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Thermal analysis of joule heating effect on micropolar nanofluid flow past a vertical stretching/Shrinking surface: Triple solutions

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Abstract: The combined usage of nanofluid and porous medium in various engineering and industrial processes provides great results in achieving the ultra-high cooling rate requirements of current industries. Further, the role of nanoliquid in the advancement of nanotechnology and electronic devices at the engineering and industrial level attracts researchers to explore this area of research. Furthermore, nonliquids are noteworthy in transportation, biomedical applications, as well as in thermal and mass transmission because of their high thermal conductivity characteristics in contrast with regular liquids. Taking this into account, the novel aspect of the current research is to explore thermal and mass transmission of micropolar nanoliquid flow through a non-linear vertical stretching/shrinking surface. Further chemical reactions and thermal radiation are utilized in energy and mass equations. Further suitable similarity transformations are introduced to convert the governing equations of micropolar nanofluid flow into non-linear ordinary differential equations. Then the resultant ODEs are solved by applying bvp4c in MATLAB software. Due to the existence of more than one solution, stability analysis is performed. It is observed that the first solution is stable and feasible. Moreover, the investigation portrays that the microrotation boundary layer thickness increases in all solutions with the growth in the material factor.
Keywords: *Chemical reaction, MHD, Micropolar nanofluid, Similarity transformations, Stability analysis, Stretching/shrinking surface, Thermal radiation, Triple solutions.*

1. Introduction

In recent years, research in the field of fluid dynamics has progressed as an area of scientific research because of its key applications in automotive industry, civil engineering, biomedical engineering, aerospace engineering, oil gas industry and many more. Progresses in nanotechnology have prompted the intensity heat transmission in nanofluid, which is alluded to as a liquid containing the nanometer sized particles. Because of its high thermal properties, it has various applications in many fields like sun oriented thermal system, aviation, biomedical, cooling and electronic cooling.

Recently researchers developed a new type of liquid called hybrid nanoliquid because it has an ability to enhance high heat transmission rate due to its higher thermal conductivity. These liquids are union of two types of nano particles. Reddy, et al. [1] investigated entropy generation in thermal analysis of nanoliquid flow via numerical technique. Moreover, Ramasekhar and Reddy [2] examined multiple shape factors of nanoparticles on energy transmission of hybrid nanoliquid flow through a disk numerically. Further Gunisetty and Reddy [3] utilized analytical and numerical approaches to study the hybrid nanoliquid flow towards a disk. Rehman, et al. [4] examined thermal diffusion along with mixed convection effect on nanoliquid and explored the thermal and mass analysis phenomenon numerically.

The unsteady flow phenomenon of bioconvection nanoliquid on a wedge was discussed by Ur Rehman, et al. [5] and found that solute and thermal qualitative behavior is same against unsteadiness.

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Further Rehman, et al. [6] examined stagnation flow of nanoliquid by considering Boungiorno model in the presence of Arrhenius energy. For more details see [7-11].

Non-Newtonian liquids known as micropolar liquids are made up of thick liquid and scattered particles. In the micropolar fluid model, a micro-rotation element and a coupling element have been gathered to investigate the kinematics of micro-transformation. Animal blood, polymeric suspensions, and liquid crystals are a few examples of these fluids. There are several applications for the movement of micropolar fluids moving towards the surface, including the production of spacecraft, plastic covers, glass blowing, consistent projecting, and fibre spinning.

The micropolar fluid theory proposed by Eringen and Suhubi [12] is the most widely accepted theory of fluid containing microstructures. The idea of a boundary layer in micropolar fluids was initially examined by Willson [13]. A comprehensive book on micropolar fluid was composed by Lukaszewicz [14] who clarified that the fluid has five additional viscosities of factors. Hassanien and Gorla [15] studied at the microfluid flow behaviour and the characteristics of heat transfer over an extended sheet. Micropolar liquid pass over an extended surface with heat generation/absorption has been analyzed by Khedr, et al. [16].

