

Investigating mathematical proficiency of elementary school students: A foundation for effective learning models

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Abstract: Mathematical proficiency is a critical mathematical competency and a primary goal in mathematics education. However, previous studies have often examined each aspect of mathematical proficiency in isolation, without comprehensively exploring the interrelationships among these aspects. This study investigates the relationships among the five aspects of mathematical proficiency, namely conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition in elementary school students. Using a descriptive design with a correlational approach, data were collected from 220 fifth-grade students in elementary schools. Mathematical proficiency was measured using a combination of tests and questionnaires, with the data analyzed through descriptive statistics, correlation analysis, and factor analysis. The results reveal significant relationships among the aspects of mathematical proficiency. Factor analysis grouped conceptual understanding and procedural fluency into one factor, while strategic competence and adaptive reasoning formed another factor. Productive disposition was found to be an independent aspect, not directly related to the other factors. These findings provide a foundation for developing effective instructional models that integrate all aspects of students' mathematical proficiency.

Keywords: Elementary education, Factor analysis, Instructional model, Mathematical proficiency.

1. Introduction

The importance of mathematics has grown significantly as the world continues to evolve dynamically and progressively due to globalization and digitalization. This evolution presents new challenges for mathematics education, particularly in preparing students to adapt and compete in a digital society [1]. In the digital era, mathematics is both pervasive and often invisible. The role of mathematics progresses alongside technological advancement, as it is central to technological functions [2]. This situation has led to the perception that mathematics is exclusive to select individuals rather than accessible to everyone [3].

A paradox exists in mathematics education, particularly at the elementary level. On one hand, mathematics is both valued and proven to be important for students, yet they often do not enjoy learning it. A synthesis of research findings indicates that students experience significant anxiety and frustration when studying mathematics, with some even describing it as akin to facing a lion [4-7]. Investigations into research findings further reveal that elementary students generally have a weak understanding of mathematical concepts and exhibit numerous misconceptions [8-10]. This weak understanding and frequent misconceptions are influenced by teachers' methods of presenting concepts

in the classroom. Studies even show that misconceptions often arise from teachers replicating the thinking processes and instructional sequences found in textbooks without modification [11, 12].

All mathematical operations today can be performed by computers. This fact tends to reinforce the perception of a gap between what happens in the real world and what is taught in schools. However, this does not imply that learning mathematics is no longer necessary; rather, it suggests that what is taught in mathematics needs to change [1]. Mathematics education should no longer equip students solely with competencies that can be handled by computers. As Polya once stated: “The first and foremost objective in mathematics education is to teach students to think” [13]. Therefore, the paradigm of mathematics education must shift to emphasize equipping students with mathematical proficiency, which is the primary goal of mathematics education [3].

Mathematical proficiency consists of five main components including Conceptual Understanding, Procedural Fluency, Strategic Competence, Adaptive Reasoning, and Productive Disposition [3]. Conceptual Understanding encompasses the ability to grasp mathematical concepts and explain the relationships among them. Procedural Fluency refers to the ability to apply mathematical procedures accurately and efficiently. Strategic Competence involves students' ability to devise strategies for solving mathematical problems. Adaptive Reasoning includes the capacity for logical thinking, evaluating solutions, and making adjustments to the strategies used. Productive Disposition reflects a positive attitude toward mathematics, including the belief that learning mathematics is valuable and important. These five aspects should be developed holistically and complementarily, as a weakness in one area can hinder progress in others [14]. Therefore, effective mathematics instruction must consider the balanced development of each aspect of mathematical proficiency in an integrated manner [15].

The development of mathematical proficiency is crucial as it forms the foundation for students' ability to face academic challenges at subsequent levels and in everyday life [16, 17]. Mathematical proficiency not only assists students in solving mathematical problems but also cultivates logical, critical, and creative thinking skills, which are essential in the digital era and the context of Industry 4.0 [18]. These skills enable students to actively engage in contextual problem-solving and make decisions relevant to real-life situations. Furthermore, a balanced development of mathematical proficiency equips students to face challenges with confidence and apply their mathematical skills across various life contexts.

