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Assessment of the impact of the lesson study approach and didactic suitability criteria on mathematics teacher training from the onto-semiotic approach

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Abstract: Improving mathematics teacher training requires evidence-based methodologies to enhance instructional quality and professional competencies. This study evaluates the impact of the EDEM Diploma Program, which integrates the Lesson Study approach and Didactic Suitability Criteria (DSC) within the Didactic-Mathematical Knowledge and Competencies Model (DMKC). The research follows a mixed-methods design, combining quantitative pre-test and post-test assessments with qualitative analysis of lesson planning and teaching reflections. The program was implemented in two phases: planning and execution of lesson study cycles, including collaborative lesson design, classroom implementation, reflective teaching discussions, and pedagogical assessment. Results indicate a 17.4 percentage point improvement in teachers' mathematical knowledge, with 82% demonstrating improved lesson structuring, 76% enhancing formative assessment strategies, and 88% gaining confidence in interactive teaching techniques. Findings highlight the strong correlation between teacher reflection and pedagogical improvement, emphasizing the effectiveness of structured, research-based professional learning. The study concludes that integrating Lesson Study and DSC enhances both mathematical knowledge and instructional quality, providing a scalable model for mathematics teacher training. Practical implications suggest the need for sustained professional development programs and longitudinal studies on teacher retention and student performance outcomes to assess the long-term impact of this approach.

Keywords: *Didactic suitability criteria, Lesson study, Mathematics teacher training, Pedagogical reflection, Professional development, Teacher self-efficacy, Teaching quality.*

1. Introduction

Mathematics teacher training faces increasing challenges, particularly in educational contexts where instructional quality is a priority. Effective mathematics education requires teachers not only to master mathematical content but also to develop specialized pedagogical competencies that support student learning. However, numerous studies indicate that current teacher training programs often fail to provide adequate preparation in mathematics didactics, resulting in instructional gaps that hinder effective mathematics teaching [1-3].

In response to these challenges, Lesson Study (LS) has emerged as a collaborative, research-based methodology that fosters structured professional reflection, peer learning, and continuous improvement of teaching practices. Originating in Japan, LS has been widely implemented in different educational

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contexts worldwide, proving effective in enhancing mathematics teachers' instructional strategies, pedagogical decision-making, and classroom management skills. LS promotes a cyclical process of planning, implementation, observation, and refinement, enabling teachers to analyze their own practices and develop adaptive teaching strategies that respond to students' learning needs [4].

Simultaneously, the Onto-Semiotic Approach to Mathematical Knowledge and Instruction (OSA) offers a theoretical and analytical framework for evaluating mathematics education processes. A key component of OSA is the Didactic Suitability Criteria (DSC), which serves as a comprehensive tool for assessing instructional effectiveness. The DSC framework evaluates six interconnected dimensions: epistemic, cognitive, interactional, mediational, affective, and ecological suitability, providing an evidence-based approach to analyzing the coherence, relevance, and impact of teaching practices [5].

The integration of Lesson Study with Didactic Suitability Criteria (LS-DSC) represents a novel approach to mathematics teacher education, bridging theory and practice through a structured framework for didactic reflection and pedagogical transformation. This study assesses the impact of implementing LS-DSC within the OSA framework on mathematics teacher training, examining how these methodologies contribute to teachers' professional competency development and instructional effectiveness. By systematically evaluating pre-service and in-service teacher preparation, this research offers empirical insights into the role of LS and DSC in improving mathematics instruction, as illustrated in Figure 1, which presents the integrated model that visually synthesizes the connection between Lesson Study, Didactic Suitability, and the OSA framework.

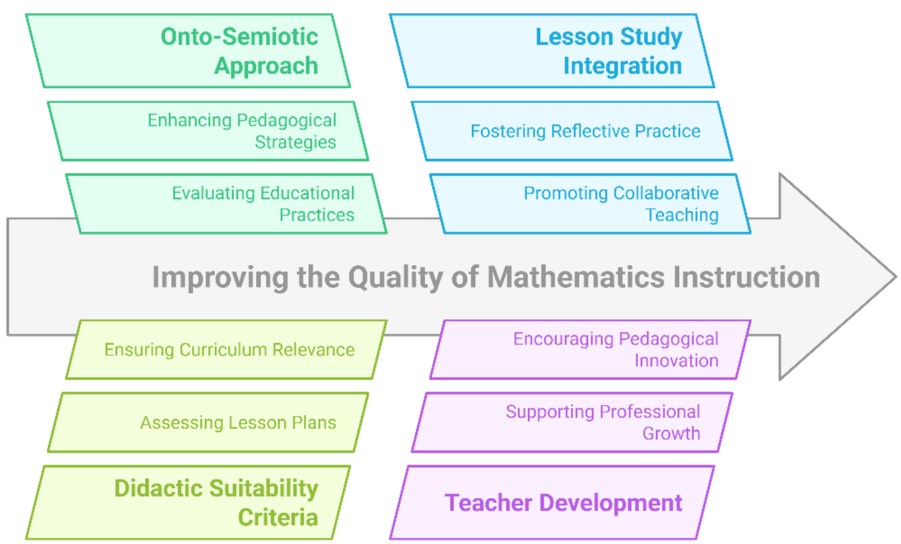

Figure 1.
Enhancing Mathematics Education through integrated frameworks.

A key concern in mathematics teacher education is the limited emphasis on mathematics didactics in university-level training programs. In many cases, mathematics teachers receive only five hours of didactics training, compared to eleven courses in general pedagogy, leaving them underprepared for the complexities of mathematics instruction. Studies suggest that this imbalance in training negatively impacts teachers' ability to implement effective instructional strategies, particularly in diverse classroom settings where differentiated instruction is essential [1]. Additionally, the devaluation of mathematical knowledge within the educational system has contributed to a decline in student engagement with STEM disciplines, further exacerbating learning gaps from the primary education level [2, 3].