Heat transport in the micropolar boundary layer flow close to a stagnation point on an expanding surface was investigated by Bhargava and Takhar [17]. The outflow of micropolar nanofluid with a stretched surface was explained by Anwar, et al. [18] using a mathematical analysis method. Further Al Faqih, et al. [19] explored the inclination impact on the micro-rotational flow of nanoliquid for a slandring sheet. Maranna, et al. [20] investigated non-Newtonian liquid flow behavior in the presence of MHD impacts analytically.

Murugan, et al. [21] considered a slanted surface for maxwell liquid flow on a thin film and employed an entropy generation for the theoretical analysis. Moreover, Jagadeesh, et al. [22] examined mathematical model of Williamson liquid 3-D flow by incorporating Joule heating effect. Abbas, et al. [23] discussed micro-rotational nanoliquid flow for the characterization of two porous disks.

Thermal radiation is the process by which heat from the heated area is immediately emitted as an electromagnetic wave in all directions. Thermal radiation can be found in everyday activities including experiencing the heat from the sun, having a campfire, and using heaters and ovens. From a technological and scientific point of view, thermal radiation plays a crucial role in engineering processes that involve high operating temperatures. Furthermore, it plays a key role in the design of nuclear plants, gas turbines, aircraft, space vehicles, reliable equipment, and satellites.

This effect is vital in solar energy, gas turbine, electrical power organization, space vehicles, satellite, and other modern fields. A boundary layer of a micropolar fluid flow in the presence of thermal radiations through a nonlinear extending sheet was discussed by Hsiao [24]. In a permeable medium, radiation in a micropolar nanofluid was examined by Izadi, et al. [25].

According to the porous medium, Abdal, et al. [26] explored the radiation on the magnetohydrodynamics flow of micropolar fluid. A computational study of the effect of thermal radiation on the boundary layer across a stretchable surface was conducted by Ghadikolaei, et al. [27]. The properties of thermal radiation and energy sources on the flow of magnetohydrodynamics micropolar fluid through an extending/shrinking surface were examined by Khader and Sharma [28].

Study on the impact of thermal radiation on viscous flow when a sheet is extended exponentially has been conducted by Sajid and Hayat [29]. The mathematical study of micropolar nanofluid through an extending surface with the effects of magnetohydrodynamic and heat radiation has been investigated by Pal and Mandal [30]. Moreover, Rafique, et al. [31] examined the radiation effects on the nanoliquid flow on a slanted sheet by utilizing numerical techniques.

Recently, Maranna, et al. [32] studied the impacts of thermal radiation on the hybrid nanoliquid flow behavior analytically. Moreover, Mahabaleshwar, et al. [33] examined analytical model of Boussinesq-Stokes suspension liquid flow analytically by incorporating thermal radiations. In addition, thermal radiations as well as injection impacts on a porous contracting surface were examined by Maranna, et al. [34].

The study of chemical reaction has become an attraction of the current area of researchers because of its vast variety of modern and industrial uses in polymer processing and electrochemistry. These applications include absorption at the surface of water organizations, food management, the manufacturing of glass, chemical production machinery, and transportation of energy in a wet cooling highpoint.

The impact of a chemically reacting fluid in permeable medium on magnetohydrodynamics flow through an extended sheet was investigated by Jonnadula, et al. [35]. The chemical reactive magnetohydrodynamics boundary layer flow of nanoliquid over deformable sheet was investigated by Kumar, et al. [36]. By taking into account the extra effect of thermal radiation, Mabood, et al. [37] examined the influence of chemical reaction on the stagnation point flow of nanofluid.

The effect of higher order chemical reaction and activation of energy in third grade magnetohydrodynamics nanofluid flow across an extended sheet was examined by Hayat, et al. [38]. The impact of a chemical reaction on viscous fluid flow on a fluctuating plate was investigated by Ram, et al. [39]. Bhattacharyya, et al. [40] explored the effect of chemical reaction on the flow of Newtonian liquid through a deformable sheet via numerical technique.

Numerous studies have explored the effects of heat sources on fluid flow over stretching and shrinking sheets. These effects have important implications in both physics and engineering. Specifically, the influence of heat sources on various flow problems is valuable for understanding particle deposition rates in nuclear reactors, electronic chips, and semiconductor wafers. In addition, it has a significant role in the manufacturing of rubber and plastic papers, to extract heat from nuclear fuel waste products, for preserving food, for the removal of radioactive material, and for many other purposes.

