

Atomistic simulation of two-dimensional 1T-MnX₂ sheets under uniaxial tension

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Abstract: We investigate the mechanical properties of two-dimensional 1T-MnX₂ materials through the molecular dynamics finite element method with the Stillinger-Weber potential. The two-dimensional Young's modulus, Poisson's coefficient, maximum stress, and strain of these materials are examined for four 1T-MnX₂ sheets. The effects of the armchair and zigzag directions on the mechanical properties under uniaxial tension are considered and discussed. Under uniaxial tension, we have determined the main mechanical properties. For 1T-MnO₂, the maximum stress (σ_t) was 16.794 N/m in the armchair direction, and the maximum elastic modulus (E_t) was 154.96 N/m in the zigzag direction. For 1T-MnTe₂, the maximum Poisson's ratio (ν) observed was 0.181 in the armchair direction. These materials are considered approximately isotropic and are characterized by brittle fracture. Simulation results will help to design and use two-dimensional 1T-MnX₂ sheet-based nanocomposites and nanodevices.

Keywords: 1T structures, 2d-materials, Atomistic simulation, Stillinger-weber. Uniaxial tension.

1. Introduction

This study focuses on determining the mechanical properties of two-dimensional materials that are compounds of transition metal Mn with non-metallic elements existing in the form of hexagonal structure 1T-MnX₂ [1] where X is a non-metallic element. This polymorph can exist in nature or be created in the laboratory [1]. The structure of the 1T-MnX₂ material is shown in Figure. 1, through the orthogonal projection and the axial projection of a basic cell with the corresponding numbered atoms. Each basic cell (the rectangle drawn with dashed lines in Figure. 1) with side size a (Å) contains 9 atoms including 3 Mn atoms (red) on a plane, each Mn atom is bonded to 6 non-metallic atoms X (blue) distributed on two planes symmetrical to the plane containing Mn, the distance between the plane containing Mn and X is h ; meanwhile, an X atom is bonded to 3 surrounding Mn atoms; X is one of the following elements: Oxygen (O), sulfur (S), selenium (Se), tellurium (Te). The material structure is more clearly shown through the material parameters including: the bond distance between two Mn and X atoms is $d_{\text{Mn-X}}$; the bond angle between X-Mn-X atoms is the angle θ_{MnXX} ; Mn-X-Mn is the angle θ_{XMnMn} (see Table 1).

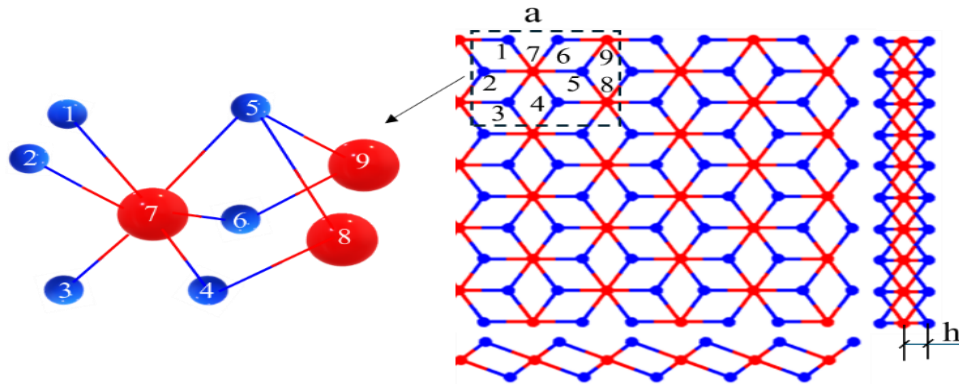


Figure 1. Schematic illustration of: 1T-MnX₂ structures (X= O, S, Se and Te are non-metallic elements).

Table 1. Material network parameters of 1T-MnX₂ materials.

No.	Materials	Lattice constant, a (Å)	d_{Mn-X} (Å)	θ_{MnXX} (°)	θ_{XMnMn} (°)
1	1T-MnO ₂	2.82	1.88	97.181	97.181
2	1T-MnS ₂	3.12	2.27	86.822	86.822
3	1T-MnSe ₂	3.27	2.39	86.330	86.330
4	1T-MnTe ₂	3.54	2.59	86.219	86.219

Source: Jiang and Zhou [1].