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Recent research highlights that the effectiveness of mathematics instruction depends on teachers' continuous professional development, emphasizing the need for rigorous, research-based training in instructional strategies, differentiated learning techniques, and neuroscience applications in education Carnell and Tillery [6]; Morales-López and Font [7] and Sousa, et al. [8]. Garderen, et al. [9] argue that structured training programs enhance teachers' ability to identify and develop effective didactic interventions, which are critical for optimizing student learning outcomes. The COVID-19 pandemic accelerated the transition to digital education, further highlighting the need for technology-enhanced training models in mathematics teacher education [10].

Mathematics teacher education should therefore be conceived as a dynamic, lifelong learning process, requiring systematic professional development. Carvalho, et al. [11] emphasize that effective mathematics didactics ensures that mathematical knowledge is accessible to all students, regardless of social, emotional, physical, or linguistic backgrounds. This approach aligns with contemporary research advocating for adaptive teaching strategies and inclusive learning environments that support diverse student populations.

Despite existing educational policies aimed at strengthening mathematics teacher training, initial teacher education programs remain insufficient in addressing specialized instructional needs Castro [12]. Mendes, et al. [13] further highlight that many teachers lack the necessary pedagogical tools to adjust instructional methods, particularly in diagnosing students' difficulties in mathematical reasoning and problem-solving. This lack of preparation often results in resistance to instructional change, negatively impacting student engagement and mathematics achievement.

This study proposes that the integration of Lesson Study and Didactic Suitability Criteria within the Onto-Semiotic Approach offers a structured and sustainable model for mathematics teacher training. By focusing on collaborative professional inquiry, reflective practice, and empirical data collection, this research contributes to the development of evidence-based teacher training programs that enhance instructional quality, support pedagogical innovation, and foster long-term improvements in mathematics education.

1.1. Knowledge and Competency Model for Mathematics Teachers (DMKC Model)

Within the framework of the Onto-Semiotic Approach to Mathematical Knowledge and Instruction (OSA), a theoretical model for the Didactic-Mathematical Knowledge and Competencies (DMKC Model) has been developed [14-16]. This model establishes analytical dimensions that underpin the knowledge and competencies required by mathematics teachers.

1.2. The Concept of Competency and Key Competencies

In the DMKC Model, competency is understood from the perspective of competent action, encompassing a combination of knowledge, skills, affective dispositions, and reflective tools that enable effective performance in professional contexts. Competency is considered a potentiality that materializes in the execution of efficient actions [17].

From this perspective, problem-solving tasks serve as the foundation for developing and accessing teacher competencies, as they generate representations of professional challenges. Teachers (or prospective teachers) mobilize their cognitive, effective, and reflective skills to address these challenges. The success of such interventions serves as an indicator of their ability to execute similar practices, providing empirical evidence of their competency level [16].

The DMKC Model identifies two core competencies for mathematics teachers:

1. Mathematical competency, which refers to teachers' mastery of mathematical content.
2. Didactic analysis and intervention competency, which is central to instructional practice. This competency involves designing, implementing, and evaluating both their own and others' learning sequences using didactic analysis techniques and quality criteria to establish iterative cycles of planning, implementation, assessment, and improvement [14].

1.3. Characterization of Didactic Analysis and Intervention Competency

Mathematics education research lacks a universally accepted paradigm for analyzing mathematical learning activities. Within the DMKC Model, it is assumed that OSA's theoretical tools enable such analysis in terms of mathematical practices, objects, and processes. The ability of teachers to identify and structure these elements provides a foundation for understanding students' learning processes and assessing their mathematical competencies.

Within OSA, the concept of Learning Configurations (LC) has been introduced as a tool to analyze teacher-student interactions during instructional activities [18]. LC represents a theoretical construct that model's teacher-student behaviors in relation to specific teaching tasks and mathematical content (primary object-process configurations). Learning, therefore, emerges from instructional interactions.

Mathematics teachers must be capable of designing and managing LC to ensure effective student learning. This process is governed by a complex network of norms and meta-norms derived from various educational, institutional, and social factors [19]. Understanding how these norms influence instruction is crucial to directing learning processes effectively.

To guide instructional decision-making, teachers must develop Normative Analysis Competency (NAC) to address key pedagogical questions:

- What norms constrain instructional development?
- How should these norms evolve over time?
- What aspects can be modified to improve learning outcomes?

1.4. Criteria for Didactic Suitability in Mathematics Education

Didactic suitability refers to the extent to which an instructional process effectively aligns with learning objectives and ensures meaningful student engagement given contextual constraints and available resources [15]. A highly suitable instructional process systematically integrates the following six didactic suitability criteria (DSC):

1. Epistemic Suitability (ES): Ensures that the mathematical content taught adheres to high-quality mathematical standards, referencing both prescribed curricula and institutionalized mathematical knowledge.
2. Cognitive Suitability (CS): Assesses the alignment between the expected and actual student learning, ensuring that instruction falls within students' proximal zone of development.
3. Interactional Suitability (IS): Evaluates the effectiveness of interaction patterns in identifying and resolving cognitive conflicts, while promoting student autonomy.
4. Mediational Suitability (MS): Examines the availability and adequacy of instructional materials and resources, as well as the temporal organization of the learning process.
5. Affective Suitability (AS): Measures students' engagement, interest, and motivation throughout the learning experience.
6. Ecological Suitability (ES): Ensures alignment between instructional processes and broader institutional, curricular, and sociocultural contexts.

