-sided linear induction motor is proposed to reduce the magnetic air gap so as to reduce the resistance of the motor running at high speed, which can provide a new solution for the design of high-speed SLIM. [10] studied the dynamic behavior of a single-sided linear induction motor by changing the basic design parameters; however, only a few studies have carried out a proper design of SLIM. The optimal design of SLIM can be achieved through the proper sizing technique, which is subject to a set of

constraints which could be thermal, mechanical or user's specification. Several optimization techniques have been applied in the design of SLIM, such as Particle Swarm Optimization (PSO), Artificial Neural Network (ANN), while the objective functions are efficiency, power factor, thrust or a combination of these [11]. The optimization procedures have yielded good geometric sizing, and majority are able to detect if the solution to the problem is feasible or infeasible at the early stage of the simulation process [12]. Since time is of the essence, simulation processes and techniques are good and reliable if they can detect infeasibility or return results in time [13]. In [14] a more efficient SLIM was developed by making three changes to the geometric parameters of the fixed and moving parts leveraging a 3-D simulation of a Single-Sided Linear Induction Motor with Transverse and Longitudinal Magnetic Flux.

The aim of this paper is to use the Geometric Programming (GP) technique in the sizing of the geometric parameters and performance indices of a three-phase single sided linear induction motor (SLIM). Two objective functions were developed; namely, material minimization and the minimization of the stator losses with the efficiency improvement and power factor as key variables. Eight constraints were applied in order to quicken the process. The end effect factors were not included in this design process as the design is for low speed application [15].

A comparison of the results of this study with existing analytic design will be carried out with the condition for selecting the best being the improvement in efficiency and power factor [16].

These two are termed the performance indices and they are derived from the independent variables which are the sizing parameters.

Interest in Geometric Programming as an optimization tool is not new; however, the advantages of this optimization technique are fast gaining attention [17]. Geometric programming does not only solve a problem but it is also able to produce design equations that can be utilized and thus avoid resolving problems when the input parameters are changed [18-19]. The Geometric Programming technique is now very efficient and reliable [20-21]. The GP technique uses the concepts of monomials and posynomials functions as the form to express the objective functions and the constraints [22-23]. None of the other solution procedures such as linear programming allows design equations to be developed in a manner similar to that of geometric programming and this is a significant advantage for geometric programming [18]. The GP technique has proven to be successful in the minimization of the cost of transformers and synchronous motors [20].

A Geometric Programme (GP) is an optimization problem in the form of Eq. (1) [22],

$$\begin{aligned} & \text{minimize } f_0(x) \text{ subject to } f_i(x) \leq 1 \quad i = 1, \dots, m \\ g_i(x) & \leq 1 \quad i = 1, \dots, p \end{aligned} \quad (1)$$

where f_i are posynomial functions, g_i are monomials, and x_i are the optimization variables.

A monomial function is defined as Eq. (2)

$$g_i(x) = c_i x_1^{a_1} x_2^{a_2} \dots x_n^{a_n} \quad (2)$$

where c is a positive real constant called the monomial coefficient; and a_1, \dots, a_n are real numbers which could be positive, negative or fractional constants that are referred to as exponents of the monomial.

The sum of monomial function is named a posynomial function presented in Eq. (3)

$$f_i(x) = \sum_{k=1}^K c_k x_1^{a_{1k}} x_2^{a_{2k}} \dots x_n^{a_{nk}} \quad (3)$$

If a posynomial is multiplied by a monomial the result is a posynomial; similarly, a posynomial can be divided by a monomial with the result being a posynomial.

In many ways, the geometric programming technique is similar to linear programming technique; however, its advantages includes: (i) it supports nonlinear objective function, (ii) the constraints can be nonlinear and the optimal value can be determined with a dual without first determining the specific value of the primal variables [24-25].

In this study, the objective functions and their corresponding constraints will be developed using the basic rules governing the formulation of monomials and posynomials as they apply to GP.

2. Method

2.1. Optimization Problem Design Variables

The following quantities were chosen as the independent design variables: stator length L_s , stator width W_s , stator yoke depth h_y , stator slot width w_s , and stator tooth width w_t ; these variables are on the primary of the SLIM. The variables considered on the secondary of the SLIM are the width of back iron W_b and the depth of the back iron T_b .

2.1.1. Objective Function

In this work the following minimization objective functions were considered: Weight of stator material M_T and Stator losses W_{SL} as presented in the following subsections:

i. Weight of Stator Material

The weight of stator iron material M_{fe} and copper material M_{cu} have considerable bearings with the independent design variables and these are presented in Eq. (4) and Eq. (5), respectively.

$$M_{fe} = \rho_{fe}(W_s(w_t h_s m p q + h_y L_s + T_b W_b L_s)) \quad (4)$$

$$M_{cu} = \frac{2mN_{ph}(W_s + \frac{\pi L_s}{2p})\rho_{cu}l}{J} \quad (5)$$

The total material required to produce one unit of SLIM is given in Eq. (6)

$$M_T = \rho_{fe}(W_s(w_t h_s S_s + h_y L_s) + T_b W_b L_s) + \frac{2mN_{ph}(W_s + \frac{\pi L_s}{2p})\rho_{cu}l}{J} \quad (6)$$

ii. Stator Loss

Stator loss W_{SL} is derived from copper loss and stator core loss presented in Equation. (7)

$$W_{SL} = 2mIJN_{ph}\rho_{cu}\left(W_s + \frac{\pi L_s}{2p}\right) + W_p\rho_{fe}\left(W_s(w_t h_s S_s + h_y L_s)\right) + W_y\rho_{fe}T_b W_b L_s \quad (7)$$

2.1.2. Constraints

The equality and inequality constraints shown in Table 1 are imposed on the optimization problems. These constraints are chosen to bring out the best in the design.