Although numerous studies have discussed aspects of mathematical proficiency separately, there remains a gap in research that comprehensively investigates the relationships among these aspects, particularly in elementary school students. Most research has focused on cognitive aspects, such as Conceptual Understanding and Procedural Fluency, while aspects like Strategic Competence, Adaptive Reasoning, and Productive Disposition have often been underemphasized [16, 19]. This study aims to address this gap by examining the interrelationships among all aspects of mathematical proficiency, with the expectation of providing new insights into how these aspects mutually support and contribute to mathematics learning at the elementary level. This research is also relevant to the current educational context, where a more holistic approach is necessary to enable students to fully develop mathematical proficiency.

This study aims to investigate the interrelationships among the components of mathematical proficiency in elementary school students, including Conceptual Understanding, Procedural Fluency, Strategic Competence, Adaptive Reasoning, and Productive Disposition. This objective is expected to provide a more comprehensive understanding of how these aspects function in an integrated manner to support students' mathematical proficiency. The findings of this study are also anticipated to serve as a foundation for designing more effective and sustainable instructional strategies, reinforcing each aspect of mathematical proficiency in a balanced way.

2. Research Methodology

2.1. Research Design

This study is a descriptive research with a correlational approach. Descriptive research allows researchers to observe, record, and analyze phenomena as they occur without intervention. A correlational approach within descriptive research is used to identify patterns of relationships between two or more variables that are neither manipulated nor intervened upon [20]. This study aims to investigate the relationships among aspects of mathematical proficiency in elementary school students, including Conceptual Understanding, Procedural Fluency, Strategic Competence, Adaptive Reasoning, and Productive Disposition.

2.2. Participants and Data Collection

This study was conducted in elementary schools located in Badung Regency, Bali, Indonesia, involving a sample of 220 fifth-grade students. A cross-sectional sampling technique was used, where data were collected at a single point in time to provide a snapshot of the population or phenomenon at the time of data collection [20]. The selection of participants was focused on schools implementing the Kurikulum Merdeka. This approach was employed to ensure that the data obtained reflect actual conditions and are relevant to the current educational context.

Data collection in this study was conducted using two primary instruments, namely a test and a questionnaire. Together, these instruments form a comprehensive assessment of students' mathematical proficiency. The test covers four main aspects including Conceptual Understanding, Procedural Fluency, Strategic Competence, and Adaptive Reasoning. Each aspect comprises 3 essay questions, totaling 12 questions, with a scoring range of 0 to 3 for each item. This test assesses students' cognitive abilities in understanding concepts, executing procedures, thinking strategically, and applying adaptive reasoning. Meanwhile, the Productive Disposition aspect is measured using a Likert-scale questionnaire with a scoring range of 1 to 5. This questionnaire consists of 25 statements assessing students' attitudes and motivation toward learning mathematics, which is an essential aspect of mathematical proficiency.

2.3. Data Analysis

The data in this study were analyzed using several statistical techniques, including descriptive analysis, correlation analysis, and factor analysis. Descriptive analysis was used to describe the distribution of data for each aspect of mathematical proficiency, namely Conceptual Understanding, Procedural Fluency, Strategic Competence, Adaptive Reasoning, and Productive Disposition. This analysis includes calculations of the mean, standard deviation, minimum, and maximum values for each component. The results of the descriptive analysis provide an initial overview of students' abilities and attitudes toward mathematics.

Correlation analysis was conducted to determine the strength and direction of relationships among the components of mathematical proficiency. Pearson correlation was used to identify the associations between components, such as Conceptual Understanding and Procedural Fluency, or Strategic Competence and Adaptive Reasoning. The hypothesis tested was that there are significant relationships among the aspects of mathematical proficiency. A significance level of $\alpha = 0.05$ was applied.