The impact of heat source/sink on MHD flow of a non-Newtonian fluid over a stretching plate has been researched by Abel and Nandeppanavar [41]. The stagnation point flow of a nanofluid into an exponentially stretched surface with heat generation/absorption was investigated by Malvandi, et al. [42]. The effects of a heat generation/absorption on MHD combined convection flow from a sloped extended surface were investigated by Abo-Eldahab and El Aziz [43].

Recently, Rafique, et al. [44] discussed the nanoliquid flow over a rotating disk by employing numerical technique. They incorporated heat generation or absorption impacts on energy and mass transmission analysis. Furthermore, Bouchireb, et al. [45] probed the shape effect of nano-paticles on hybrid nanoliquid flow numerically for a convergent divergent channel. The impact of heat generation/obsorption was discussed by Murugan, et al. [46] by utilizing entropy analysis techniques for hybrid nanoliquid flow. The three-dimension flow of Casson liquid was discussed by Tarakaramu, et al. [47] numerically and analyzed the impacts of heat generation. Pandey, et al. [48] examined the Molybdenum disulfide-water nanoliquid flow on a moving surface by incorporating chemical reaction effect.

In view of the above literature many researchers have examined boundary layer flow for a stretching/shrinking surface but there is still a gap. Therefore, to fill this gap the micropolar nanofluid flow under the effects of radiation, heat generation/absorption, and chemical reaction over a nonlinearly vertical stretching/shrinking surface has been conducted.

This type of flow has gotten importance in the industrial and engineering field because of practical applications, especially in systems involving biomedical, lubricating, and cooling flows, as well as in areas including environmental, and aerospace engineering. The research under investigation will be very useful for the scholars to find multiple solutions for micropolar nanoliquid flow with the help of stability analysis, where stability analysis select which solution is feasible.

For the numerical computation the built-in MATLAB solver, bvp4c is employed. The effects of the parameters involved are comprehensively discussed through graphical illustrations and tabulated data. The declared goal of the analysis is:

1. To determine the impact of velocity slip condition on the flow, momentum boundary layer of micropolar nanofluid fluid.
2. To determine the significance of heat generation or absorption effects on the thermal boundary layer flow of micropolar nanofluid fluid.
3. Explore the significance of chemical reactions on Brownian motion, thermophoresis and concentration boundary layer.
4. Examine the significance of the thermal radiation on the thermal boundary layer of micropolar nanofluid fluid.
5. To examine the MHD effects on chemical reactive flow of micropolar nanofluid using Buongiorno nanofluid model.

This paper provides a platform for researchers to find the answers of the following questions

- How does the presence of a magnetic field affect the fluid dynamics in this study?
- How does thermal radiations affect the flow phenomenon of the current model?
- How nonlinear parameters affect the velocity of the liquid.

2. Problem Formulation

Micropolar nanofluid flow over a non-linear vertical stretching/shrinking surface with chemical reaction effect considered. In addition, the effects of thermophoresis and Brownian motion are considered. Moreover in heat equation thermal radiations along with heat generation or absorption effects are utilized. The stretching velocity can be expressed in the as $u_w(x) = ax^m$ here 'a' act as a constant. The temperature of the surface is T_w while the temperature on ambient nanoliquid is T_∞. In this research the variable magnetic field is considered as $B(x) = B_0^2 x^{2m-1}$ in which B_0 is the strength of magnetic field. Figure 1.