Table 1.
Constraints.

Equality constraints	Inequality constraints
$L_s/pq = h_s$	$0.115 \leq \eta \cos\theta \leq 0.2$
$L_s B_{mg}/S_s = w_t$	$pV_r/(2(1-s)f) + w_t \leq L_s$
$L_s/(2S_s) = w_s$	$F_x(1-s)/(0.5Qk_w L_s \eta \cos\theta) \leq W_s$
$0.3B_{mg}L_s/p = h_y$	$(W_s p + 0.1L_s)/p \leq W_p$
	$pE/(\sqrt{2}fL_s W_s B_{mg} k_w) \leq N_{ph}$

2.1.3. Design of Single Sided Linear Induction Motor

In this paper, a geometric sizing or design variables of SLIM using the GP technique is proposed and the SLIM specifications are taken from [15] and are presented in Table 2. The specifications used in [15] were applied in this work so that the independent and dependent variables as well as the performance indices obtained from the optimization carried out in this work can be compared with the literature on the same basis.

Table 2.
SLIM specifications and design constraints.

SLIM specification		Design constraints	
Thrust $F_x(N)$	100	$\rho_{cu}(\Omega - m)$	17.24×10^{-9}
No of phase m	3	$\rho_{fe}(kg/m^3)$	7.87×10^3
Voltage (V)	220	$B_{mg}(T)$	0.36
Frequency $f(Hz)$	50	$Q(A/m)$	32000
Velocity $V_r(m/s)$	3.5	$B_{mly}(T)$	1.2
Slip s	0.3	$W_p(W/kg)$	23
No of poles p	4	$W_y(W/kg)$	16.89
		$T_b(m)$	0.003
		k_w	1

3. Results

Equation (8) and Equation (9) are the posynomial expressions with five (5) monomial terms obtained from the simulation of the Geometric Programming technique using [20] in MATLAB/Octave environment.

$$M_T = 94440 \times W_p w_t h_y + 7870 \times W_p L_p h_y + 23.6 \times W_s L_p + 247905.1383 \times N_{ph} W_p J^{-1} + 97352.1 \times N_{ph} L_p J^{-1} \quad (8)$$

These expressions provide the minimization of both the weight of stator material and the resulting stator losses of a SLIM if the sizing parameters presented in Table 3 and Table 4 as obtained from the simulation are used. All of the outputted expressions are in terms of the independent and dependent variables. None of the other solutions procedures such as linear programming allows design equations to be developed in a manner similar to that of geometric programming and this is a significant advantage for geometric programming. The independent variables are compared with those in [15] and are within the acceptable variation limits as [15] provides only the main dimensions (stator length and stator width) of the proposed designed SLIM.

The outputted independent and dependent variables of the objective functions are same and this is accepted because the two objective functions are similar but for the input constants. The values of the weight of stator materials for the minimized M_T is lower than that obtained in the minimized W_{SL} by 33.11% while the value of the stator losses for the minimized W_{SL} is lower than that obtained in the minimized M_T by 24.94%. The product of efficiency and power factor as depicted in Table V is the same for both objective functions and it improves appreciably when compared with that obtained in [12] by 54%.

$$W_{SL} = 2172120 \times W_p w_t h_p + 132790.8789 \times W_p L_p h_p + 663.9544 \times W_s L_p + 4.77 \times 10^{-7} \times N_{ph} W_p J + 1.8732 \times 10^{-7} \times N_{ph} L_p J \quad (9)$$

Table 3.
Independent variables.

Independent variables	M_T	W_{SL}	Analytic	% Error
Stator length $L_s(m)$	0.2062	0.2062	0.2065	0.15
Stator width $W_s(m)$	0.0849	0.0849	0.1000	1.51
Back iron width $W_b(m)$	0.0900	0.0900	0.1050	1.51
Stator tooth width $w_t(m)$	0.0062	0.0062	0.0065	0.15
Stator slot depth $h_s(m)$	0.0164	0.0164	–	
Stator yoke depth $h_y(m)$	0.0056	0.0056	–	
Stator slot width $w_s(m)$	0.0086	0.0086	–	

Table 4.
Dependent variables.

Dependent variables	M_T	W_{SL}	Analytic	% Error
Stator current density $J_s(A/m^2)$	4×10^6	2.5×10^6	–	
Gap flux density $B_{mg}(T)$	0.36	0.36	0.3600	
Maximum Yoke flux density $B_{ymax}(T)$	1.2	1.2	–	
Maximum stator teeth flux density $B_{tmax}(T)$	1.6	1.6		
Number of Turns $N_{ph}(Turns)$	469	469	455	

Table 5.
Performance index.

Performance index	M_T	W_{SL}	Analytic	% error
Product of efficiency and power factor $\eta \cos \theta$	0.25	0.25	0.115	54
Stator losses (W)	203.3872	152.6552	–	24.94
Weight of stator (kg)	6.4351	9.6203	–	33.11

4. Conclusions

Presently, the computational trend is to subject an initial engineering design concept to some robust optimization techniques such as the Particle Swarm Optimization (PSO) and the artificial Neural Network (ANN), and thereafter, validate the resulting outputs. The strength of the optimization tool to bring out the best design concept is also subjected to further analysis.

In this article, the Geometric Programming technique as an optimization tool has been applied on two objective functions namely the stator weight and the stator losses in the design of single sided linear induction motor (SLIM). Nine constraints sourced from existing SLIM expressions were applied and simulated using a GP tool in MATLAB/Octave environment.

The results obtained suggest that the stator losses was 203.4 W, while the weight of stator is 6.44 kg. Comparison with the literature showed that the findings of this work are within specified limits

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