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Multifractal detrended cross-correlation analysis of Islamic stock markets in the pacific Asia region

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Abstract: This study aims to uncover the multifractal characteristics and complex interactions among six Islamic stock markets in the pacific Asia region, enhancing understanding of their dynamics. Employing Multifractal Detrended Cross-Correlation Analysis (MF-DCCA), this research analyzes a dataset covering the period from January 1, 2011, to August 1, 2024, with approximately 3315 observations. In the preliminary analysis, the application of the DCCA Cross-Correlation Coefficient method revealed that the cross-correlations are persistent and exhibit long-term stability across most index pairs. Utilizing core components of the MF-DCCA method, such as Generalized Hurst Exponents, Rényi Exponents, and the Hölder Singularity Spectrum, further confirmed that the index pairs exhibit long-range persistent cross-correlations and multifractal behavior. Surrogate and shuffling transformations showed that the observed multifractality is influenced by both long-term cross-correlations and heavy-tailed distributions. The findings have important implications for investors, policymakers, and financial analysts concerned with portfolio diversification, risk management, and the efficiency of Islamic financial markets in this economically vital region. Understanding the interconnectedness of these markets can aid in developing more effective investment strategies and regulatory frameworks. This study contributes original insights into the dynamics of Islamic stock markets in the Pacific Asia region, offering a nuanced perspective on their complex behaviors and interactions through the lens of multifractality.

Keywords: *Cross-correlation, Generalized hurst exponents, Hölder singularity spectrum, Multifractal, Rényi exponents.*
JEL Classification: *C10; C22; C58; C63; G10; G15.*

1. Introduction

In today's highly interconnected global financial landscape, grasping the complex interactions between various markets is essential. Conventional correlation methods, usually based in linear models, often fall short in capturing the intricacies of financial systems, especially when dealing with non-linear behaviors, non-stationary processes, and multiple scaling properties. As a result, the analysis of multifractal cross-correlations has gained prominence, providing a more detailed understanding of market relationships across different time horizons.

Multifractality in financial markets arises from diverse scaling behaviors over varying time frames, revealing intricate dynamics like long-range dependencies and fat-tailed distributions. Unlike traditional monofractal models, which rely on a uniform scaling exponent, multifractal analysis introduces a range of scaling exponents, offering deeper insights into market behaviors such as efficiency variations and patterns of volatility clustering. When utilized in cross-correlation studies, this method unveils the hidden complexities of relationships between financial indices, which may not be apparent through more conventional analytical techniques.

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In an increasingly globalized financial environment, markets are highly interdependent, and disturbances in one can propagate to others, creating unpredictable consequences. Analyzing the multifractal nature of these cross-correlations allows researchers and investors to better comprehend these intricate connections, leading to more strategic decisions in both risk management and investment planning. Furthermore, Multifractal cross-correlation analysis enhances portfolio diversification by revealing fluctuations in asset correlations that traditional methods often miss, allowing for more resilient portfolios. It also improves risk management by uncovering non-linear dependencies between assets, offering deeper insights into joint risks during volatile market conditions. Additionally, this approach helps identify market inefficiencies and potential arbitrage opportunities, making it valuable for investors and regulators aiming to manage systemic risks.

Several studies employed Multifractal Detrended Cross-Correlation Analysis MF-DCCA to investigate the multifractal properties of cross-correlations between assets across various financial markets. This study aims to fill a crucial gap in the literature by applying MF-DCCA to Islamic stock markets in the Pacific Asia region. By focusing on these markets, the research provides a more comprehensive understanding of their dynamic interactions and contributes to the broader field of multifractal analysis. Islamic stock markets, shaped by Shariah principles, exhibit unique risk profiles and behaviors that traditional methods may miss. MF-DCCA can reveal intricate interactions and inefficiencies, offering deeper insights into market dynamics, efficiency, and stability.

The MF-DCCA method is an advanced technique that extends traditional financial time series analysis by incorporating multifractality - a feature that reflects diverse scaling behaviors across different time scales. This approach enables a nuanced investigation of cross-correlation patterns between time series, capturing both transient fluctuations and enduring dependencies.

Multifractality, characterized by the presence of multiple scaling exponents within a time series, reveals the intricate structure of its fluctuations. The foundational concept of Detrended Fluctuation Analysis (DFA), introduced by Peng, et al. [1] was designed to detect long-range correlations in non-stationary time series. Kantelhardt, et al. [2] subsequently extended this framework with Multifractal Detrended Fluctuation Analysis (MF-DFA), which allows for the examination of multifractal properties over a range of scales.

To explore cross-correlations between two time series, Podobnik and Stanley [3] introduced Detrended Cross-Correlation Analysis (DCCA), an extension of DFA. Zhou [4] further developed this concept into Multifractal Detrended Cross-Correlation Analysis (MF-DCCA), combining DCCA and MF-DFA. This integrated methodology provides a comprehensive analysis of the multifractal characteristics of cross-correlations, enhancing the understanding of the complex interactions between financial time series.

The article is structured as follows: Section 2 reviews existing literature on cross-correlation multifractality specifically within financial markets. Section 3 outlines the data and methodology used in the study. Section 4 presents and discusses the empirical results. Finally, Section 5 offers conclusion, implications, and recommendations based on the findings.

2. Literature Review

The application of multifractal analysis in understanding cross-correlations between financial markets has been increasingly recognized for its ability to capture complex, long-term dependencies that traditional methods often overlook. A significant body of research has employed Multifractal Detrended Cross-Correlation Analysis (MF-DCCA) to explore the dynamic relationships between various financial markets.

Studies have shown that multifractal cross-correlations are prevalent across both developed and emerging markets. For instance, the examination of stock markets in the MENA region (Morocco, Tunisia, Egypt, and Jordan) revealed significant multifractal cross-correlations, highlighting the interdependence between these markets [5]. Similarly, research on the Chinese Renminbi (RMB) markets, comparing onshore and offshore markets, found strong short-term correlations with the

British Pound and long-term cross-correlations with the Malaysian Ringgit, further emphasizing the global reach and persistence of these relationships [6].

The multifractal behavior of cross-correlations has also been observed in the context of major financial hubs, such as Hong Kong and Shanghai. For example, the relationship between the Hang Seng China Enterprises Index and RMB exchange markets revealed that onshore RMB markets exhibit stronger multifractality than offshore markets, suggesting that domestic factors may play a larger role in market dynamics [7]. Other studies on the Shanghai-Hong Kong Stock Connect also highlighted that cross-correlations between these two markets became more persistent and stronger post-liberalization, underscoring the impact of financial reforms on market integration [8].

Further evidence of multifractality in financial market relationships is found in the volatility interactions between Mainland China, the US, and Hong Kong stock markets. These studies indicated that significant market events, such as fluctuations in the Hang Seng Index, can have substantial ripple effects across the region, with the strongest cross-market conductivity observed between Hong Kong and Mainland China stock markets [9]. The identification of long-range correlations, particularly in times of large fluctuations, underscores the importance of understanding multifractality when evaluating market stability.

Moreover, research on market behavior during economic crises has demonstrated how cross-correlations intensify in response to financial shocks. For example, a study on the Shanghai Stock Exchange Composite and the S&P 500 indices found that the relationship between these markets became significantly stronger during the financial crisis, with heightened multifractality and volatility reflecting increased market risk [10]. This highlights how extreme market events, such as economic crises, can lead to more pronounced and persistent cross-correlations, further challenging traditional market assumptions.

The multifractal approach has also provided critical insights into the dynamic relationships between exchange rates and stock market liquidity. For instance, in the case of the RMB exchange index and stock market liquidity in Shanghai and Shenzhen, the study found that cross-correlations exhibited strong positive persistence, especially during periods of tightening monetary policy [11]. This multifractality challenges the traditional efficient market hypothesis, demonstrating the complex, non-linear nature of market interactions and the need for more sophisticated models to capture these dynamics.

Recent studies on the RMB exchange rate reform and its impact on cross-correlations between different RMB markets further illustrate the evolving nature of market interdependence. The reform led to a decrease in the persistence and degree of multifractality, highlighting the role of policy changes in reshaping market relationships [12]. Similarly, the analysis of soybean futures and spot prices in China revealed a strong, persistent multifractal cross-correlation, illustrating how commodity markets also exhibit long-range dependencies, further extending the application of multifractal analysis to broader asset classes [13].