Various the two-dimensional (2D) materials are existed in many different structural forms such as Pucked structure Lê Minh, et al. [2] planar hexagonal graphene Geim [3], low-bucked silicene, BN and AlN [4], 1H and 1T structures [1]. These materials exhibit exceptional properties, including high electrical and thermal conductivity, and remarkable mechanical strength. They exhibit good mechanical properties with high elastic modulus of ~ 0.5 -1 TPa and tensile strength of ~ 61 GPa [5]. They possess distinguishable chemical and thermal stability with high oxidation resistance up to 900° C in air [6], wide band-gaps independent of tube structures [7, 8]. Excellent thermal conductivity [9]. They are also an effective violet and ultra-violet light emission material [10, 11]. Interestingly, their mechanical behavior at the nanoscale can differ significantly from macroscopic observations, showcasing phenomena like a negative Poisson's ratio [2, 12]. And extreme hardness (e.g., graphene being harder than diamond) [13]. Furthermore, their bandgap can be tailored through the application of mechanical strain [14-16] or external electric fields, without requiring any chemical alterations [16-18]. Their versatility has led to their incorporation into electronic devices for manufacturing nanometer-sized transistors [1, 3, 16]. electronic storage devices [19, 20]. And applications within the energy [21, 22]. And medical fields [23, 24]. Such applications require a deep comprehension of the mechanical properties and performance of 1T-MnX₂ materials under various loading conditions. Characterizing the mechanical properties of nanoscale materials presents significant experimental challenges due to difficulties in manipulation, high costs, and often infeasibility. Consequently, researchers often rely on computational methods like experimental simulation (e.g., atomic finite element method (AFEM) [25-27]. Molecular dynamics (MD) [28]. First-principles [29, 30]. And ab initio calculations [31] to predict these properties.

The Mechanical properties of 1T-MnX₂ materials seem still unexplored by MDFEM method. It should be noted that the Mechanical properties of these materials has been investigated by molecular dynamics (MD) method see e. g. Jiang and Zhou [1] and references therein. The present work investigates through molecular dynamics finite element method (MDFEM) the mechanical properties of two-dimensional 1T-MnX₂ materials under uniaxial tension. The effects of the armchair and zigzag directions to the mechanical properties of these materials under uniaxial tension are studied and discussed.

2. Framework for Analysis

While density functional theory (DFT) calculations and molecular dynamics (MD) simulations are time-consuming, molecular dynamic finite element methods (MDFEM), sometime known as atomic-scale finite element methods or atomistic finite element methods, have been developed to analyze nanostructured materials in a computationally efficient way, see e. g. [25, 28]. To achieve the atomic positions of the BN-NT under specific boundary conditions, molecular dynamic finite element method (MDFEM) is here adopted. In MDFEM, atoms and atomic displacements are considered as nodes and translational degrees of freedom (nodal displacements), respectively. Both first and second derivatives of system energy are used in the energy minimization computation, hence it is faster than the standard conjugate gradient method which uses only the first order derivative of system energy as discussed in [32]. The stiffness matrices of these elements are established based upon interatomic potentials. Similar to conventional finite element method, global stiffness matrix is assembled from element stiffness matrices. Hence, relations between atomic displacement and force can be derived by solving a system of equations.

For each 1T-MnX₂ sheet, the atomic interaction potential is determined through the parameters of the Stillinger-Weber potential function including the direct binding energy of two adjacent atoms (E_r , eV) and the binding energy of three adjacent atoms (E_θ , eV) through the following equations:

$$E = E_r + E_\theta \quad (1)$$

$$E_r = \sum_{i=1}^m V_2 \quad (2)$$

$$E_\theta = \sum_{i=1}^n V_3 \quad (3)$$

$$V_2 = A e^{\left[\rho / (r_{ij} - r_{\max ij})\right]} \left(B / r_{ij}^4 - 1 \right) \quad (4)$$

$$V_3 = K e^{\left[\rho_{ij} / (r_{ij} - r_{\max ij}) + \rho_{ik} / (r_{ik} - r_{\max ik})\right]} \left(\cos(\theta_{ijk}) - \cos(\theta_0) \right)^2 \quad (5)$$