Each of these criteria includes specific components and indicators that can be assessed using structured evaluation rubrics. The DSC framework provides a systematic methodology for conducting comprehensive and balanced assessments of mathematics learning processes, facilitating evidence-based improvements in instructional design.

2. Methodology

This study examines the design, implementation, and impact of a specialized Mathematics Teacher Training Diploma (EDEM: Estrategias Didácticas para la Enseñanza de las Matemáticas) for primary and secondary school educators in Panama. The program was developed and delivered through the virtual learning platform of the University of Panama (UP), leveraging blended learning methodologies to provide a flexible and accessible professional development opportunity.

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The training program follows a two-phase structure:

1. Phase 1 – Research and Development (2 months): Participants engage in lesson design focused on key mathematical competencies, collaboratively developing instructional strategies. This phase emphasizes theoretical and pedagogical foundations, ensuring that teachers gain a deep understanding of mathematical instruction before classroom implementation. Educators iteratively refine their lesson plans based on peer feedback and expert evaluations. A core component of this phase is the Learning Analysis Methodology, a framework that includes five sub-analyses: conceptual clarity, content development, cognitive guidelines, instructional strategies, and assessment design. At the end of each month, participants submit comprehensive pedagogical reports outlining their refined lesson plans and analytical reflections.
2. Phase 2 – Classroom Implementation and Evaluation (4 months): Teachers implement their improved lesson plans in real classroom settings while receiving continuous mentoring from national and international mathematics education experts. This phase includes structured classroom observations, peer reviews, and didactic analysis cycles. Teachers document their experiences, challenges, and successes, contributing to a collaborative learning community aimed at strengthening professional teaching competencies.

The methodology is rooted in the Lesson Study (LS) framework, which promotes a structured approach to lesson planning, implementation, observation, and refinement. Additionally, the program aligns with the Didactic Suitability Criteria (DSC) under the Onto-Semiotic Approach (OSA), ensuring a systematic and research-based evaluation of teaching practices [5].

2.1. Competency Framework and Alignment with the DMKC Model

The EDEM Diploma Program is designed for in-service primary and secondary school teachers under the jurisdiction of the Ministry of Education (MEDUCA) in Panama. Its primary objective is to develop and strengthen didactic-mathematical competencies aligned with the Didactic-Mathematical Knowledge and Competencies Model (DMKC) [14].

A pilot study was conducted to assess the alignment of the DMKC Model with national educational standards. This involved analyzing the graduate profiles of Panama's two leading teacher-training institutions and MEDUCA's official pedagogical competency frameworks. The post-pilot version of EDEM was redesigned based on the strengths and limitations identified in the original program.

The final structure of the diploma consists of five six-month modules (blended learning format, 320 hours total) facilitated by a team of experts, including an international instructor from Brazil and three national mathematics education specialists. The program utilizes Moodle as its virtual learning platform, developed and hosted by the University of Panama's Virtual Campus. To enhance accessibility, synchronous sessions are scheduled primarily during evening hours.

2.2. Participants and Demographics

The study involved a total of 100 secondary school mathematics teachers from various regions across Panama, with a diverse representation in terms of age, gender, geographical location, and institutional affiliation.

2.3. Age Distribution

The participant group was composed of individuals across different age brackets, reflecting a balance of early-career and experienced educators. 29.7% of the participants were between 34 and 41 years old, 27% were between 42 and 46 years old, and 23% were within the 26 to 33 age range. This distribution indicates a substantial proportion of teachers in their mid-career phase, with a significant presence of younger professionals who could benefit from advanced pedagogical training. *(See Figure 1 – Age Distribution of Participants)*

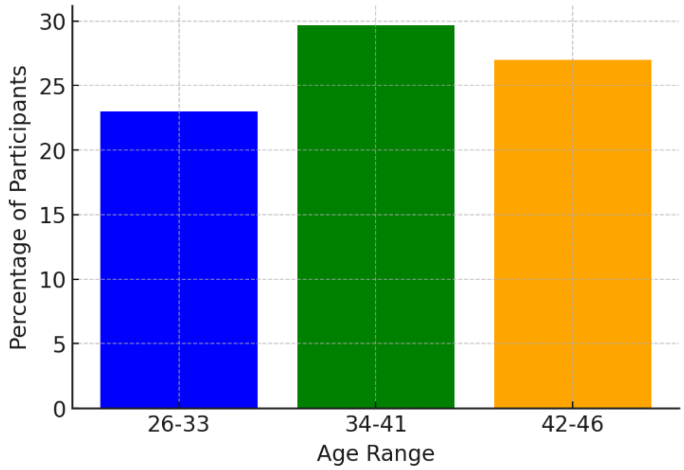

Figure 2.
Age Distribution of Participants.

2.4. Gender Distribution

The gender distribution was 55.4% female and 44.6% male, illustrating a relatively balanced participation of educators. The higher representation of female teachers aligns with broader trends in education, where women often comprise a larger proportion of the teaching workforce, particularly in primary and secondary education. *(See Figure 2 - Gender Distribution of Participants)*.

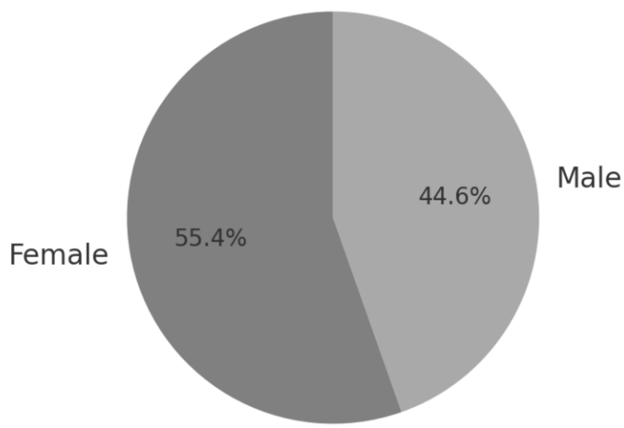

Figure 3.
Gender Distribution of Participants.