Factor analysis was used to identify the latent structure of the components of mathematical proficiency. This analysis aimed to discover patterns of relationships and group these components into the primary dimensions of mathematical proficiency. Before conducting factor analysis, prerequisite tests were applied to ensure that the data met the criteria for factor analysis. The first prerequisite test was Bartlett's Test of Sphericity, used to examine whether there is sufficient correlation among variables. Additionally, the Kaiser-Meyer-Olkin (KMO) Measure of Sampling Adequacy was used to assess sample adequacy. The KMO value must be greater than 0.5 for the data to proceed with factor analysis.

3. Results

This study provides a description of elementary school students' mathematical proficiency and explains the relationships among its aspects. The description of mathematical proficiency was conducted by calculating the mean for each aspect as well as the overall mathematical proficiency. Meanwhile, the relationships among each aspect of mathematical proficiency were examined using correlation analysis, followed by factor analysis. The results of the statistical analysis are presented in Tables 1, 2, 3, 4, and 5.

Table 1.
Descriptive Statistics of Elementary School Students' Mathematical Proficiency

Descriptives						
	Conceptual Understanding	Procedural Fluency	Strategic Competence	Adaptive Reasoning	Productive Disposition	Mathematical Proficiency
N	220	220	220	220	220	220
Missing	0	0	0	0	0	0
Mean	3.32	3.46	2.04	2.68	89.8	62.9
Median	3.00	3.00	2.00	3.00	90.0	63.4
Standard deviation	1.98	2.41	1.90	2.29	14.1	10.6
Minimum	0	0	0	0	36	29.8
Maximum	9	9	8	9	128	90.7

Table 1 shows an average mathematical proficiency score of 62.90 for students, which falls within the high category. In more detail, the average scores for each aspect indicate that conceptual understanding and procedural fluency are in the moderate category. Meanwhile, strategic competence and adaptive reasoning fall into the low category. The productive disposition aspect, however, is in the high category. This suggests that the high overall mathematical proficiency score is primarily contributed by a high level of productive disposition, even though other aspects remain in the moderate or low categories.

Table 2.
Pearson Correlation Analysis of Mathematical Proficiency Aspects.

Correlation Matrix						
		Conceptual Understanding	Procedural Fluency	Strategic Competence	Adaptive Reasoning	Productive Disposition
Conceptual Understanding	Pearson's r	—				
	df	—				
	p-value	—				
Procedural Fluency	Pearson's r	0.571	—			
	df	218	—			
	p-value	<.001	—			
Strategic Competence	Pearson's r	0.443	0.486	—		
	df	218	218	—		
	p-value	<.001	<.001	—		
Adaptive Reasoning	Pearson's r	0.363	0.440	0.555	—	
	df	218	218	218	—	
	p-value	<.001	<.001	<.001	—	
Productive Disposition	Pearson's r	0.207	0.199	0.209	0.149	—
	df	218	218	218	218	—
	p-value	0.002	0.003	0.002	0.027	—

Table 2 presents the results of the Pearson correlation analysis. At a significance level of $\alpha = 0.05$, the results indicate that there are significant relationships among the aspects of mathematical proficiency. In detail, the analysis reveals a significant correlation between Conceptual Understanding and Procedural Fluency ($r = 0.571$, $p < 0.05$), suggesting that students with better conceptual

understanding also tend to be more proficient in performing mathematical procedures. Additionally, the correlation between Strategic Competence and Adaptive Reasoning ($r = 0.555$, $p < 0.05$) highlights that students who are able to think strategically also tend to have strong adaptive reasoning skills.

The relationship between Productive Disposition and other aspects of mathematical proficiency shows lower correlations compared to the relationships among the other aspects. The highest correlation was found between Productive Disposition and Strategic Competence ($r = 0.209$, $p < 0.05$). This finding suggests that a positive attitude contributes to supporting strategic thinking skills, although its influence is not as strong as the relationships among the other aspects of mathematical proficiency.