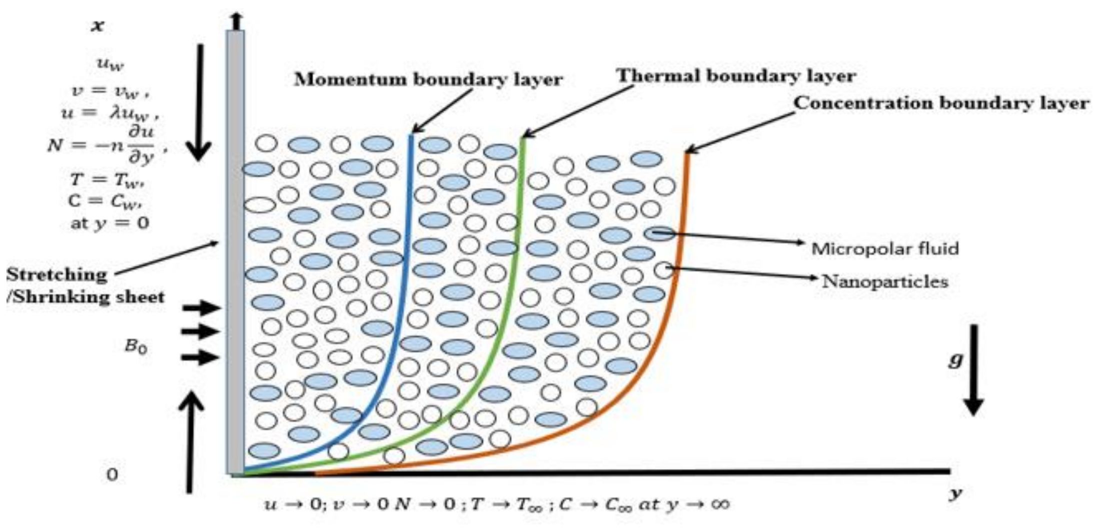

Figure 1.
Flow structure with coordinate system.

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The flow equations including momentum equation, micro-rotations, energy and mass transfer , in vector form in view of above mentioned asumptions can be expressed in the form.

$$\nabla . V = 0, \tag{1}$$
$$\rho \frac{dV}{dt} = (\mu + \mathrm{k})\nabla^2 V + k\nabla N - \sigma B^2 V cos\gamma + g[\,\beta_T(T - T_\infty) + \ \beta_C(C - C_\infty)], \tag{2}$$
$$\rho j \frac{dN}{dt} = \gamma \nabla^2 N - k(2N - \nabla \times \mathrm{V}), \tag{3}$$
$$\rho c_p \frac{dT}{dt} = (k)\nabla^2 T + \frac{\partial q_r}{\partial y} + (\rho c_p)[D_B \nabla \mathrm{C}.\nabla \mathrm{T} + \frac{D_T}{T_\infty}(\nabla \mathrm{T})^2] + Q_o(T - T_\infty), \tag{4}$$
$$\frac{dC}{dt} = D_B \nabla^2 C + \frac{D_T}{T_\infty}\nabla^2 T - R^*(C - C_\infty), \tag{5}$$

The flow equations governed the current flow problem in view of Dero, et al. [49] are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{6}$$
$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = (\mu + k)\frac{\partial^2 u}{\partial y^2} + k\frac{\partial N}{\partial y} - \sigma_m B^2 u cos\gamma + \rho g[\,\beta_T(T - T_\infty) + \ \beta_C(C - C_\infty)], \tag{7}$$
$$u\frac{\partial N}{\partial x} + v\frac{\partial N}{\partial y} = \frac{1}{\rho_j}\left[\gamma\frac{\partial^2 N}{\partial y^2} - k\left(2N + \frac{\partial u}{\partial y}\right)\right], \tag{8}$$
$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \left(\alpha + \frac{16\sigma^* {T_\infty}^3}{3k^*\rho C_p}\right)\frac{\partial^2 T}{\partial y^2} + \sigma_m B^2 u^2 cos\gamma + \ \tau_w\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y} + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right] + \frac{Q_0}{\rho C_p}(T - T_\infty), \tag{9}$$
$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = \ D_B\frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2} - \ R^*(C - C_\infty). \tag{10}$$

Subject to boundary conditions

$$v = v_w\,;u = \lambda u_w(x)\,;N = \ -n\frac{\partial u}{\partial y}\,;T = \ T_w\,;C = \ C_w \quad at\ y = 0,$$
$$u \to 0; N \to 0\,;T \to T_\infty\,;C \to C_\infty \ as\ y \to \infty. \tag{11}$$