2.5. Geographic Representation

Participants were drawn from six key regions of Panama, ensuring an inclusive representation from both urban and rural areas. The highest percentage of participants were from Panamá Oeste (38.3%) and Panamá City (29.6%), followed by Colón (12.3%) and Chiriquí (11.1%). A smaller fraction of teachers hailed from Darién (2.5%) and the Ngäbe-Buglé indigenous region (1.2%). This geographic diversity highlights the national reach and accessibility of the training program, ensuring that educators from various socio-economic and cultural backgrounds could participate. *(See Figure 3 - Geographic Distribution of Participants)*

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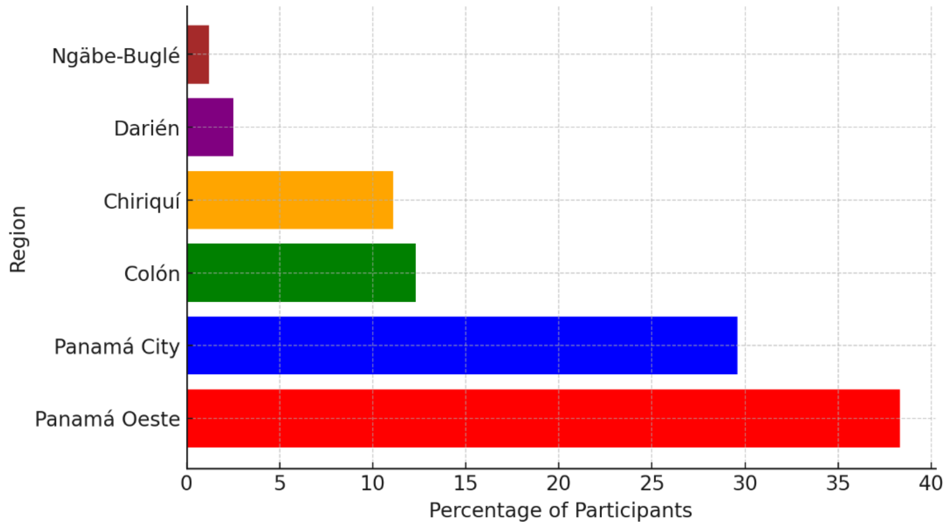

Figure 4.
Geographic Distribution of Participants.

2.6. Institutional Affiliation

The majority of participants were affiliated with public schools (78.4%), while 18.9% were from private institutions. This distribution is significant given that public schools in Panama often face greater challenges in terms of resource allocation, teacher training opportunities, and technological integration. By targeting a higher percentage of public-school educators, the program aimed to bridge existing educational gaps and enhance mathematics instruction across diverse learning environments. *(See Figure 4 and 5).*

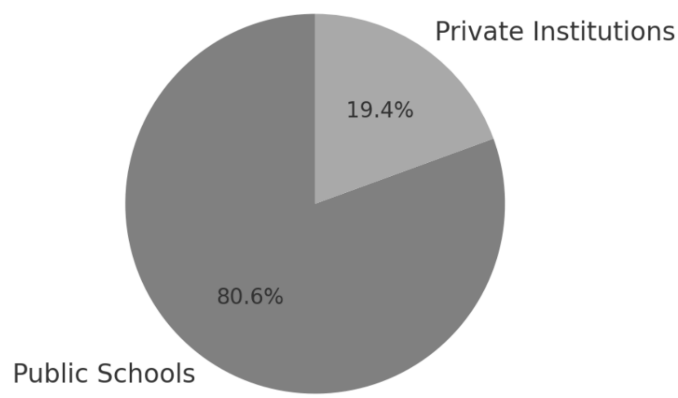

Figure 5.
Institutional Distribution of Participants.

These demographic insights provide a comprehensive understanding of the participant profile, which is crucial for assessing the impact of the training program and tailoring future professional development initiatives to address the specific needs of different educator groups.

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2.7. Structure and Implementation of the Teacher Training Program

The Mathematics Teacher Training Program (EDEM) was designed to enhance the professional competencies of mathematics educators through a structured two-phase approach. The program integrates lesson study methodologies, didactic suitability criteria (DSC), and the Didactic-Mathematical Knowledge and Competencies Model (DMKC) to ensure effective pedagogical development.

Phase 1: Planning and Lesson Design

The first phase, known as the "Planning and Design Phase," spanned six months and focused on the development of instructional design competencies. Teachers initially designed a lesson centered around specific mathematical skills, collaborating with their peers and instructors to refine pedagogical strategies. The training sessions, facilitated through the virtual platform of the University of Panama, provided theoretical and pedagogical support to ensure that participants could effectively implement these strategies in their classrooms.

As a result of this phase, teacher groups reorganized classroom structures, integrating educational resources and instructional materials with a strong pedagogical and didactic foundation. The iterative improvement process continued as each group conducted a lesson analysis, linking their instructional design to the curriculum and presenting findings to peers and instructors. Through this process, teachers revised and enhanced their lesson plans based on the feedback provided.

In the DMKC Model, mathematics teachers are expected to develop two core competencies:

1. Mathematical Competency, which refers to their mastery of mathematical content.
2. Didactic Analysis and Intervention Competency, which involves the ability to analyze, assess, and improve the teaching-learning process.

To develop the second competency, teachers must acquire:

- Descriptive and explanatory knowledge to analyze what happens in the classroom during instruction (Didactic Dimension of the DMKC Model).
- Evaluative knowledge to assess instructional effectiveness and propose future improvements (Metadidactic-Mathematical Dimension of the DMKC Model) [20].