In which, E is total atomic bond energy; E_θ (eV) is total angular bond energy of 3 atoms on the entire membrane; E_r (eV) is total linear bond energy between two atoms of the membrane; V_2 (eV) is linear bond energy of two adjacent atoms; V_3 (eV) is angular bond energy of 3 adjacent atoms; m and n are number of linear bonds and angular bonds in a calculation model; A (eV) and K (eV) are material coefficients; ρ (Å), B (Å⁴), ρ_{ij} (Å), ρ_{ik} (Å), θ^0 (degrees) are geometric parameters of the material; r_{ij} (Å), r_{ik} (Å) are bond length between two atoms i, j and k respectively; θ_{ijk} (degrees) is bond angle between three atoms i, j, k (where i is the middle atom) (Figure. 2). These parameters are summarized in Tables 2 and 3.

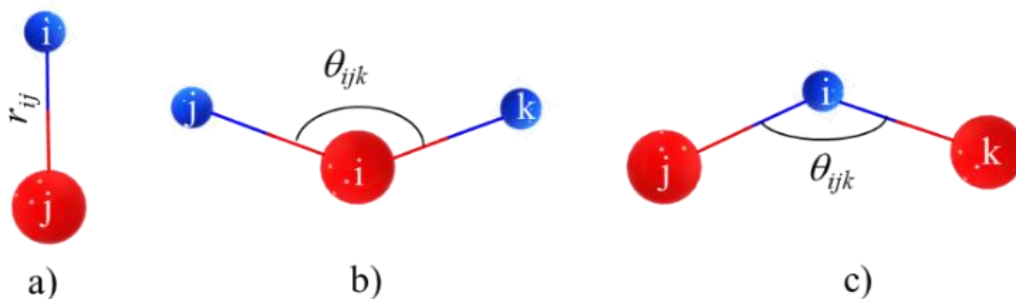


Figure 2.

Element model when using Stillinger-Weber potential function: a) Straight bond between two atoms; b,c) Angular bond between 3 atoms.

Table 2.Stillinger-Weber potential parameters for the straight bond of material 1T-MnX₂ [1].

No.	Materials	Bond	A , eV	ρ , Å	B , Å ⁴	r_{\min} , Å	r_{\max} , Å
1	1T-MnO ₂	Mn-O	9.675	1.212	6.246	0.0	2.635
2	1T-MnS ₂	Mn-S	3.127	1.111	13.276	0.0	3.064
3	1T-MnSe ₂	Mn-Se	3.422	1.153	16.314	0.0	3.220
4	1T-MnTe ₂	Mn-Te	4.007	1.246	22.499	0.0	3.488

Table 3.Stillinger-weber potential parameters for angle bond of the 1T-MnX₂ [1].

No.	Materials	Angular bond	K , eV	θ , (°)	ρ , Å	ρ , Å	$r_{\min 12}$, Å	$r_{\max 12}$, Å	$r_{\min 13}$, Å	$r_{\max 13}$, Å	$r_{\min 23}$, Å	$r_{\max 23}$, Å
1	1T-MnO ₂	θ_{MnOO}	60.755	97.181	1.212	1.212	0.0	2.635	0.0	2.635	0.0	3.852
		θ_{MnMn}	60.755	97.181	1.212	1.212	0.0	2.635	0.0	2.635	0.0	3.852
2	1T-MnS ₂	θ_{MnSS}	19.765	86.822	1.111	1.111	0.0	3.064	0.0	3.064	0.0	4.262
		θ_{MnMn}	19.765	86.822	1.111	1.111	0.0	3.064	0.0	3.064	0.0	4.262
3	1T-MnSe ₂	θ_{MnSeSe}	19.390	86.330	1.153	1.153	0.0	3.220	0.0	3.220	0.0	4.467
		θ_{MnMn}	19.390	86.330	1.153	1.153	0.0	3.220	0.0	3.220	0.0	4.467
4	1T-MnTe ₂	θ_{MnTeTe}	19.307	86.219	1.246	1.246	0.0	3.488	0.0	3.488	0.0	4.836
		θ_{MnMn}	19.307	86.219	1.246	1.246	0.0	3.488	0.0	3.488	0.0	4.836