Where σ^* denoted electrical conductivity. μ denotes viscosity, ϑ represents kinematic viscosity, ρ shows density, k denotes vortex viscosity, N depicts angular velocity, γ is the spin gradient viscosity, $\alpha = \frac{k}{\rho C_p}$ is thermal conductivity parameter, D_T stands for thermophoresis diffusion coefficient and D_B stands for Brownian motion parameter. The stream function $\psi = \psi(x, y)$ is follows for this use $u = \frac{\partial \psi}{\partial y}, v = \ -\frac{\partial \psi}{\partial x}$

In view of Rafique, et al. [50] and Rajput, et al. [51] the following similarity variables are utilized in this research work.

$$\psi = \sqrt{\frac{2\vartheta a x^{m+1}}{m+1}}f(\eta)\,, \qquad \eta = y\sqrt{\frac{(m+1)a x^{m-1}}{2\vartheta}}, \qquad N = \ a x^m \sqrt{\frac{a(m+1)x^{m-1}}{2\vartheta}}\ h(\eta)\,,$$

$$\theta(\eta) = \ \frac{(T - T_\infty)}{(T_w - T_\infty)}\,, \phi(\eta) = \frac{(C - C_\infty)}{(C_w - C_\infty)}. \tag{12}$$

The following ODEs are generated by utilizing similarity transformations(12) in Equations(6)-(11)

$$(1+K)f''' + Kh' + ff'' + \frac{2}{m+1}(Gr\theta + Gc\phi) - \frac{2m}{m+1}f'^2 - \frac{2M}{m+1}f'cos\gamma = 0, \tag{13}$$
$$\left(1+\frac{K}{2}\right)h'' + fh' - \left(\frac{3m-1}{m+1}\right)hf' - \frac{2K}{m+1}(2h + f'') = 0, \tag{14}$$
$$\frac{1}{P_r}\left(1+\frac{4}{3}Rd\right)\theta'' + N_b\theta'\phi' + N_t\theta'^2 + EcMf'^2cos\gamma + Q\theta + f\theta' = 0, \tag{15}$$
$$\phi'' + \phi' f\, Sc + \frac{N_t}{N_b}\theta'' - RSc\phi = 0. \tag{16}$$

Where,

$$K = \frac{k}{\mu}, M = \frac{\sigma_m B^2}{\rho a},\ Sc = \frac{\vartheta}{D_B},\ P_r = \frac{\vartheta}{\alpha},\ N_b = \frac{\tau_w D_B(C_w - C_\infty)}{\vartheta},\ N_t = \frac{\tau_w D_T(T_w - T_\infty)}{\vartheta T_\infty}, Gr_x = \frac{g\beta_T(T_w - T_\infty)x^{-2m+1}}{a^2},\ Gc_x = \frac{g\beta_c(C_w - C_\infty)x^{-2m+1}}{a^2},\ Q = \frac{Q_o}{\rho C_p u_w},\ Rd = \frac{4\sigma^* T_\infty^3}{kK^*},\ R = \frac{R_0}{a}. \tag{17}$$

In this section primes signify the distinction with regard to η, K is the micropolar material parameter, Gr_x local Grashof number, Gc_x is local modified Grashof number, M magnetic factor, P_r denotes the Prandtl number, Sc for Schmidt number, C_r is chemical reaction factor, Q depicts heat generation/absorption factor, Rd represents the radiation factor. To make Gc_x and Gr_x free from x, which is possible only in the case when β_T and β_c are proportional to x^{2m-1}. Thus, in view of Makinde and Olanrewaju [52] G_r and G_c becomes

$$Gc = \frac{gn(C_w - C_\infty)}{a^2},\ \ Gr = \frac{gn_1(T_w - T_\infty)}{a^2}.$$

The associative boundary conditions are changed to:

$$f(0) = S,\ f'(0) = \lambda,\ h(\eta) = -nf''(\eta),\ \theta(\eta) = 1,\ \phi(\eta) = 1 \text{ at } \eta = 0,$$
$$f'(\eta) \to 0, h(\eta) \to 0, \theta(0) \to 0, \phi(0) \to 0 \qquad as\ \eta \to \infty. \tag{18}$$

Where,

$v_w = -\sqrt{\frac{a\vartheta(m+1)}{2}}\, x^{\left(\frac{m-1}{2}\right)}$ is the suction/injection parameter $S > 0$ for suction and $S < 0$ for injection.