The increasing relevance of multifractal analysis extends to more recent financial instruments, such as green bonds. Studies have shown that green bonds, like traditional financial assets, exhibit significant multifractal characteristics, indicating that multifractal analysis can be a valuable tool for understanding the interactions within emerging financial markets [14].

In summary, the literature highlights the widespread applicability of multifractal methods in capturing the complex, long-term dependencies between financial markets. These studies provide a nuanced understanding of market interconnections, particularly in the context of financial liberalization, policy changes, and market crises. The findings underscore the need for advanced analytical techniques, such as MF-DCCA, to monitor and assess market stability, risk, and investor behavior in an increasingly interconnected global financial system.

3. Methodology

3.1. Data

The dataset for this study comprises daily closing prices from Islamic indices across six stock markets in the Asia-Pacific region: China, India, Indonesia, Pakistan, Malaysia, and Thailand. These indices are designed in accordance with Islamic finance principles, excluding financial activities that are not Sharia-compliant. The indices include:

- The FTSE Shariah China which is designed to measure the performance of Shariah-compliant companies listed in China. It is a part of the FTSE Shariah Global Equity Index Series.
- The Nifty 500 Shariah Index is an Islamic equity index derived from the broader Nifty 500 Index, which tracks the performance of the top 500 companies listed on the National Stock Exchange of India (NSE) based on market capitalization.
- The Jakarta Islamic Index, which is a stock market index established on the Indonesia Stock Exchange (IDX).
- The Karachi Meezan 30 is a stock market index that tracks the performance of the top 30 Shariah-compliant companies listed on the Pakistan Stock Exchange (PSX).
- The FTSE Bursa Malaysia EMAS Shariah Index is a benchmark index that tracks the performance of Shariah-compliant companies listed on Bursa Malaysia, the Malaysian stock exchange. It is part of the broader FTSE Bursa Malaysia EMAS Index, which includes large and mid-cap companies.
- The FTSE SET Shariah is designed to track the performance of Shariah-compliant companies listed on the Stock Exchange of Thailand (SET).
- In the following, we will denote the six Islamic indices as China, India, Jakarta, Karachi, Malaysia and Thailand. The data span from 01/01/2011 to 01/08/2024, comprising nearly 3315 observations. All data were downloaded from the website www.investing.com.

The index prices were then converted into logarithmic returns $r_t = ln\left(\frac{P_t}{P_{t-1}}\right) = ln(P_t) - ln(P_{t-1})$, where P_t denotes the index daily price and ln corresponds to the natural logarithm.

3.2. Method

3.2.1. Multifractal Detrended Cross-Correlation Analysis

In this section, we introduce the Multifractal Detrended Cross-Correlation Analysis (MF-DCCA).

Consider two time series $x = \big(x(k)\big)_{1 \le k \le N}$ and $y = \big(y(k)\big)_{1 \le k \le N}$, for $1 \le k \le N$, where N is the length of the series. It is assumed that these series have compact supports, meaning that $x(k) = 0$ and $y(k) = 0$ for only a negligible fraction of the values k.

Step 1: We determine the profiles $X = \big(X(i)\big)_{1 \le i \le N}$ and $Y = \big(Y(i)\big)_{1 \le i \le N}$ of the series x and y defined by :

$$X(i) = \sum_{k=1}^{N}(x(k) - \bar{x}) \qquad Y(i) = \sum_{k=1}^{N}(y(k) - \bar{y}) \tag{1}$$

where $\bar{x}$ and $\bar{y}$ are the means of the series X and Y.

Step 2: For a given time scale s, we divide the profiles X and Y into $N_S = Int(N/s)$ non-overlapping segments of the same length s, where $Int(.)$ represents the function that gives the integer part of a real number. Based on the recommendations of Peng, et al. [1] $5 \le s \le N/4$ is traditionally selected. Since N is generally not a multiple of s, a short part at the end of the profiles may be neglected. To incorporate all the ignored parts of the series, the same procedure is repeated starting from the end of the profile. Thus, we obtain $2N_S$ segments. For $1 \le i \le s$, we have two segmentations: $X\big((v-1)s + \mathrm{i}\big)$ for $1 \le v \le N_S$ and $X\big((N - v - N_S)s + \mathrm{i}\big)$ for $N_S + 1 \le v \le 2N_S$.

Step 3: In each segment, we use the Ordinary Least Squares (OLS) method to properly fit data with a local trend. We denote by $p_{X,v}^{m}(i)$ and $p_{Y,v}^{m}(i)$ the fitting polynomials of respectively the profile X and the profile Y for the v-th segment. For $1 \leq v \leq 2Ns$:

$$p_{X,v}^{m}(i) = \alpha_0^v + \alpha_1^v . i + \cdots + \alpha_m^v . i^m \quad (2)$$

$$p_{Y,v}^{m}(i) = \beta_0^v + \beta_1^v . i + \cdots + \beta_m^v . i^m \quad (3)$$

In the empirical study, the order m of the fitting polynomial can be linear, quadratic, cubic, or even of a higher order. Choosing an appropriate value of m can avoid overfitting the series.

Step 3: After determining the fitting polynomial $p_{X,v}^{m}(i)$ and $p_{Y,v}^{m}(i)$, we calculate the detrended covariances $f_{X,Y}^2(v,s)$ for all time scales s and for every segment $1 \leq v \leq 2Ns$.

- The covariance $f_{X,Y}^2(v,s)$ is defined by:

$$f_{XY}^2(v,s) = \frac{1}{s}\sum_{i=1}^{s}\left|X\big((v-1)s+i\big) - p_{X,v}^{m}(i)\ \right| . \left|Y\big((v-1)s+i\big) - p_{Y,v}^{m}(i)\right| \quad (4)$$

for $1 \leq v \leq N_S$, and:

$$f_{XY}^2(v,s) = \frac{1}{s}\sum_{i=1}^{s}\left|X\big((N-v-N_S)s+i\big) - p_{X,v}^{m}(i)\ \right| . \left|Y\big((N-v-N_S)s+i\big) - p_{Y,v}^{m}(i)\right| \quad (5)$$

for $N_S + 1 \leq v \leq 2N_S$.

Step 4: By averaging the covariances over all segments, we obtain the fluctuation functions $F_q^{XY}(s)$ of order q defined by:

$$F_q^{XY}(s) = \left[\frac{1}{2N_S}\sum_{v=1}^{2N_S}\left(f_{XY}^2(v,s)\right)^{\frac{q}{2}}\right]^{\frac{1}{q}} \quad (6)$$

for $q \neq 0$, and:

$$F_0^{XY}(s) = exp\left[\frac{1}{4N_S}\sum_{v=1}^{2N_S} ln\left(f_{XY}^2(v,s)\right)\right] \quad (7)$$

for $q = 0$.

The purpose of the MF-DCCA procedure is primarily to determine the behavior of the fluctuation functions $F_q^{XY}(s)$ as a function of the time scale s for various values of q. To this end, steps 2 through 4 must be repeated for different time scales s.

Step 5: We analyze the multi-scale behavior of the fluctuation functions $F_q^{XY}(s)$ by estimating the slope of the log-log plots of $F_q^{XY}(s)$ versus s for different values of q. If the analyzed time series X and Y exhibits long-range cross-correlation according to a power-law, such as fractal properties, the fluctuation function $F_q^{XY}(s)$ will behave, for sufficiently large values of s, according to the following power-law scaling law:

$$F_q^{XY}(s) \sim s^{H_{XY}(q)} \quad (8)$$

or

$$log\left(F_q^{XY}(s)\right) = log\left(s^{H_{XY}(q)}\right) + log(C) \quad (9)$$

where $H_{XY}(q)$ is called the generalized Hurst exponent, which is the power-law cross-correlation of the two series X and Y.