Factor analysis was then conducted to more clearly identify the relationships among the five aspects of mathematical proficiency. Prior to conducting factor analysis, prerequisite tests were performed to ensure that the data met the necessary criteria. Table 3 and 4 presents the results of the prerequisite tests, which include Bartlett's Test of Sphericity and the Kaiser-Meyer-Olkin (KMO) Measure of Sampling Adequacy. Table 3 shows that Bartlett's Test yielded a p-value of < 0.001 , indicating sufficiently strong and significant correlations among the variables, thus justifying the use of factor analysis. Table 4 displays a Kaiser-Meyer-Olkin (KMO) value of 0.762, which exceeds the threshold of 0.5. This indicates that the sample size is adequate for factor analysis.

Table 3.
Results of Bartlett's Test of Sphericity.

Bartlett's Test of Sphericity		
χ^2	df	p
263	10	<.001

Table 4.
Results of KMO Measure of Sampling Adequacy.

KMO Measure of Sampling Adequacy	
	MSA
Overall	0.762
Conceptual Understanding	0.757
Procedural Fluency	0.755
Strategic Competence	0.758
Adaptive Reasoning	0.760
Productive Disposition	0.880

Table 5.
Factor Loadings for the Five Aspects of Mathematical Proficiency.

Factor Loadings			
	Factor		Uniqueness
	1	2	
Conceptual Understanding		0.847	0.323
Procedural Fluency		0.519	0.472
Strategic Competence	0.731		0.400
Adaptive Reasoning	0.750		0.480
Productive Disposition			0.926

Note: 'Minimum residual' extraction method was used in combination with a 'oblimin' rotation.

Based on the results of the factor analysis shown in Table 5, two main factors emerged, each reflecting a different dimension of mathematical proficiency. The first factor, labeled Understanding and Procedural, consists of two components: Conceptual Understanding with a loading factor of 0.847 and Procedural Fluency with a value of 0.519. These results indicate that students with strong conceptual

understanding tend to have higher procedural skills. A deep understanding of concepts enhances students' ability to perform mathematical procedures more accurately and efficiently.

The second factor, labeled Strategic and Adaptive, consists of two components: Strategic Competence with a loading factor of 0.731 and Adaptive Reasoning with a value of 0.750. These results emphasize the importance of the relationship between strategic thinking and logical reasoning in mathematical problem-solving. Both skills support each other and contribute to the development of students' mathematical proficiency in more complex contexts.

Meanwhile, Productive Disposition has a uniqueness value of 0.926, indicating that this variable is more independent and not closely related to the other mathematical proficiency factors. This suggests that developing a positive attitude toward mathematics requires a distinct intervention, separate from the development of mathematical skills.

4. Discussion

The descriptive analysis reveals variations in students' abilities across each aspect of mathematical proficiency, including Conceptual Understanding, Procedural Fluency, Strategic Competence, Adaptive Reasoning, and Productive Disposition. Certain aspects, such as Procedural Fluency, recorded higher average scores, indicating that students tend to be more proficient in performing mathematical procedures compared to strategic thinking or adaptive reasoning. However, aspects like Strategic Competence and Adaptive Reasoning showed lower averages, suggesting that students' abilities in strategic and adaptive thinking still need improvement. These data provide insights into areas requiring greater focus in instruction and can aid in designing more effective strategies to strengthen less developed aspects.

The correlation findings between Conceptual Understanding and Procedural Fluency indicate that conceptual understanding plays an important role in developing students' procedural skills. This aligns with previous findings suggesting that enhancing conceptual understanding can facilitate the development of procedural skills, and conversely, practicing procedural skills can iteratively strengthen students' conceptual comprehension [21-23]. The interdependence between these two aspects implies that mathematics instruction should be sequenced, with a focus on conceptual understanding at the outset before students practice procedural skills. An imbalance between these aspects may lead students to memorize procedures without understanding the underlying meaning, or, conversely, to struggle in applying procedures without a deep understanding [24-26].