The following terms refer to the skin friction, Sherwood number, and Nusselt number for the current problem:

$$Nu_x = \frac{xq_w}{k(T_w - T_\infty)},\ Sh_x = \frac{xq_m}{D_B(C_w - C_\infty)},\ C_f = \frac{\tau_w}{u_{w^2}\rho}. \tag{19}$$

Where,

$$q_w = -\left(k + \frac{16\sigma^* T_\infty^3}{3k^*}\right)\frac{\partial T}{\partial y},\ q_m = -D_B\frac{\partial C}{\partial y},\ \tau_w = \left[(\mu + k)\frac{\partial u}{\partial y} + kN\right]_{y=0}. \tag{20}$$

The dimensionless decreased Nusselt number $-\theta'(0)$, decreased Sherwood number $-\varphi'$, and Skin friction constant C_{fx} are defined as:

$$C_{fx}(0) = (1+K)f''(0),$$

$$-\theta'(0) = \frac{Nu_x}{\left(1+\frac{4}{3}R\right)\sqrt{\frac{m+1}{2}Re_x}}, \quad -\phi'(0) = \frac{Sh_x}{\sqrt{\frac{m+1}{2}Re_x}}, \quad C_{fx}(0) = \frac{C_f}{2}\sqrt{\frac{2}{m+1}Re_x}\,. \tag{21}$$

Where,

$$Re_x = \frac{u_w x}{\vartheta}.$$

3. Numerical Solution

The numerical outcomes of the ODE's (8) to (11) in the presence of boundary settings equation (13) have been scrutinized via bvp4c built in function in Matlab. Many researchers employed bvp4c function for the detection of multiple solutions because of the efficiency of this solver. At the early mesh point an initial guess was considered which is suitable for the solution.

The corresponding initial guesses and the thickness of the boundary layer η_∞ must be tolerable as they may expose the performance of the solution. The tolerance taken up to 10^{-5} for the correctness of the results in this research work. In addition, it is noted that bvp4c solver is a very efficient and suitable technique for the numerical solutions of complex boundary value problems. Figure 1a displays the flow chart of Bvp4c method utilized in this study.

4. Results and Discussion

Numerical solutions have been established for radiations effects on micropolar nanofluid flow across a vertical non-linear stretching/shrinking surface by incorporating chemical reaction and heat generation/absorption. The bvp4c (MATLAB) function is the main application used for investigating data as shown in Fig 2. Similarity differential equations as given in equations (13) - (16) are computed along with the condition (18), to investigate the flow and heat transfer properties.