When $H_{XY}(q)$ depend on q, the cross-correlation of the two-time series is multifractal, otherwise it is monofractal. To estimate the values of $H_{XY}(q)$ for different values of q, we perform a semi-logarithmic regression of the time series $H_{XY}(q)$ on the time series $F_q^{XY}(s)$. For positive q, $H_{XY}(q)$

describes the scaling behavior of intervals with large fluctuations. On the contrary, for negative q, $H_{XY}(q)$ describes the scaling behavior of segments with wavelet fluctuations.

$H_{XY}(q)$ is a decreasing function and to measure the degree of multifractality between the two series, we can use the variation ΔH_{XY} between the minimum and maximum values as defined below:

$$\Delta H_{XY} = H_{XY-Max} - H_{XY-Min} = H_{XY}(q_{min}) - H_{XY}(q_{max}) \tag{10}$$

The larger ΔH_{XY} is, the stronger the degree of multifractality will be.

For positive values of q, the average fluctuation function $F_q^{XY}(s)$ is dominated by segments v with large covariances $f_{XY}^2(v,s)$. Thus, for $q > 0$, the generalized Hurst exponents $H_{XY}(q)$ describe the scaling properties of large fluctuations. In contrast, for $q < 0$, the exponents $H_{XY}(q)$ describe the scaling properties of small fluctuations.

It is well known that the generalized Hurst exponent $H_{XY}(q)$ method is directly related to the multifractal scaling exponent $\tau_{XY}(q)$, commonly known as the Rényi exponent:

$$\tau_{XY}(q) = q.H_{XY}(q) - 1 \tag{11}$$

If the Rényi exponent $\tau_{XY}(q)$ increase nonlinearly with q, the cross-correlation of the two series is multifractal. Otherwise, if the Rényi exponent $\tau_{XY}(q)$ is a linear function of q, then the cross-correlation is monofractal.

Another interesting way to characterize the multifractality of the time series cross-correlations, is to use the Hölder spectrum or the singularity spectrum $f_{XY}(\alpha_{XY})$ of the Hölder exponent α_{XY}. It is well known that the singularity spectrum $f_{XY}(\alpha_{XY})$ is related to the Rényi exponent $\tau_{XY}(q)$ through the Legendre transform:

$$\begin{cases} \alpha_{XY} = \tau'_{XY}(q) \\ f_{XY}(\alpha_{XY}) = q.\alpha_{XY} - \tau_{XY}(q) \end{cases} \tag{12}$$

where $\tau'_{XY}(q)$ is the derivative of the function $\tau_{XY}(q)$.

When the cross-correlation between the two series is multifractal, then the singularity spectrum $f_{XY}(\alpha_{XY})$ present a concave bell-shaped curve.

The richness of the multifractality can be determined by the width of the spectrum defined by:

$$\Delta\alpha_{XY} = \alpha_{XY-max} - \alpha_{XY-min} \tag{13}$$

Thus, the wider the spectrum, the richer the multifractal behavior of the cross-correlation of the analyzed time series.

We can easily deduce the relationship between the generalized Hurst exponent $h(q)$ and the singularity spectrum $f_{XY}(\alpha_{XY})$:

$$\begin{cases} \alpha_{XY} = H_{XY}(q) + q.H'_{XY}(q) \\ f_{XY}(\alpha_{XY}) = q.\left(\alpha_{XY} - H_{XY}(q)\right) + 1 \end{cases} \tag{14}$$

3.2.2. DCCA Cross-Correlation Coefficient

To assess quantitively the cross-correlations between two non-stationary series, Zebende [15] proposed a DCCA cross-correlation coefficient. This method is based on the DCCA method of Podobnik and Stanley [3] and the DFA method [1]. In contrast to the original DCCA cross-correlation coefficient method, which uses non-overlapping segments, we will apply a version that uses overlapping segments, similar to the MF-DCCA method. Using the notations from section 3.2.1, the method is described below.

The covariance $f_{XY}^2(v,s)$ is defined in (8) and (9). The covariances $f_X^2(v,s)$, $f_Y^2(v,s)$ are defined by:

$$f_X^2(v,s) = \frac{1}{s}\sum_{i=1}^{s}\left(X\left((v-1)s+i\right) - p_{X,v}^m(i)\right)^2 \tag{15}$$

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$$f_Y^2(v,s) = \frac{1}{s}\sum_{i=1}^{s}\Big(Y\big((v-1)s+i\big) - p_{Y,v}^{m}(i)\Big)^2 \tag{16}$$

for $1 \leq v \leq Ns$, and

$$f_X^2(v,s) = \frac{1}{s}\sum_{i=1}^{s}\Big(X\Big(\big((N-v-Ns)s+i\big)\Big) - p_{X,v}^{m}(i)\Big)^2 \tag{17}$$

$$f_Y^2(v,s) = \frac{1}{s}\sum_{i=1}^{s}\Big(Y\Big(\big((N-v-Ns)s+i\big)\Big) - p_{Y,v}^{m}(i)\Big)^2 \tag{18}$$

for $Ns+1 \leq v \leq 2Ns$.

By averaging the covariances over all segments, we obtain the DFA-variance fluctuation functions $F_{DFA-X}^2(s)$ and $F_{DFA-Y}^2(s)$, and the DCCA-covariance fluctuation function $F_{DCCA}^2(s)$ defined by:

$$F_{DFA-X}^2(s) = \frac{1}{2Ns}\sum_{v=1}^{2Ns} f_X^2(v,s) \tag{19}$$

$$F_{DFA-Y}^2(s) = \frac{1}{2Ns}\sum_{v=1}^{2Ns} f_Y^2(v,s) \tag{20}$$

$$F_{DCCA}^2(s) = \frac{1}{2Ns}\sum_{v=1}^{2Ns} f_{XY}^2(v,s) \tag{21}$$

The DCCA cross-correlation coefficient $\rho_{DCCA}(s)$ is defined by:

$$\rho_{DCCA}(s) = \frac{F_{DCCA}^2(s)}{\sqrt{F_{DFA-X}^2(s)} \times \sqrt{F_{DFA-Y}^2(s)}} \tag{22}$$

The cross-correlation coefficient $\rho_{DCCA}(s)$ is an effective measure with properties similar to those of the standard correlation coefficient. It is a dimensionless quantity that ranges from -1 to 1. When $\rho_{DCCA}(s) = 0$, there is no cross-correlation between the two series. A value of $-1 < \rho_{DCCA}(s) < 0$ indicates an anti-persistent cross-correlation, while $0 < \rho_{DCCA}(s) \leq 1$ suggests a persistent cross-correlation between the two series. If $\rho_{DCCA}(s) = -1$, the two series are perfectly anti-persistent cross-correlated. Conversely, if $\rho_{DCCA}(s) = 1$, the two series are perfectly persistent cross-correlated.

3.2.3. Sources of Cross-Correlation Multifractality

Kantelhardt, et al. [2] identified two primary sources of multifractality in the cross-correlation of bivariate time series: long-term temporal correlations and heavy-tailed distributions. To assess the contribution of each source to the overall cross-correlation multifractality, we apply two transformations to the original return series: Shuffling (random permutation) and surrogation (phase randomization).

The shuffling technique maintains the distribution of the data's moments but removes any long-term correlations. After permutation, the data retain their statistical distribution but lack temporal correlations or memory. The surrogation technique, on the other hand, isolates the effect of long-term correlations on multifractality. This method involves randomly altering the temporal phases of the data, disrupting long-term correlations while preserving the overall fluctuation behavior. Several techniques for surrogation are discussed in the literature:

- Inverse Fast Fourier Transform (IFFT) [16].
- Iterated Algorithm (iAAFT) [17].
- Statically Transformed Autoregressive Process (STAP) [18].

4. Empirical Results

4.1. Test for Non-Stationarity

We applied the Augmented Dickey-Fuller (ADF) test to both the daily price series and the logarithmic return series of the six Pacific Asia Islamic indices. Table 1 shows the results.