This overarching competency is further divided into four subcompetencies:

1. Mathematical Activity Analysis
2. Analysis and Management of Interactions and Conflicts
3. Analysis of Norms and Meta-Norms
4. Evaluation of Didactic Suitability in Instructional Processes

Phase 2: Course Implementation and Instructional Reflection

The second phase, known as the "Course Learning Phase," was structured around iterative cycles of implementation, observation, and reflection. Teachers implemented their lesson plans in real classroom settings, collecting data on instructional effectiveness. Each group followed a lesson study model, executing their designed lessons over a structured period and documenting implementation outcomes for group analysis (see Figure 4).

On a monthly basis, teachers and instructors engaged in collaborative reflection sessions to analyze classroom experiences and identify key instructional elements for further development. These sessions incorporated the seven didactic principles outlined in the DMKC and Onto-Semiotic Approach (OSA) frameworks:

- Principle 1: Focus on Teaching – The program was designed to strengthen teachers' pedagogical practice and specialized mathematical knowledge.
- Principle 2: Focus on Mathematical Knowledge – The training centered on the four core mathematical competencies outlined in Panama's national curriculum.
- Principle 3: Expert Facilitation – Each synchronous session featured two experts, one specializing in modeling competencies and the other in problem-solving skills, ensuring expertise-driven instruction.

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- Principle 4: Inquiry and Reflection – The program emphasized curriculum-based inquiry, promoting a culture of critical reflection and professional dialogue.
- Principle 5: Interdisciplinary Engagement – Teachers collaborated across subject areas, integrating mathematical instruction with other disciplines to enhance cross-curricular connections.
- Principle 6: Mathematical Knowledge Development – The structured development of mathematical sequences facilitated progressive learning.
- Principle 7: Classroom Data Collection – The second phase emphasized data-driven instructional improvements, ensuring that lesson adaptations were based on empirical classroom evidence.

The second phase reinforced iterative instructional refinement, ensuring that teachers engaged in continuous professional learning cycles. Data collection and analysis played a central role in the program, allowing educators to develop evidence-based improvements in their instructional practice.

By incorporating collaborative professional inquiry, interdisciplinary exchange, and expert-led facilitation, the training program established a systematic approach to mathematics teacher development. The findings suggest that structured reflection, guided expertise, and a strong curricular framework significantly enhance teachers' pedagogical effectiveness and subject-matter proficiency.

3. Results and Discussion

A comparative analysis of the pre-test and post-test assessments revealed substantial progress in teachers' conceptual understanding of mathematics. The mean pre-test score was 65.2%, indicating moderate proficiency, whereas the post-test results showed an average of 82.6%, reflecting an improvement of 17.4 percentage points. This increase suggests that the program effectively enhanced teachers' mathematical reasoning and analytical skills, equipping them with the necessary knowledge to deliver higher-quality instruction, as visually represented in Figure 6, which illustrates the upward trajectory in participants' performance.

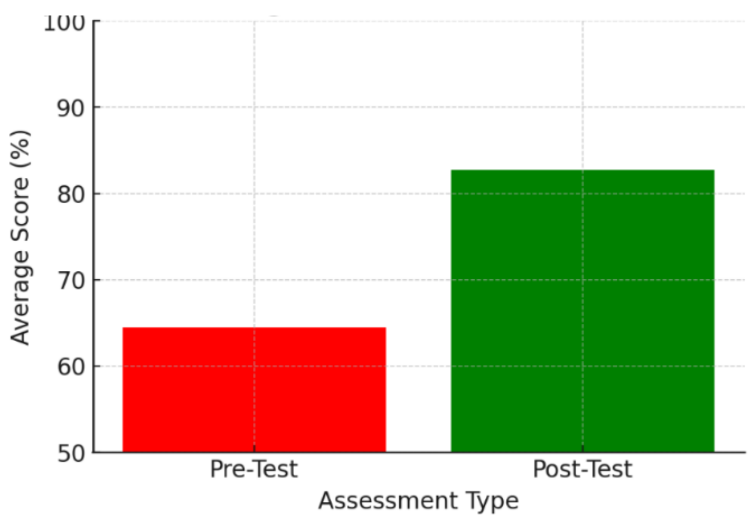

Figure 6.
Mathematical Knowledge Evolution before and after the program.

3.1. Improvement in Teachers' Mathematical Knowledge

A comparative analysis of pre- and post-test results revealed significant gains in teachers' conceptual understanding of mathematics. The average score increased from 65.2% to 82.6%, reflecting a 17.4 percentage point improvement. This indicates that the program strengthened teachers'

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mathematical reasoning and analytical skills, enabling more effective instruction. Additionally, participants showed greater fluency in problem-solving and mathematical modeling—key areas of focus, as depicted in Figure 7. These results support existing research on the value of structured teacher training in enhancing both content knowledge and pedagogical practice [21, 22].