For each 1T-MnX₂ membrane, the total number of atoms is N ; \mathbf{X}_i and \mathbf{u}_i are the initial coordinates and displacement of the i -th atom; then, the coordinates of the atom after deformation are $\mathbf{x}_i = \mathbf{X}_i + \mathbf{u}_i$. The atomic interaction potential of the entire membrane calculated by formula (1) is an equation that depends on the coordinates of each atom on the membrane as follows:

$$E = E(\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N) \quad (6)$$

On the other hand, when the membrane is subjected to an external force f_i on the i -th atom (considered as nodes), the atoms in the membrane have displacements u_i correspondingly, the potential energy of the applied external force is calculated as follows

$$E_{ext} = \sum_{i=1}^N f_i u_i \quad (7)$$

The total potential energy of the membrane is

$$\Pi = E - E_{ext} \quad (8)$$

The membrane is in equilibrium when the total potential energy reaches its minimum value according to the principle of minimum potential energy. Therefore, the first derivative of the total potential energy will be zero as shown in Eq. (9)

$$\frac{\partial \Pi}{\partial \mathbf{u}_i} = 0; \quad i = 1 \div N$$

(9)

Solving the system of equations (9), the displacement of all atoms in the membrane is determined. In the experimental simulation using the atomic finite element method, solving the system of equations (9) using the Newton-Raphson iteration method, the use of this method has been clearly shown in [25, 27, 28], then the equation in the form of finite elements is

$$\mathbf{K}^{(k)} \cdot \mathbf{u}^{(k)} = \mathbf{F}^{(k)} \quad (10)$$

With

$$\mathbf{K}_{ij}^k = \frac{\partial^2 \Pi^{(k)}}{\partial \mathbf{u}_i \partial \mathbf{u}_j}; \mathbf{F}_i^{(k)} = -\frac{\partial \Pi^{(k)}}{\partial \mathbf{u}_i} = \mathbf{f}_i - \frac{\partial E^{(k)}}{\partial \mathbf{u}_i}; \quad (11)$$

Equations (10) and (11) are the basic equations of the finite element. With $\mathbf{K}^{(k)}$ being the global stiffness matrix, $\mathbf{u}^{(k)}$ being the nodal displacement vector and $\mathbf{F}^{(k)}$ being the nodal force vector. If each atom is considered as a node, the displacement of the atom is the displacement of the node. Due to the structure of the orthogonal hexagonal membrane, two types of elements are formed on the membrane, i.e., 3-node elements (Valence) are molecules on the boundary of the membrane, each element has 3 bonded atoms, 4-node elements (Improper) are elements inside the membrane, and each element is bonded by 4 atoms (Figure. 3). Since each atom can move in 3 directions, formula (11) is used to calculate the stiffness matrix of each element with dimensions $[\mathbf{K}_{Val}]_{9 \times 9}^{(e)}$; $[\mathbf{K}_{Imp}]_{12 \times 12}^{(e)}$, corresponding to 3-node and 4-node elements. From the element stiffness matrix, calculate the global stiffness matrix $[\mathbf{K}]_{3N \times 3N}^{(k)}$ which is the composite matrix of the element stiffness matrices over the entire membrane. The displacement vectors and force vectors have dimensions corresponding to the global stiffness matrix.