The findings indicate that Strategic Competence and Adaptive Reasoning have a significant relationship, underscoring the need for developing both strategic thinking and adaptive reasoning together. These two components play a crucial role in solving complex mathematical problems [27, 28]. Strategic Competence enables students to formulate and select effective strategies, while Adaptive Reasoning is required to validate or revise solutions when initial strategies do not succeed [29-31]. These findings have important implications for instructional design, suggesting that students should engage in ongoing problem-solving activities that allow them to develop and test strategies reflectively.

Meanwhile, Productive Disposition was found to be an independent component, not directly related to cognitive aspects, yet still relevant in supporting student engagement in the learning process [28, 32-34]. In the context of instructional design, this finding suggests that specific strategies are needed to foster motivation and a positive attitude toward mathematics. Contextual and problem-based learning approaches can be used to enhance student engagement and cultivate a positive disposition, thereby encouraging the development of other mathematical components more effectively [35, 36].

The factor analysis results indicate that several components of mathematical proficiency are interrelated and can be grouped into simpler dimensions. This analysis helps identify patterns of relationships that highlight connections among cognitive aspects, such as Conceptual Understanding and Procedural Fluency, as well as between Strategic Competence and Adaptive Reasoning [14]. With these factor patterns, we can design more focused instructional strategies. For instance, conceptual understanding should be prioritized at the beginning of instruction, before students are introduced to

more complex procedural skills. Subsequently, strategic competence and adaptive reasoning should be developed through problem-solving tasks that challenge students to think critically and adjust strategies independently. These factor analysis results provide a solid foundation for designing structured instructional phases, ensuring that each component of mathematical proficiency is systematically strengthened.

The findings of this study provide a critical foundation for designing more effective and evidence-based instructional models. Understanding the relationships among the components of mathematical proficiency enables educators to develop structured and sustainable learning strategies [37, 38]. The findings on the connections between Conceptual Understanding and Procedural Fluency, as well as between Strategic Competence and Adaptive Reasoning, suggest that instruction should be sequenced in a progressive and mutually supportive manner [39]. Mathematics instruction should not focus on a single component alone but instead integrate multiple components to cultivate comprehensive mathematical proficiency [40]. Thus, these findings serve as an initial basis for research and the development of instructional models that emphasize a balance between cognitive skills and a positive disposition toward mathematics.

This study provides an initial foundation for developing effective instructional models to enhance students' mathematical proficiency. Future research is recommended to focus on the development, implementation, and evaluation of instructional models based on these findings. The models developed should consider the interconnections among the components of Conceptual Understanding, Procedural Fluency, Strategic Competence, and Adaptive Reasoning, while explicitly integrating the enhancement of Productive Disposition within each phase of instruction. Furthermore, subsequent research should test the effectiveness of these models in various educational contexts, such as schools with different social backgrounds and curricula, to ensure their relevance and sustainability in educational practice. Thus, the instructional models developed will not only be empirically based but also make a tangible contribution to improving students' mathematical proficiency.

5. Conclusion

The descriptive analysis results indicate variations in students' abilities across each aspect of mathematical proficiency, with procedural fluency achieving a higher average score compared to other aspects. The correlation analysis shows that all aspects of mathematical proficiency are significantly interrelated, suggesting that conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition cannot be developed in isolation, as they mutually support one another.

The factor analysis results show that conceptual understanding and procedural fluency are grouped into one factor, while strategic competence and adaptive reasoning form another factor. This indicates a close relationship among aspects within each factor, particularly between procedural skills and conceptual understanding, as well as between strategic thinking and adaptive reasoning. Productive disposition was found to be an independent aspect not directly related to the other factors, emphasizing that a positive attitude toward mathematics remains important as a support for successful learning. These findings provide a foundation for developing effective and structured instructional models that reinforce each aspect in a balanced manner to enhance students' overall mathematical proficiency.

Transparency:

The authors confirm that the manuscript is an honest, accurate, and transparent account of the study; that no vital features of the study have been omitted; and that any discrepancies from the study as planned have been explained. This study followed all ethical practices during writing.

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