Table 1.
ADF test for prices and log-returns of the six Pacific Asia Islamic indices.

	t-statistic					
	China	**India**	**Indonesia**	**Pakistan**	**Malaysia**	**Thailand**
ADF test statistics for prices	-2.159	0.269	-2.995	-1.208	-2.780	-3.154
ADF test statistics for returns	-56.211	-64.822	-58.080	-51.849	-37.498	-38.973
Test critical values	Level 1%: -3.961; Level 5%: -3.411; Level 10%: -3.127					

We observe that all the ADF test statistics for the daily prices of the six Islamic indices exceed the critical values at the 1%, 5%, and 10% significance levels. Therefore, we fail to reject the null hypothesis of a unit root, indicating that the daily price series of the six Islamic indices are generated by non-stationary processes. It is also noteworthy that all the ADF test statistics for the logarithmic returns of the six Islamic indices are below the critical values at the 1%, 5%, and 10% significance levels. Therefore, we reject the null hypothesis of a unit root, indicating that the daily logarithmic returns series of the six Islamic indices are generated by a stationary process.

4.2. DCCA Cross-Correlation Coefficient

In this section, the DCCA cross-correlation coefficient is applied to quantify the cross-correlation between the six Pacific Asia Islamic indices.

Figure 1 shows the plots of the DCCA cross-correlation coefficient $\rho_{DCCA}(s)$ as a function of the variable s for the 15 pairs of Pacific Asia Islamic indices.

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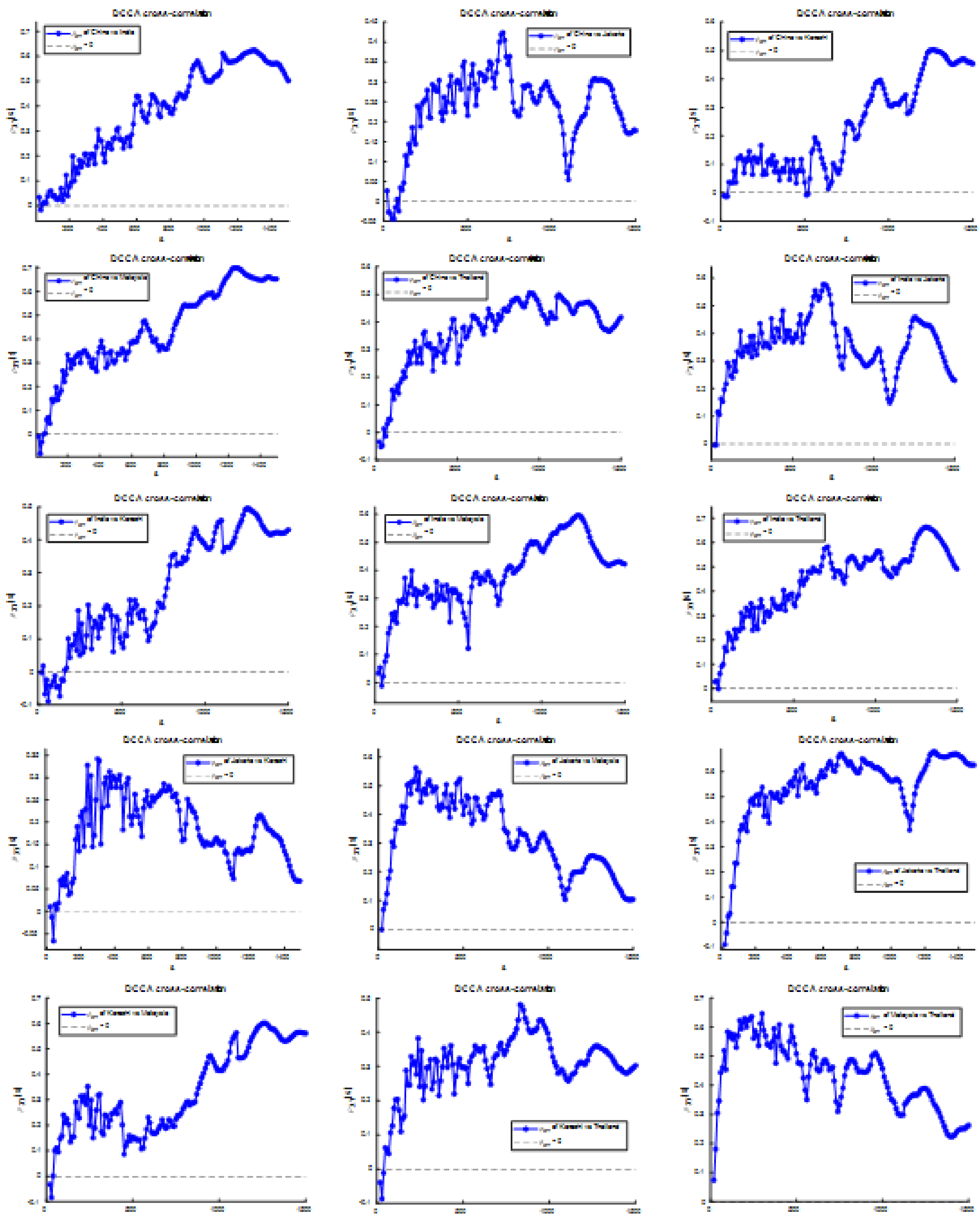

Figure 1.
DCCA cross-correlation coefficient $\rho_{DCCA}(s)$ vs. s for all pairs of indices, $s \in S = [20{:}\,10{:}\,1500]$.

We can observe that all pairs of the six Pacific Asia Islamic indices show a DCCA cross-correlation with $0 < \rho_{DCCA}(s) < 1$, indicating persistent cross-correlation. This suggests that these indices do not behave independently; rather, their movements are interlinked, with past movements in one index having a lasting effect on the others. This persistent relationship implies that the indices are influenced by similar macroeconomic forces, investor behavior, or regional events that affect Islamic financial markets in the Pacific Asia region. Factors such as shared geopolitical conditions, trade relationships, or economic policies may contribute to similar market responses across these indices. Importantly, since the DCCA method removes long-term trends, the observed cross-correlation reflects short- to medium-term fluctuations in the indices, rather than being driven by broader economic trends like growth or inflation.

4.3. Application of MF-DCCA

In this section, the MF-DCCA technique is applied to analyze the multifractal cross-correlation of the six Pacific Asia Islamic indices.

- *Generalized Hurst Exponents* $H_{XY}(q)$

Figure 2 shows the plots of the generalized Hurst functions $H_{XY}(q)$ as function of the variable $q \in [-45:5:-5, -4.1:0.1:-0.1, 0.1:0.1:4.1, 5:5:45]$ for all the pairs of Pacific Islamic indices.