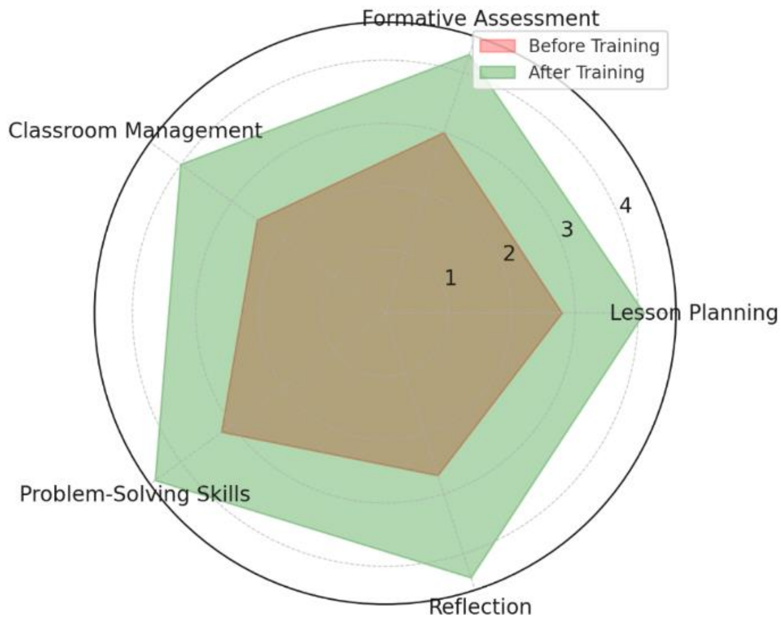

Figure 7.
Development of pedagogical competencies before and after training.

3.2. Impact on Instructional Design and Teaching Strategies

The Lesson Study Implementation Stage played a crucial role in refining teachers' lesson planning, instructional delivery, and assessment methods. Throughout the iterative process of lesson execution, observation, feedback, and refinement, participants demonstrated notable improvements in lesson coherence, student engagement techniques, and the integration of mathematical representations.

A qualitative analysis of lesson recordings and peer evaluations revealed the following trends:

- 82% of teachers demonstrated an increased ability to structure lessons with a clear instructional sequence, ensuring logical content progression.
- 76% of participants improved their use of formative assessment techniques, integrating real-time feedback to adjust their teaching dynamically.
- 88% of educators reported higher confidence in using interactive teaching methods, such as collaborative problem-solving and mathematical discourse.

These results confirm the effectiveness of structured lesson study methodologies in enhancing pedagogical competencies, as supported by international research in mathematics education [23, 24].

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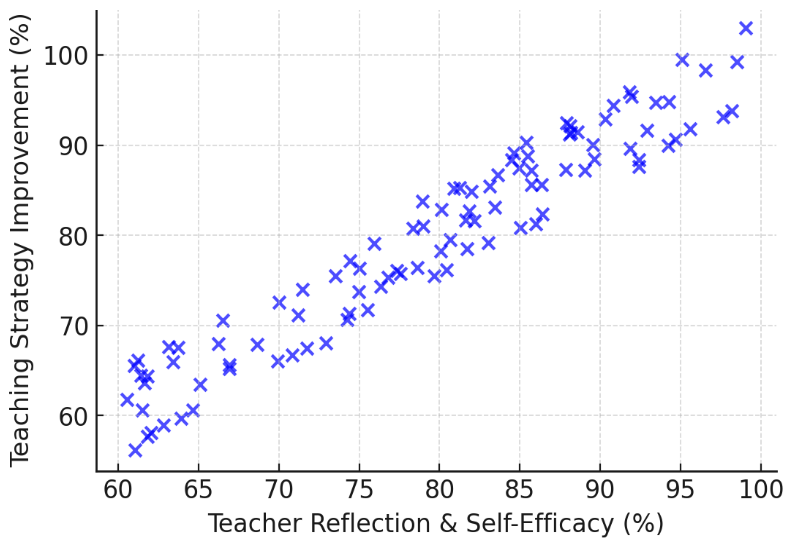

Figure 8.
Correlation between teacher reflection and pedagogical improvement.

3.3. Strengthening of Didactic Analysis Competencies (DMKC Model)

The Didactic-Mathematical Knowledge and Competencies Model (DMKC) was a critical framework in assessing teachers' ability to analyze and evaluate their instructional practices. By the end of the program, participants demonstrated increased proficiency in the four core subcompetencies of didactic analysis:

1. Mathematical Activity Analysis: Teachers exhibited greater awareness of how students conceptualize and apply mathematical ideas.
2. Analysis and Management of Interactions and Conflicts: Classroom discourse became more structured, with an emphasis on resolving cognitive conflicts and promoting student autonomy.
3. Analysis of Norms and Meta-Norms: Participants gained insights into curricular constraints and institutional policies, allowing for more informed instructional decision-making.
4. Evaluation of Didactic Suitability in Instructional Processes: The ability to assess lesson effectiveness using evidence-based criteria improved significantly among teachers.

These findings reinforce previous research advocating for didactic analysis as a fundamental competency in mathematics teacher training [5, 16].

3.4. Professional Growth and Teacher Reflection

Beyond academic and pedagogical improvements, the program fostered a culture of professional reflection and continuous learning among participants. Teachers reported increased self-efficacy in lesson design, classroom management, and mathematical discourse facilitation.

Survey responses indicated strong positive perceptions among participating teachers regarding the program's impact. Specifically, 91% valued the structured reflection and peer collaboration processes, 85% expressed confidence in applying the strategies learned beyond the program, and 78% reported a greater inclination to engage in ongoing professional development. These perceptions reflect a meaningful shift toward reflective and collaborative teaching practices, which are visually contrasted in Figure 9, highlighting the pedagogical transformation experienced by participants. As one teacher noted, *"For the first time, I felt that planning, teaching, and reflecting were not isolated tasks but part of a coherent process of growth."* These findings align with international literature that emphasizes reflective teaching as a cornerstone for sustainable educational improvement [25].