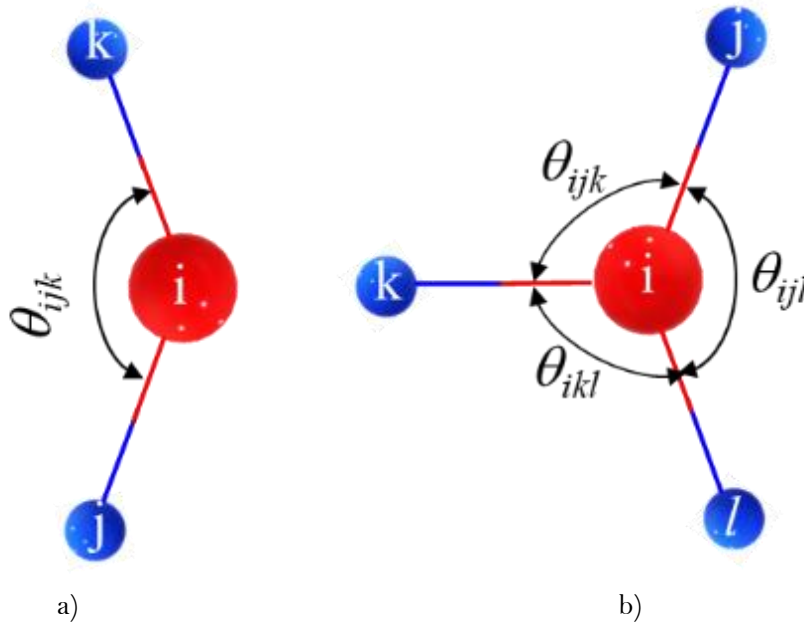


Figure 3. Element model using the atomic finite element method: a) the Valence element is formed by 3 adjacent atoms; b) the Improver element is formed by 4 adjacent atoms.

The system of equations (10) is solved by using the Newton–Raphson iteration method with displacement boundary conditions i.e., atoms on the tensile boundary have displacement equal to $u(0)$, and atoms on the retaining boundary (Figure. 4) have no displacement in the tensile direction. This is the experimental simulation method using molecular dynamic finite element method (MDFEM).

The The experimental model of the sheet tensile test is shown in Figure. 4 through the displacement method.

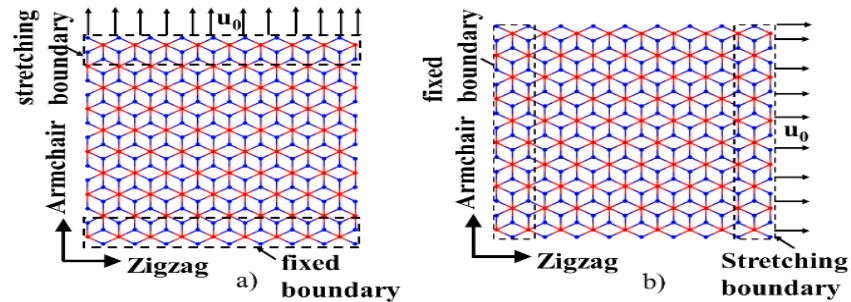


Figure 4.
The sheet with displacement boundary conditions under uniaxial tension: a) the armchair direction; b) the zigzag direction.

In the simulation, pull the rectangular sheet with approximately equal side dimensions (the rectangle is considered to be a square); each sheet has 4200 atoms; each stretching step causes the atoms on the stretching edge to move with an increment of $\delta\varepsilon = 0.001$, repeating until the sheet has a broken bond (the sheet is destroyed as shown in Figure. 6). At each step, equation (10) is solved to determine the displacement and nodal force of all atoms on the sheet at this step. The position of atoms in the next steps is determined as follows:

$$x_{(k+1)} = x_{(k)} + u_{(k)} \quad (12)$$

The iterative process continues until , with a given error, at which point the sheet is considered to have failed. The image of the failed sheet is shown in Figure. 6. The data of the entire experimental process, of all steps, is synthesized to determine the mechanical parameters that need to be calculated.

The tensile results of the 1T-MnX₂ materials are shown through the stress-strain relationship graph; the two-dimensional elastic modulus is determined by linearizing their relationship with strain in the range from 0 to 0.1; the Poisson's ratio is determined based on the ratio of transverse strain and axial strain; $\nu = -\varepsilon_y/\varepsilon_x$; the two-dimensional tensile stress and tensile strain occur at the end of the tensile program. The stress-strain relationship when stretching the intact sheet of the four materials is shown in Figure. 5.