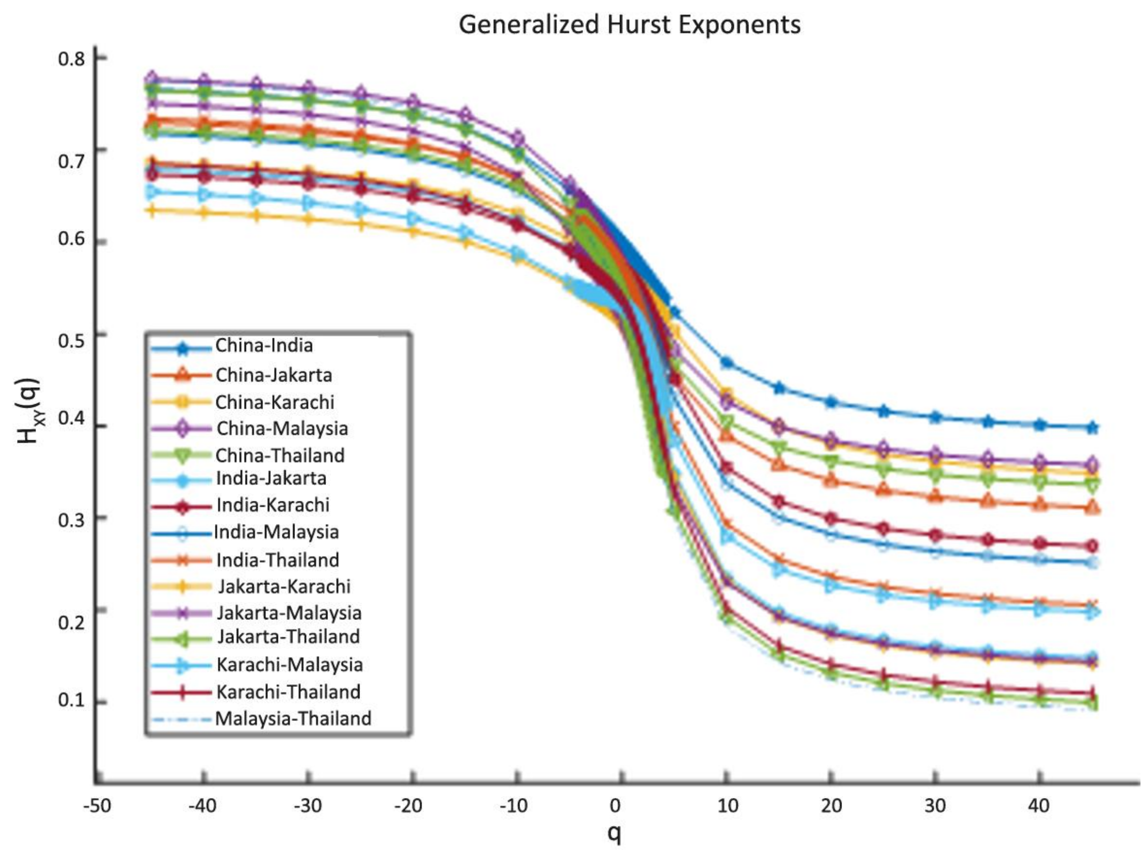

Figure 2.
Generalized Hurst exponents $H_{XY}(q)$ for all pairs of Pacific Islamic indices.

As shown in the previous figure, as q increases from -45 to 45, the generalized Hurst exponent $H_{XY}(q)$ decreases non-linearly for all pairs of indices. This indicates that the cross-correlations between each pair of indices exhibits multifractal nature.

We observed also that for $q < 0$, $H_{XY}(q) > 0.5$, whereas for $q > 0$, $H_{XY}(q) < 0.5$. This indicates a multifractal structure in the bivariate time series with different scaling behaviors for small and large fluctuations. $H_{XY}(q) > 0.5$ for $q < 0$ indicates persistence in the smaller fluctuations, meaning that small changes in the series are likely to continue in the same direction. For example, small positive returns are likely to be followed by further small positive returns, showing trend-following behavior at smaller scales. This persistence suggests that the bivariate time series has long-range correlations in its smaller movements, and it may be more predictable at this level. This could suggest inefficiency, as small fluctuations exhibit memory, potentially allowing for short-term arbitrage or trading strategies based on these small movements. On the contrary, $H_{XY}(q) < 0.5$ for $q > 0$ indicates anti-persistence in the larger fluctuations, meaning that large changes in one direction are likely to be followed by a reversal or opposite movement. For example, a large positive return is more likely to be followed by a negative return, and vice versa. This anti-persistent behavior at large scales suggests that the time series tends to correct itself after significant movements. This could imply overreaction to big news or shocks, leading to a reversion in prices. Large movements are less predictable, and the market may be more efficient at correcting extreme events.

This multifractality implies that the underlying process driving the time series is complex, with varying dynamics depending on the magnitude of the fluctuations. This could indicate that different types of traders (e.g., short-term vs. long-term traders) might be influencing the market differently, leading to diverse scaling behaviors across different time horizons or fluctuation sizes.

The degree of multifractality of the cross-correlations could be measured by the difference between the smallest and largest values of $H_{XY}(q)$:

$$\Delta H_{XY} = H_{XY-Max} - H_{XY-Min} = H_{XY}(q_{min}) - H_{XY}(q_{max}) \qquad (23)$$

The table below present the degree of multifractality for the 15 pairs of indices in decreasing order of ΔH_{XY}.

Table 2.
Degrees of multifractality of the 15 pairs cross-correlations based on the generalized Hurst exponent in decreasing order of ΔH_{XY}.

Rank	Pairs of indices	ΔH_{XY}
1	Malaysia vs Thailand	0.683
2	Jakarta vs Thailand	0.622
3	Jakarta vs Malaysia	0.607
4	Karachi vs Thailand	0.576
5	India vs Jakarta	0.530
6	India vs Thailand	0.529
7	Jakarta vs Karachi	0.493
8	India vs Malaysia	0.466
9	Karachi vs Malaysia	0.457
10	China vs Thailand	0.428
11	China vs Malaysia	0.419
12	China vs Jakarta	0.419
13	India vs Karachi	0.404
14	China vs India	0.367
15	China vs Karachi	0.337

We observed that all the cross-correlations between each pair of indices display multifractal behavior, as $\Delta H_{XY} = 0$ signifies that the bivariate time series demonstrate monofractal behavior.

- *Rényi Exponent* $\tau_{XY}(q)$

Figure 3 shows the plots of the Rényi Exponent $\tau_{XY}(q)$ as a function of the variable $q \in [-45{:}5{:}-5, -4.1{:}0.1{:}-0.1, 0.1{:}0.1{:}4.1, 5{:}5{:}45]$ for the 15 pairs of Islamic indices.

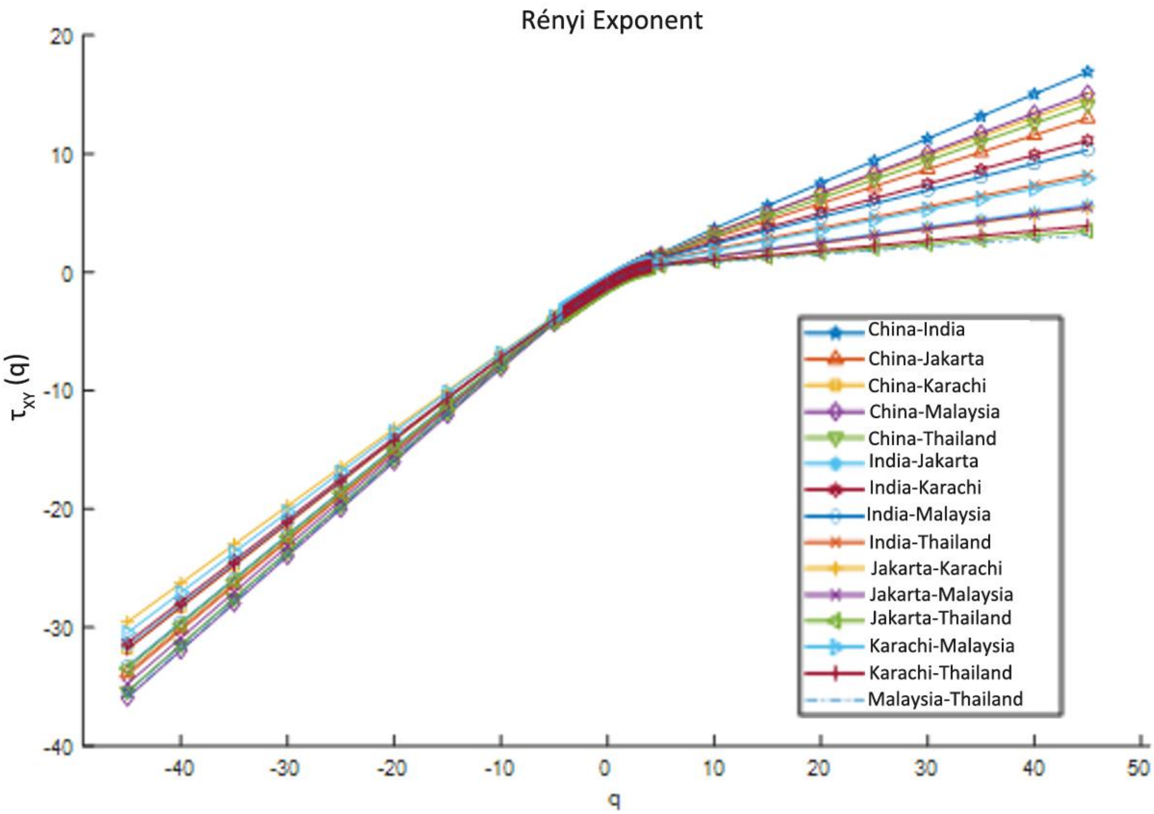

Figure 3.
Rényi exponents $\tau_{XY}(q)$ for all pairs of indices.