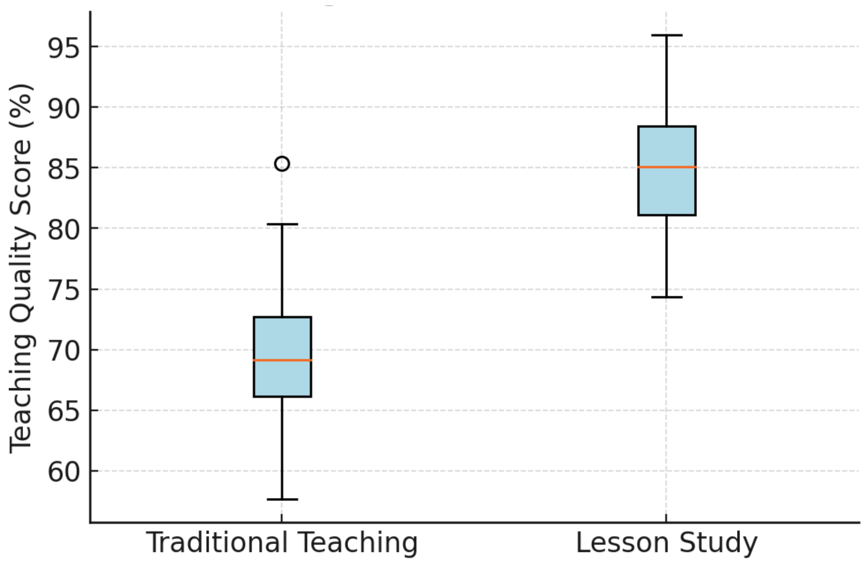

Figure 9.
Comparison of teaching methods: traditional vs. Lesson Study.

4. Discussion

The results of this study confirm that the EDEM Diploma Program successfully addressed key challenges in mathematics teacher education by integrating evidence-based instructional frameworks such as Lesson Study, the Didactic Suitability Criteria (DSC), and the DMKC Model.

Compared to traditional teacher training models, which often focus on content delivery rather than pedagogical refinement, the EDEM approach provided a more holistic and iterative method of professional development. The combination of theory, guided practice, and reflective analysis resulted in measurable gains in mathematical knowledge, instructional quality, and didactic analysis competencies.

These findings align with global best practices in teacher education, as documented in studies on lesson study effectiveness [26] teacher knowledge development [22] and the impact of professional learning communities on instructional improvement [27].

5. Conclusion and Implications

The findings of this study highlight the transformative impact of integrating Lesson Study (LS) and Didactic Suitability Criteria (DSC) within the Onto-Semiotic Approach (OSA) in mathematics teacher training. The success of the EDEM program reinforces the importance of structured, research-based professional development models, providing empirical evidence that collaborative learning, reflective practice, and instructional analysis significantly enhance teachers' professional competencies. The results confirm that mathematics teacher education must be conceived as a continuous, iterative process, integrating theory, practice, and systematic evaluation to ensure long-term improvements in instructional quality.

The quantitative and qualitative analyses presented in this study reveal key areas for future refinements in mathematics teacher training programs. The Lesson Study cycles proved highly effective in developing teachers' analytical skills, improving lesson planning strategies, and fostering peer collaboration. Additionally, the implementation of DSC as an evaluative framework provided teachers with a structured methodology for assessing and refining their instructional approaches, leading to a more evidence-based, student-centered pedagogy. However, the findings also indicate that certain program components require further optimization to maximize long-term impact.

Based on the empirical findings, the EDEM program can be further strengthened through the following enhancements:

5.1. Expanding the Lesson Study Component

- Future iterations should incorporate classroom-based action research projects, allowing teachers to systematically document and analyze their instructional interventions over extended periods.
- The integration of longitudinal lesson analysis cycles would provide deeper insights into the evolution of teaching strategies and student learning outcomes.
- Collaborative inquiry should be reinforced through cross-institutional partnerships, enabling teachers to exchange best practices and research findings beyond their immediate professional networks.

5.2. Integrating Technology-Enhanced Learning Strategies

- The increasing role of digital education in mathematics teaching necessitates the integration of technology-enhanced instructional strategies.
- Blended learning approaches—combining synchronous and asynchronous digital platforms—should be incorporated into the training framework to provide greater accessibility and flexibility for in-service teachers.
- Data-driven teaching analytics should be explored to evaluate and personalize instructional methods, ensuring that teachers can adapt their pedagogical approaches based on student performance trends.

5.3. Strengthening Institutional Collaborations for Sustainable Professional Growth

- The study emphasizes the need for stronger university-school partnerships to sustain long-term professional development efforts.
- Universities should play a proactive role in mentoring and supporting teachers beyond the training program, ensuring ongoing access to professional learning communities, instructional coaching, and peer support networks.
- Policymakers should prioritize the institutionalization of LS-DSC models as a standard teacher training framework, fostering a national and international movement towards competency-based mathematics teacher education.

The EDEM model has the potential to serve as a global benchmark for mathematics teacher training, offering a replicable framework adaptable to diverse educational contexts. By integrating Lesson Study, Didactic Suitability Criteria, and the Onto-Semiotic Approach, the program provides a systematic, data-informed model for enhancing instructional quality in mathematics education. The study's findings contribute to ongoing discussions on STEM education, teacher competency development, and evidence-based pedagogical reform.

Ultimately, structured professional development programs that integrate reflective practice, data-driven assessment, and collaborative inquiry are crucial for improving educational quality and student learning outcomes. The EDEM program underscores the importance of sustained, research-based teacher training models that not only equip educators with theoretical and practical competencies but also foster continuous professional growth, ensuring long-term improvements in mathematics education.

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Transparency:
The authors confirm that the manuscript is an honest, accurate, and transparent account of the study; that no vital features of the study have been omitted; and that any discrepancies from the study as planned have been explained. This study followed all ethical practices during writing.