3. Results and Discussion

Figure 5 shows the stress-strain curves of the 1T-MnX₂ sheets in armchair and zigzag directions calculated by molecular dynamic finite element method (MDFEM). It can be seen from Figure 5 that the stress increases monotonously with an increase of the strain up to a maximum value and then drops suddenly. Therefore, the sheets exhibit a brittle fracture mechanism. Then, the maximum stress and strain at the maximum stress location replace the critical stress and critical strain, respectively.

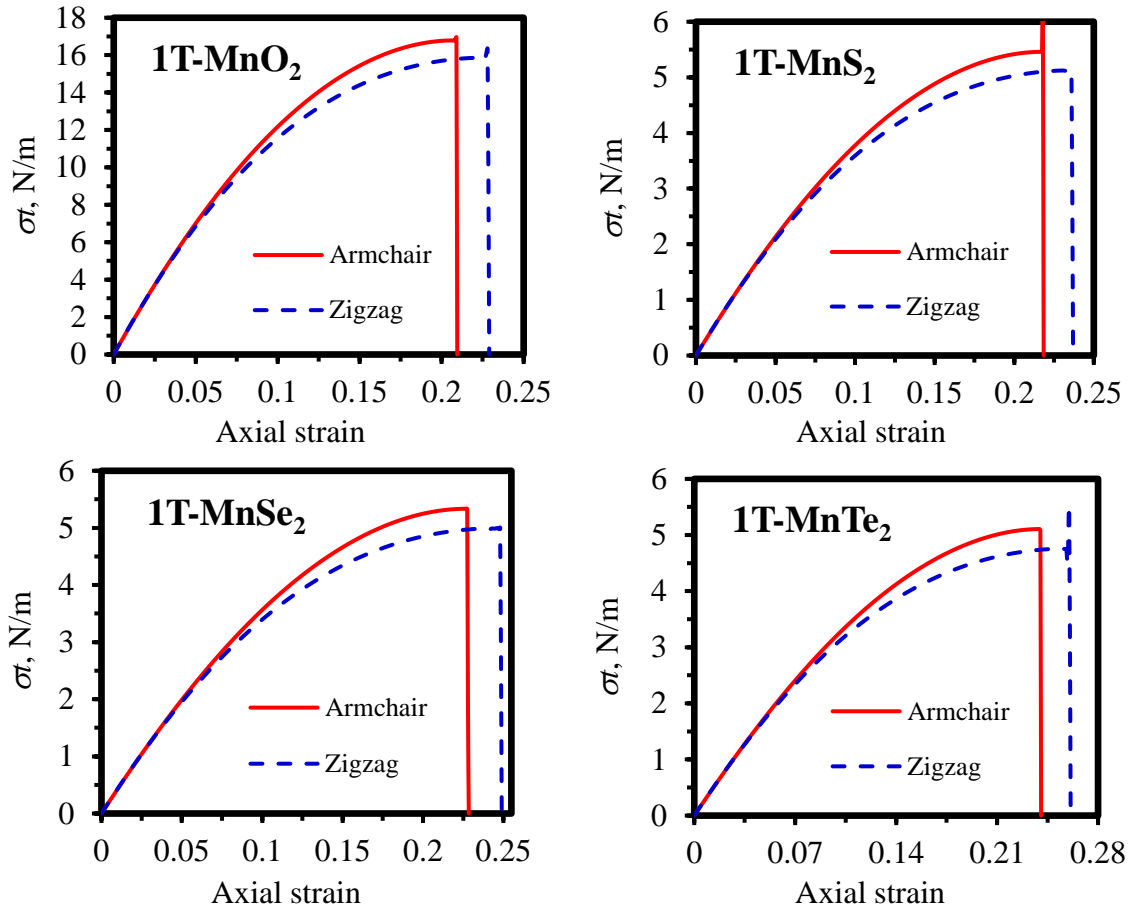


Figure 5. The stress-strain curves of 1T-MnX₂ material sheets under uniaxial tension according to armchair and zigzag directions.