As shown in the previous figure, the Rényi Exponent $\tau_{XY}(q)$ increases non-linearly as q increases from -45 to 45 for all pairs of indices. This indicates that the 15 pairs of index returns exhibit cross-correlation with multifractal features.

- *Hölder Singularity Spectrum* $f_{XY}(\alpha)$

Another interesting way to characterize the multifractality of the cross-correlations of the 15 pairs of indices is to use the Hölder spectrum $f_{XY}(\alpha)$.

Figure 4 shows the plots of the singularity spectra $f_{XY}(\alpha)$ for the 15 pairs of indices.

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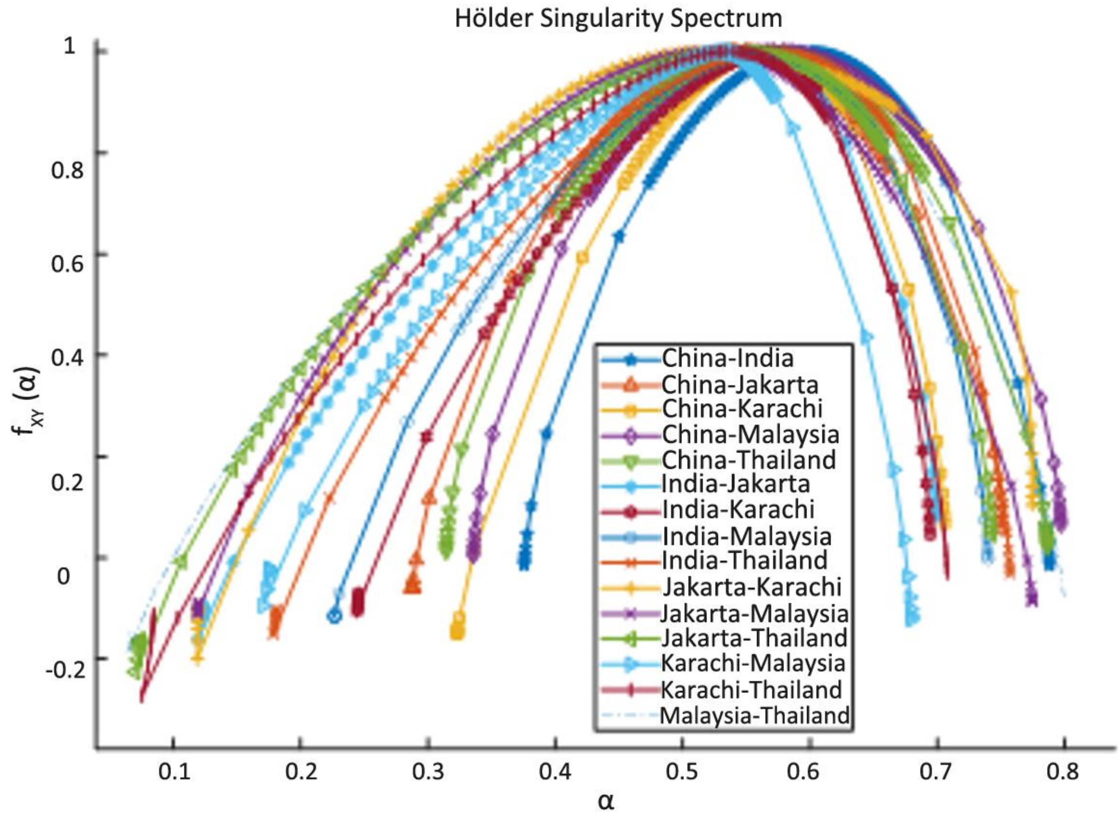

Figure 4.
Hölder Singularity Spectrum $f_{XY}(\alpha)$ for all pairs of indices.

We can observe in this figure that for 15 pairs of index returns, the curves of the singularity spectrum functions $f_{XY}(\alpha)$ have an inverted parabolic shapes. This indicates that the cross-correlations of the 15 pairs of indices exhibit multifractal nature.

The degree of multifractality can be measured by calculating the width of the spectrum, given by:

$$\Delta\alpha_{XY} = \alpha_{XY-max} - \alpha_{XY-min} \tag{24}$$

The table below present the degree of multifractality for the 15 pairs based on the generalized the singularity spectrum in decreasing order of $\Delta\alpha_{XY}$.

Table 3.
Degrees of multifractality based on the singularity spectrum for the15 pairs of indices in decreasing order of $\Delta\alpha_{XY}$.

Rank	Pairs of indices	$\Delta\alpha_{XY}$
1	Malaysia vs Thailand	0,732
2	Jakarta vs Thailand	0,669
3	Jakarta vs Malaysia	0,655
4	Karachi vs Thailand	0,623
5	India vs Jakarta	0,576
6	India vs Thailand	0,575
7	Jakarta vs Karachi	0,537
8	India vs Malaysia	0,513
9	Karachi vs Malaysia	0,504
10	China vs Thailand	0,472
11	China vs Malaysia	0,462
12	China vs Jakarta	0,462
13	India vs Karachi	0,449

14	China vs India	0,413
15	China vs Karachi	0,383

We observed that all the cross-correlations between each pair of indices display multifractal behavior, as $\Delta\alpha_{XY} = 0$ signifies that the bivariate time series demonstrate monofractal behavior.

In conclusion, the analysis, which includes the evaluation of generalized Hurst exponents, Rényi exponents, and singularity spectrum functions, confirms the presence of multifractal cross-correlations among the Islamic stock markets in the Pacific Asia region. These multifractal cross-correlations suggest that the interdependence between these markets is nonlinear and varies across different time scales. Specifically, the relationship between the markets may differ at short, medium, and long-term horizons. For example, short-term co-movements may be driven by factors such as regional news or investor sentiment, while at longer time scales, macroeconomic conditions or global events could play a more dominant role in shaping the correlations.

4.4. Source of Multifractality for the Cross-Correlations

As previously noted, there are two different sources of multifractality, long-term temporal cross-correlations and the heavy tails distributions. To determine how each source contributes to the overall cross-correlations multifractality, two transformations are applied on the original geometric return series: the shuffling and the surrogation.

In this study, two shuffling techniques were used, namely "randperm" and "randi". For phase surrogation, the Inverse Fast Fourier Transform (IFFT) method [16] is applied.

Figure 5 and Figure 6 compare the curves of the generalized Hurst exponent $H_{XY}(q)$ and the curves of the singularity spectrum $f_{XY}(\alpha)$ for the 15 original pairs of index returns series with those of the surrogate and the shuffled series.