References
[1] J. Katz, M. Stupel, and G. Ron, "The mathematical training of teachers: A global perspective," *Mathematics Education Review*, vol. 15, no. 2, pp. 167-181, 2002.
[2] A. Breda, L. Pino-Fan, and V. Font, "Analyzing mathematics teachers' didactic-mathematical knowledge using the DMKC model," *Journal of Mathematics Teacher Education*, vol. 24, no. 3, pp. 295-318, 2021.
[3] V. Font, A. Breda, and L. Pino-Fan, "Understanding mathematics education through the onto-semiotic approach," *International Journal of Research in Mathematics Education*, vol. 37, no. 1, pp. 67-89, 2024.
[4] R. Velásquez Castillo, "Lesson study as a strategy for enhancing mathematics teacher training," *International Journal of Pedagogical Innovation*, vol. 15, no. 1, pp. 99-117, 2023.
[5] J. D. Godino, C. Batanero, and V. Font, "The onto-semiotic approach to research in mathematics education," *ZDM Mathematics Education*, vol. 39, no. 1-2, pp. 127-135, 2007. https://doi.org/10.1007/s11858-006-0004-1
[6] E. Carnell and B. Tillery, *Teacher development and educational reform: Rethinking professional learning*. United Kingdom: Routledge, 2005.
[7] J. Morales-López and V. Font, "Neuroscience and mathematics education: Implications for teacher training," *Educational Neuroscience*, vol. 4, no. 3, pp. 189-203, 2019.
[8] D. A. Sousa, J. Sweller, and E. Jenson, "Cognitive load theory and mathematics instruction," *Educational Psychology Review*, vol. 32, no. 3, pp. 457-478, 2020.
[9] D. Garderen, A. Scheuermann, and C. Jackson, "The impact of structured training on teachers' problem-solving instruction," *Mathematics Education Research Journal*, vol. 21, no. 4, pp. 327-345, 2009.
[10] M. Alfaro-Ponce, C. Hernández, and R. Salazar, "Digital transition in mathematics education: Post-pandemic teacher training challenges," *Educational Technology & Society*, vol. 26, no. 1, pp. 45-62, 2023.
[11] C. Carvalho, R. Santos, and F. Mendes, "Inclusive education in mathematics: Addressing diverse student needs," *International Journal of Mathematical Education in Science and Technology*, vol. 49, no. 6, pp. 897-915, 2018.
[12] M. Castro, "Challenges in mathematics teacher education: A policy perspective," *Educational Policy Analysis Archives*, vol. 15, no. 8, pp. 1-22, 2007.
[13] F. Mendes, C. Carvalho, and R. Santos, "Evaluating teacher education programs in mathematics: A qualitative approach," *International Journal of Educational Research*, vol. 50, no. 2, pp. 123-135, 2011.
[14] A. Breda, V. Font, and L. Pino-Fan, "The role of mathematical and didactic knowledge in the professional development of teachers: A competency-based approach," *Educational Studies in Mathematics*, vol. 94, no. 2, pp. 223-240, 2017.
[15] J. D. Godino, B. Giacomone, and V. Font, "The didactic suitability criteria: A framework for evaluating mathematics instruction," *Journal of Mathematics Teacher Education*, vol. 20, no. 3, pp. 257-281, 2017.
[16] L. Pino-Fan, J. D. Godino, and V. Font, "Developing teacher competencies through problem-solving-based training," *Journal of Mathematics Teacher Education*, vol. 25, no. 2, pp. 317-341, 2022.
[17] M. Seckel and V. Font, "Competency-based teacher education in mathematics: Theoretical perspectives and practical applications," *Journal of Teacher Education*, vol. 66, no. 2, pp. 159-172, 2015.
[18] J. D. Godino, C. Batanero, R. Roa, and M. R. Wilhelmi, "Learning configurations: An instrument for the didactic analysis of mathematics instruction processes," *Journal of Mathematics Education*, vol. 20, no. 1, pp. 5–32, 2006.

[19] J. D. Godino, H. Rivas, and M. R. Wilhelmi, "Learning configurations: A theoretical construct for analyzing classroom interactions," *Educational Studies in Mathematics*, vol. 72, no. 2, pp. 203-221, 2009.
[20] L. Pino-Fan, A. Assis, and W. Castro, "The role of didactic analysis in mathematics teacher training: A study within the DMKC model," *International Journal of Science and Mathematics Education*, vol. 13, no. 4, pp. 793-814, 2015.
[21] L. S. Shulman, "Those who understand: Knowledge growth in teaching," *Educational Researcher*, vol. 15, no. 2, pp. 4-14, 1986.
[22] D. L. Ball, M. H. Thames, and G. Phelps, "Content knowledge for teaching: What makes it special?," *Journal of Teacher Education*, vol. 59, no. 5, pp. 389–407, 2008. https://doi.org/10.1177/0022487108324554
[23] J. W. Stigler and J. Hiebert, *The teaching gap: Best ideas from the world's teachers for improving education in the classroom*. New York, USA: Free Press, 1999.
[24] C. Lewis and R. Perry, "Lesson study and the improvement of mathematics teaching: The role of inquiry and reflection," *Teachers College Record*, vol. 119, no. 5, pp. 1-27, 2017. https://doi.org/10.5951/jresematheduc.48.3.0261
[25] D. A. Schön, *The reflective practitioner: How professionals think in action*. New York, USA: Basic Books, 1983.
[26] R. Huang and Y. Shimizu, "Examining lesson study in mathematics education: International perspectives," *ZDM Mathematics Education*, vol. 48, no. 4, pp. 393-398, 2016. https://doi.org/10.1007/s11858-016-0795-7
[27] V. Vescio, D. Ross, and A. Adams, "A review of research on professional learning communities: What do we know?," *Educational Researcher*, vol. 37, no. 3, pp. 80-89, 2008. https://doi.org/10.1016/j.tate.2007.01.004