Table 4 shows the elastic modulus, maximum stress and maximum strain of 1T-MnX₂ material sheets according to armchair and zigzag directions calculated by MDFEM method. The results showed that maximum stress: $\sigma_t=16.794\text{N/m}$ under uniaxial tension 1T-MnO₂ in the armchair direction with strain $\varepsilon=0.208$; minimum stress: $\sigma_t=4.752\text{ N/m}$ under uniaxial tension 1T-MnTe₂ in the armchair direction with strain $\varepsilon=0.256$; maximum poisson's ratio: $\nu=0.181$ under uniaxial tension 1T-MnTe₂ in the armchair direction; minimum poisson's ratio: $\nu=0.121$ under uniaxial tension 1T-MnO₂ in the zigzag direction; maximum elastic modulus: $E_t=154.96\text{ N/m}$ under uniaxial tension 1T-MnO₂ in the zigzag direction; minimum elastic modulus: $E_t=37.385\text{ N/m}$ under uniaxial tension 1T-MnTe₂ in the armchair direction. These results showed that two-dimensional elastic modulus and poisson's ratio are approximately equal value when stretched in two directions armchair and zigzag. They are considered as approximately isotropic materials. In Table 4, these results were compared with the research results by Jiang under the same experimental simulation conditions but other method [1]. The comparison results showed that the errors were good agree.

Table 4.
The mechanical parameters of four 1T-MnX₂ materials by MDFEM method.

No.	Materials	Directions	Elastic modulus E_t , N/m	Poisson's ratio	Maximum stress σ_t , N/m	Tensile strain	Evaluations
1	1T-MnO ₂	AC	153.900	0.123	16.794	0.208	Our results
			156.300	0.120	16.800	0.210	by MD at 1 ^o K [1]
			-1.536	2.250	-0.035	-1.190	error (%) with [1]
		ZZ	154.960	0.121	15.883	0.224	Our results
			155.400	0.120	16.200	0.240	by MD at 1 ^o K [1]
			-0.283	1.083	-1.957	-6.667	error (%) with [1]
2	1T-MnS ₂	AC	46.159	0.147	5.468	0.218	Our results
			47.100	0.150	5.500	0.210	by MD at 1 ^o K [1]
			-1.998	-1.800	-0.590	3.571	error (%) with [1]
		ZZ	46.500	0.145	5.143	0.235	Our results
			46.800	0.150	5.300	0.250	by MD at 1 ^o K [1]
			-0.641	-3.400	-2.959	-6.000	error (%) with [1]
3	1T-MnSe ₂	AC	42.726	0.162	5.338	0.227	Our results
			43.200	0.170	5.400	0.220	by MD at 1 ^o K [1]
			-1.097	-4.765	-1.147	3.182	error (%) with [1]
		ZZ	43.083	0.160	4.996	0.246	Our results
			42.900	0.170	5.200	0.260	by MD at 1 ^o K [1]
			0.427	-6.059	-3.916	-5.385	error (%) with [1]
4	1T-MnTe ₂	AC	37.835	0.181	5.107	0.239	Our results
			38.500	0.190	5.200	0.240	by MD at 1 ^o K [1]
			-1.727	-4.947	-1.792	-0.417	error (%) with [1]
		ZZ	38.206	0.178	4.752	0.256	Our results
			38.400	0.190	5.000	0.280	by MD at 1 ^o K [1]
			-0.505	-6.105	-4.963	-8.571	error (%) with [1]

Figure 6 shows the destruction of the 1T-MnSe₂ sheet when stretching in the armchair direction with a strain of $\varepsilon = 0.227$; when stretching in the zigzag direction, at the step $\varepsilon = 0.246$, many bonds were broken and the sheet was destroyed.