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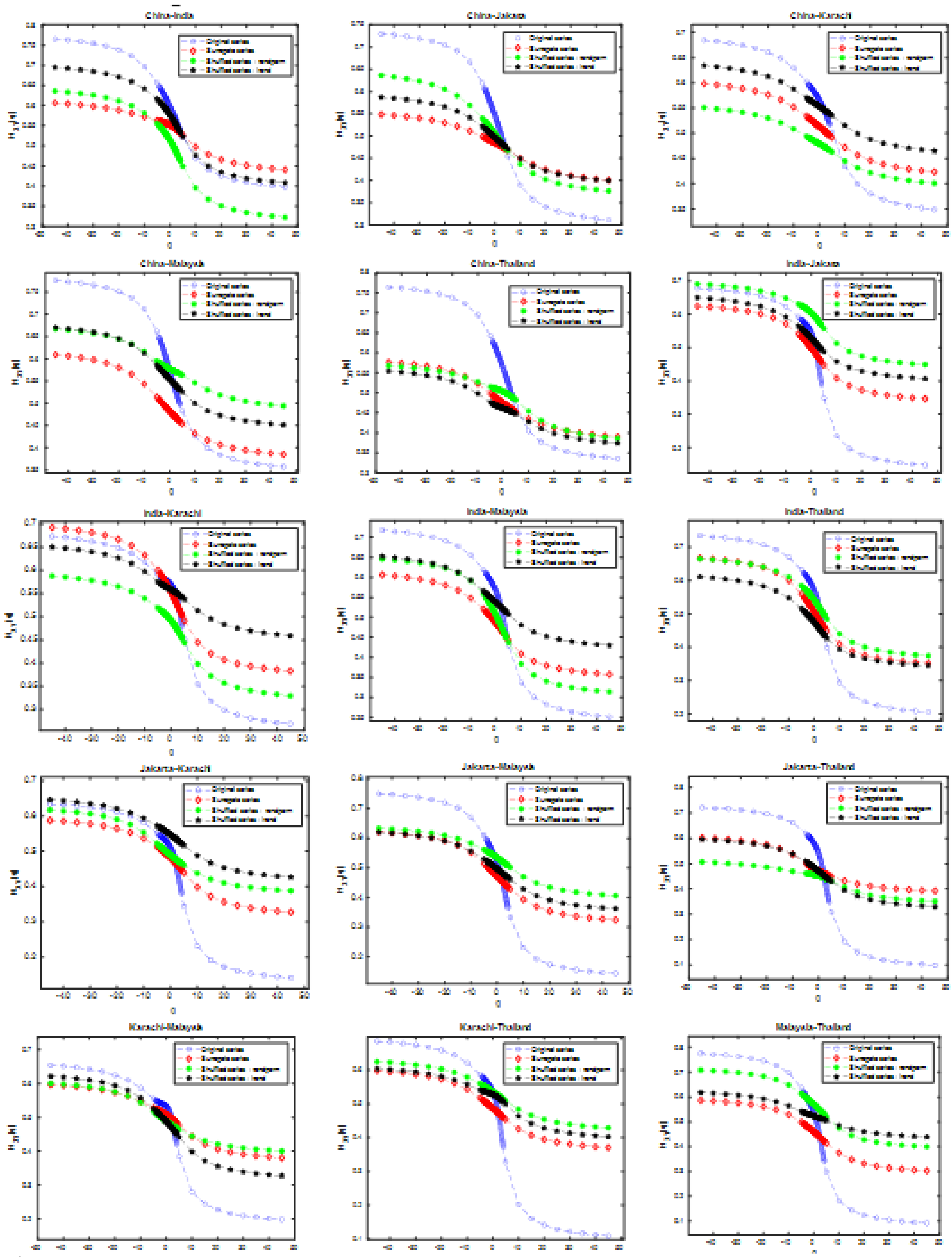

Figure 5.
Generalized Hurst exponent $H_{XY}(q)$ vs. q for all the pairs of series of original, surrogate and shuffled.

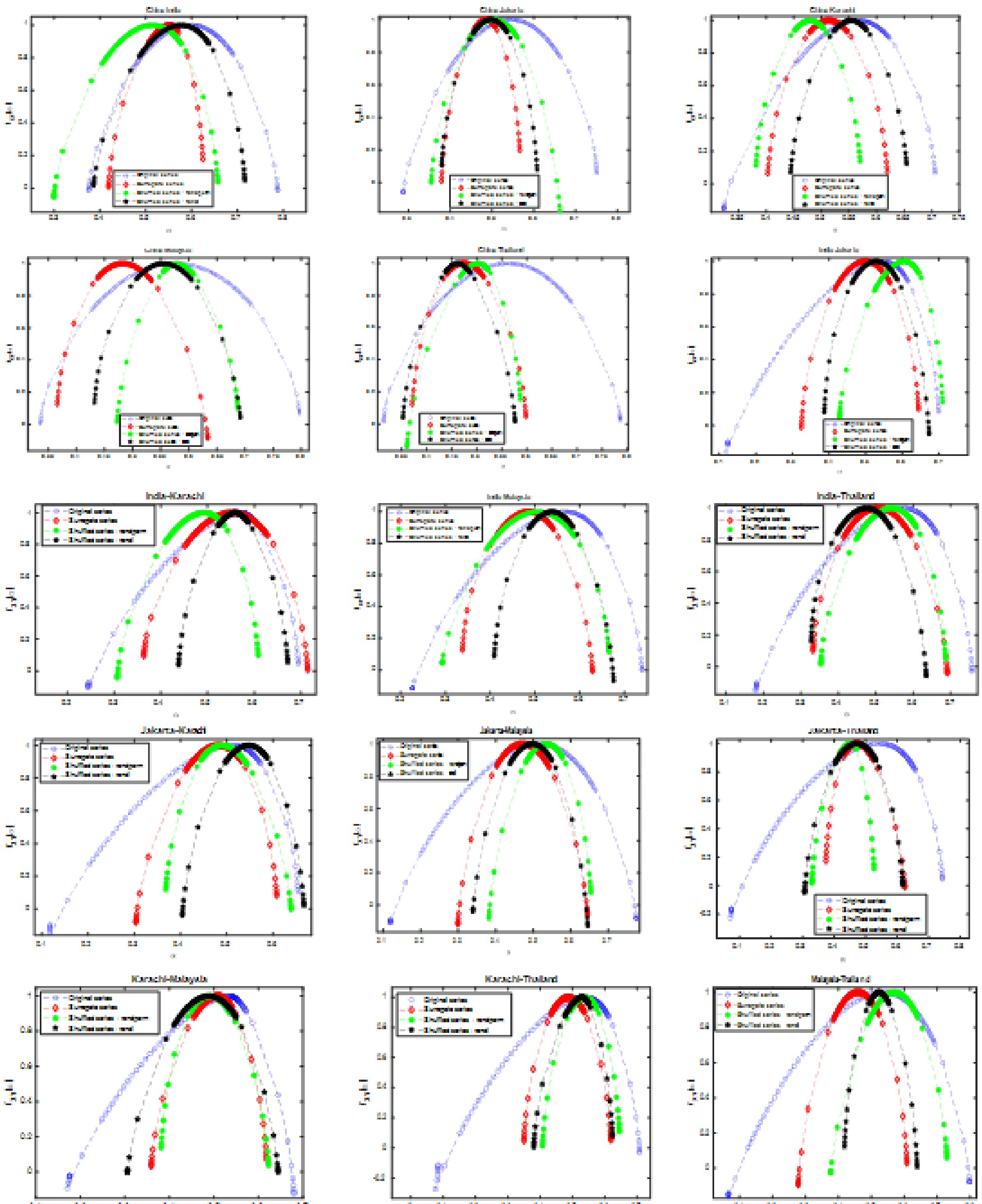

Figure 6.
Singularity spectrum $f_{XY}(\alpha)$ vs. α for all the original, surrogate and shuffled pairs of indices.

Figure 5 and Figure 6 show that the application of shuffling and surrogate transformations has reduced the degree of multifractality in the original series.

To precisely measure how much multifractality has been reduced, the values of $\Delta H_{XY} = H_{XY-max} - H_{XY-min}$ and $\Delta\alpha_{XY} = \alpha_{XY-max} - \alpha_{XY-min}$ are calculated for the 15 pairs of indices.

The MF-DCCA program is ran 100 times for each pair, and each time we obtained different results for the surrogate series and the two shuffled series, the results for the original series remained consistent. This variability is due to the algorithms generating the surrogate and shuffled series using random permutations.

However, in all 100 simulations, the ΔH_{XY} and $\Delta\alpha_{XY}$ of the original series are consistently greater than the ΔH_{XY} and $\Delta\alpha_{XY}$ of the surrogate series and the two shuffled series. Table 4 presents the results.

Table 4.
Degrees of multifractality of original, surrogate and shuffled series based ΔH_{XY} and $\Delta\alpha_{XY}$.

	Original		**Surrogate**		**Shuffled-randperm**		**Shuffled-randi**	
Pairs	ΔH_{XY}	$\Delta\alpha_{XY}$	ΔH_{XY}	$\Delta\alpha_{XY}$	ΔH_{XY}	$\Delta\alpha_{XY}$	ΔH_{XY}	$\Delta\alpha_{XY}$
China vs India	0.367	0.413	0.166	0.206	0.313	0.358	0.287	0.205
China vs Jakarta	0.418	0.462	0.148	0.188	0.260	0.309	0.188	0.142
China vs Karachi	0.337	0.383	0.175	0.217	0.150	0.189	0.170	0.122
China vs Malaysia	0.419	0.462	0.224	0.268	0.173	0.215	0.219	0.173
China vs Thailand	0.428	0.472	0.186	0.227	0.182	0.226	0.180	0.131
India vs Jakarta	0.530	0.575	0.278	0.320	0.241	0.281	0.243	0.194
India vs Karachi	0.404	0.449	0.309	0.351	0.258	0.302	0.191	0.137
India vs Malaysia	0.466	0.513	0.249	0.290	0.331	0.372	0.223	0.175
India vs Thailand	0.529	0.576	0.315	0.359	0.291	0.334	0.266	0.219
Jakarta vs Karachi	0.493	0.537	0.260	0.305	0.229	0.271	0.219	0.157
Jakarta vs Malaysia	0.607	0.655	0.297	0.346	0.227	0.272	0.258	0.192
Jakarta vs Thailand	0.622	0.669	0.209	0.250	0.156	0.197	0.266	0.197
Karachi vs Malaysia	0.457	0.504	0.217	0.259	0.200	0.241	0.294	0.211
Karachi vs Thailand	0.575	0.623	0.227	0.269	0.196	0.237	0.201	0.130
Malaysia vs Thailand	0.683	0.732	0.284	0.330	0.309	0.353	0.179	0.138

The results indicate that $\Delta H_{XY-Originate} > \Delta H_{XY-Surrogate}$, $\Delta\alpha_{XY-orginale} > \Delta\alpha_{XY-Surrogate}$, $\Delta H_{XY-Originate} > \Delta H_{XY-Shuffled}$ and $\Delta\alpha_{XY-orginale} > \Delta h_{XY-shuffled}$ for all 15 pairs of index returns. This indicates that the multifractality of the cross-correlations has been reduced by both the surrogate and shuffled transformations. We conclude that both long-term cross-correlations and heavy-tailed distributions play a significant role in the multifractal behavior of the cross-correlations between the six indices. The presence of long-term cross-correlations indicates a complex, interconnected market structure where shocks and market behaviors persist over time. This creates increased systemic risk, challenges traditional approaches to risk management and forecasting, and necessitates more dynamic and multifaceted strategies in portfolio management, regulation, and asset pricing. Understanding and effectively managing these long-term correlations is essential for investors, regulators, and policymakers striving to maintain stability and predictability in these interconnected markets. Additionally, the presence of heavy-tailed distributions suggests that extreme market events occur more frequently than conventional models predict. These events have significant implications for risk management, asset pricing, portfolio diversification, and overall financial stability. To mitigate the risks associated with these extreme events, investors and policymakers must adopt more advanced models and strategies that account for these outlier events and better manage the heightened likelihood of large-scale disruptions in the market.

4.5. Discussion

In comparing the results of this study with previous literature, we observe a number of consistent findings as well as some unique insights.

Similar to previous studies, this research highlights the presence of multifractal behavior in financial markets, specifically in the context of cross-correlations among six Islamic stock markets in the Pacific Asia region. Like the study by El Alaoui and Benbachir [5] on the MENA region, which identified significant multifractal cross-correlations, this study also demonstrates long-range persistent cross-correlations among the Pacific Asia Islamic indices. This suggests that markets in this region are similarly interconnected, with market movements in one country potentially influencing others, a finding in line with the observations made in earlier studies.

Further, our study supports the work of Xinsheng, et al. [6] who found long-term cross-correlations between the Chinese Renminbi (RMB) markets and other currencies, specifically highlighting the persistence of these relationships. The MF-DCCA method employed in our study revealed that, like the cross-correlations in the RMB markets, the Pacific Asia Islamic indices exhibit not only long-term correlations but also multifractal dynamics, signifying the persistent and complex interactions between these markets.

Additionally, our results resonate with the findings of Cao, et al. [9] who observed strong cross-market conductivity between Mainland China and Hong Kong, particularly during market fluctuations. Our analysis also indicated that cross-correlations among the six Pacific Asia Islamic stock markets are robust and long-lasting, even when considering various time scales. This points to the broader regional integration and interconnectedness of these markets, similar to what has been observed in studies of the Shanghai-Hong Kong Stock Connect [7] where post-liberalization cross-correlations between the two markets became more persistent.

Our study further builds on the work of Yanjun and Cheng [10] and Wei, et al. [11] who highlighted how financial crises and economic events can amplify the intensity of cross-correlations and multifractality. While this study did not focus explicitly on crisis periods, the findings suggest that the interconnectedness of these markets could be amplified under conditions of economic shocks or financial instability. As observed by Yanjun and Cheng [10] the degree of multifractality in market relationships often intensifies during crises, further reinforcing the argument that multifractal behavior plays a significant role in understanding market dynamics during extreme market events.

Moreover, the findings of this study align with the evolving application of multifractal analysis to broader asset classes, as seen in the work of Jia, et al. [13] on commodity markets and Acikgoz, et al. [14] on green bonds. Just as commodity markets such as soybean futures exhibit long-range dependencies, our study shows that the Islamic stock markets in Pacific Asia display similar multifractal characteristics. This indicates that the method of MF-DCCA can effectively capture the complexities of market interdependencies not only in traditional asset classes but also in emerging financial instruments and markets.

This study contributes to the growing body of literature on multifractality and cross-correlations in financial markets. While similar results have been observed in studies of other regions and asset classes, this research adds new insights by focusing on the Pacific Asia Islamic stock markets, highlighting the role of both long-term cross-correlations and heavy-tailed distributions in the cross-correlation multifractal nature of these markets. This research reinforces the importance of incorporating advanced, multifractal-based models in portfolio management, risk assessment, and financial market regulation, especially in interconnected markets such as those in the Pacific Asia region.

5. Conclusion

This study provides a detailed investigation of cross-correlations among six Pacific Asia Islamic stock markets - China, India, Indonesia, Pakistan, Malaysia, and Thailand - employing the Multifractal Detrended Cross-Correlation Analysis (MF-DCCA) method. The dataset includes daily closing prices of Pacific Asia Islamic indices, covering the period from January 1, 2011, to August 1, 2024, with approximately 3315 observations.

As a preliminary analysis, the application of the DCCA Cross-Correlation Coefficient method highlighted that the cross-correlations are not only persistent but also exhibit long-term stability across most index pairs.

Utilizing core components of the MF-DCCA method, such as generalized Hurst exponents, Rényi exponents, and Hölder singularity spectrum, further confirmed that the index pairs display long-range persistent cross-correlations and multifractal behavior. These findings indicate that the markets are deeply interconnected, with multifractal dynamics influencing their interactions.

Moreover, the investigation into the sources of multifractality through surrogation and shuffling transformations revealed that both long-term cross-correlations and heavy-tailed distributions play significant roles in the multifractal nature of the observed cross-correlations.

This study provides a detailed investigation of the cross-correlations among six Pacific Asia Islamic stock markets, revealing important practical implications for investors, policymakers, and financial market regulators. The presence of long-range cross-correlations and multifractal behavior indicates that these markets are deeply interconnected, meaning that market movements in one country could have persistent effects on others. For investors, this interdependence highlights the need for dynamic and adaptive portfolio strategies. Traditional diversification approaches, which assume market independence, may not be effective, and investors should incorporate cross-correlation and multifractal analyses into their decision-making processes to improve risk management and portfolio efficiency. Policymakers and regulators in the region must recognize the systemic risks posed by the interconnectedness of these markets. Financial stocks in one market could easily spread across the region, affecting overall market stability. By understanding these long-term correlations, regulators can better monitor and mitigate risks, ensuring that financial crises in one country do not have amplified effects across others. Moreover, the findings emphasize the importance of advanced, multifractal-based models for monitoring market stability, as traditional models may fail to capture the full scope of market interdependence and extreme events.

The interconnected nature of these markets also opens the door for financial innovation, such as the development of regional investment products or instruments that consider cross-market dynamics. Such financial products would need to account for the multifractal and cross-correlation behaviors observed to enhance investor returns and minimize risk exposure. Additionally, geopolitical or macroeconomic events, such as policy changes or economic crises, could have ripple effects across the entire region, making it crucial for financial institutions to understand these correlations to better predict market reactions and respond effectively to such events.

In summary, the study underscores the importance of incorporating multifractal and cross-correlation analyses into financial strategies and regulatory frameworks. By recognizing the complex, long-term interdependencies among these markets, stakeholders can better manage risks, improve market stability, and enhance the effectiveness of investment strategies in the Pacific Asia Islamic stock markets.

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