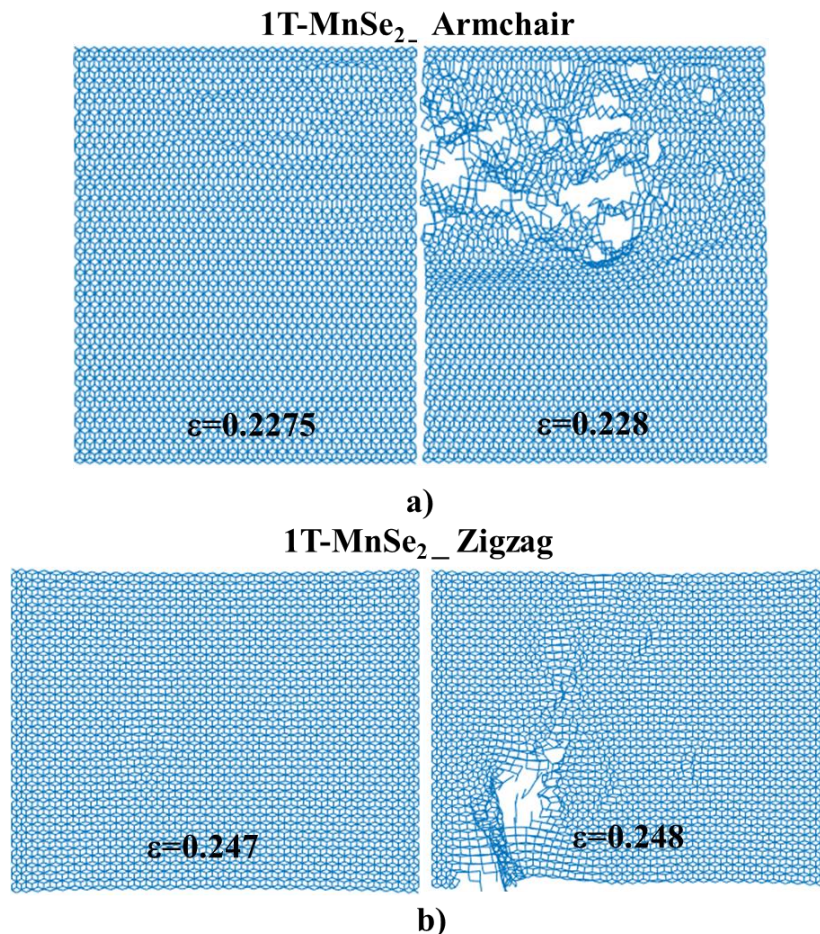


Figure 6.
Destroyed shapes of 1T-MnSe₂ sheets: a) uniaxial tension according to armchair; b) uniaxial tension according to zigzag.

4. Conclusions

We present the simulation results of the mechanical properties of 1T-MnX₂ nanosheets under uniaxial tension with the use of MDFEM with Stillinger-Weber potential. We have found maximum stress: $\sigma_t=16.794\text{N/m}$ under uniaxial tension 1T-MnO₂ in the armchair direction with strain $\varepsilon=0.208$; minimum stress: $\sigma_t=4.752\text{ N/m}$ under uniaxial tension 1T-MnTe₂ in the armchair direction with strain $\varepsilon=0.256$; maximum poisson's ratio: $\nu=0.181$ under uniaxial tension 1T-MnTe₂ in the armchair direction; minimum poisson's ratio: $\nu=0.121$ under uniaxial tension 1T-MnO₂ in the zigzag direction; maximum elastic modulus: $E_t=154.96\text{ N/m}$ under uniaxial tension 1T-MnO₂ in the zigzag direction; minimum elastic modulus: $E_t=37.385\text{ N/m}$ under uniaxial tension 1T-MnTe₂ in the armchair direction. These materials generally behave as approximately isotropic substances and exhibit brittle fracture characteristics. These results will help to design and use two-dimensional 1T-MnX₂ sheets based nanocomposites and nanodevices.

Transparency:

The authors confirm that the manuscript is an honest, accurate, and transparent account of the study; that no vital features of the study have been omitted; and that any discrepancies from the study as planned have been explained. This study followed all ethical practices during writing.

Authors' Contributions:

The idea of the paper was proposed by Nguyen Van Trang and Nguyen Huu Tu. The simulation and the calculation were conducted by Nguyen Van Thanh and Dang Van Manh. The manuscript was written by Nguyen Van Trang with support from Nguyen Huu Tu. All authors have read and agreed to the manuscript.

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