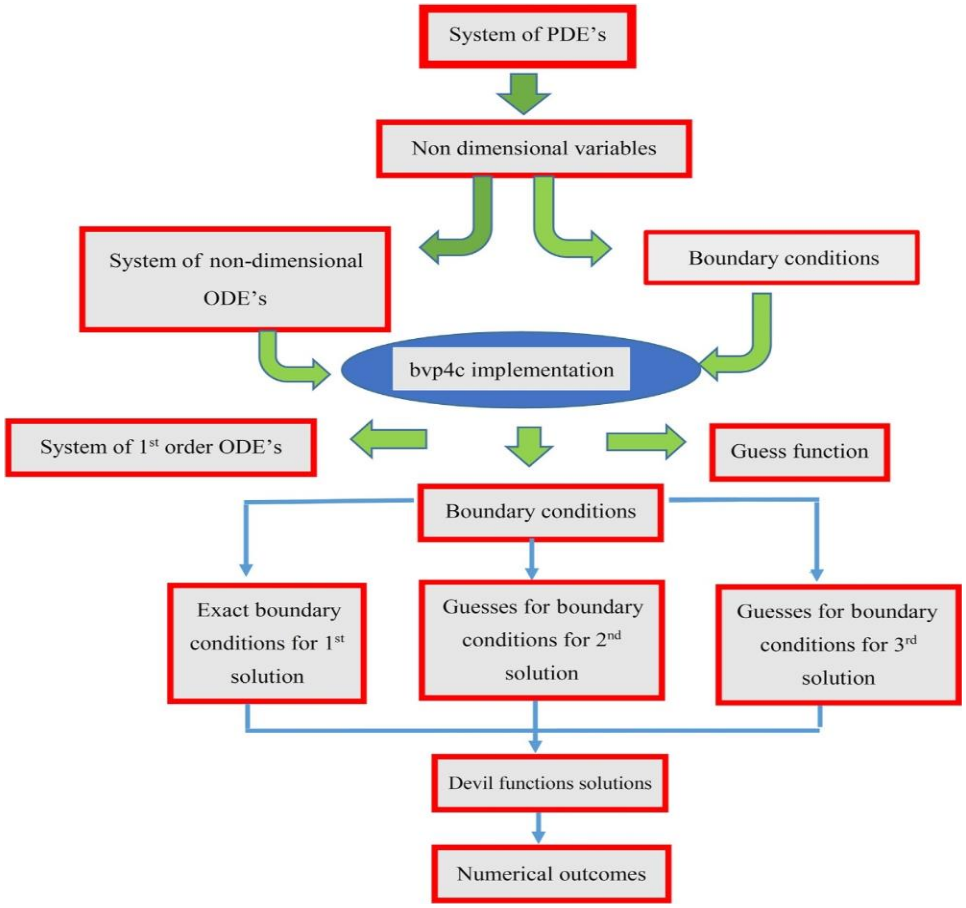

Figure 1a.
Flow chart of Bvp4c method.

Graphical representation for the momentum, rotational profiles, temperature and concentration are provided for a variety of mathematical parameters, such as micropolar material, Brownian movement, heat radiation and thermophoresis. Applying the stability analysis technique, the acquired results for the values of skin friction factors shown in Table 1 are compared to those derived from Hayat, et al. [53]. The published works and the frequency of the solutions are taken into consideration while using these values. Moreover, for every profile, the numerical results show that there are triple solutions to the micropolar nanofluid flow problem.

Table 1.
Comparison of $(C_f(Re_x)^{\frac{1}{2}})$ for $n = 0, 0.5$ and for different values of K at $\lambda, m = 1$ and $S, Gr, Gc, Nb, Nt, Q, R, Rd, M = 0$, when $\gamma \to 90^0$.

	$n = 0$		$n = 0.5$	
K	Hayat, et al. [53]	**Present**	Hayat, et al. [53]	**Present**
0	−1.00000	−1.00000	−1.00000	−1.00000
1	−1.367996	−1.367996	−1.224819	−1.224819
2	−1.621570	−1.621570	−1.414479	−1.414479
4	−2.005420	−2.005420	−1.733292	−1.733292

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According to Lund, et al. [54] and Lund, et al. [55] "this association along with the combined equation leads in a continuous result that is constantly fourth order precise in [a, b]." "The residual of the continuous result is used as the basis for both mesh selection and error control." The values of the least eigenvalue are listed in Table 2. As a result of the least eigenvalue having greater than zero, with the assistance of these values, it is possible to show that the first solution is both feasible and consistent. When the least eigenvalue is smaller than zero, the second and third solutions are incorrect and not physically possible.

Table 2.
Various values of K and S for smallest eigenvalue.

ε				
K	S	1st Solution	2nd Solution	3rd Solution
0	3	0.55221	-1.04392	-1.05492
0	2.5	0.3837	-0.76741	-0.66841
0	2	0.03158	-0.13790	-0.12770
1	3	0.47821	-0.75206	-0.73106
1	2.5	0.13271	-0.48301	-0.47301
2	3	0.25082	-0.42360	-0.51280

Figures 2 and 3 display the differences of the velocity distribution for the various values of M and n. The velocity distribution against increment in the magnetic field decreases because the presence of magnetic field in an electrically conduction liquid promotes a resistive force known as Lorentz force which cause resistive barrier in the liquid flow. Physically the resistance in the way of liquid flow decreases the velocity of the liquid. The velocity distribution in the second and third solution shows the decreasing behavior as like first solution on the growth of magnetic effect. The variations in the velocity profile for different values of n are displayed in Figure 3. The investigation indicates that in all three solutions, when the variable n grows, thickness of the momentum boundary layer increases in return the liquid velocity display increasing behavior.

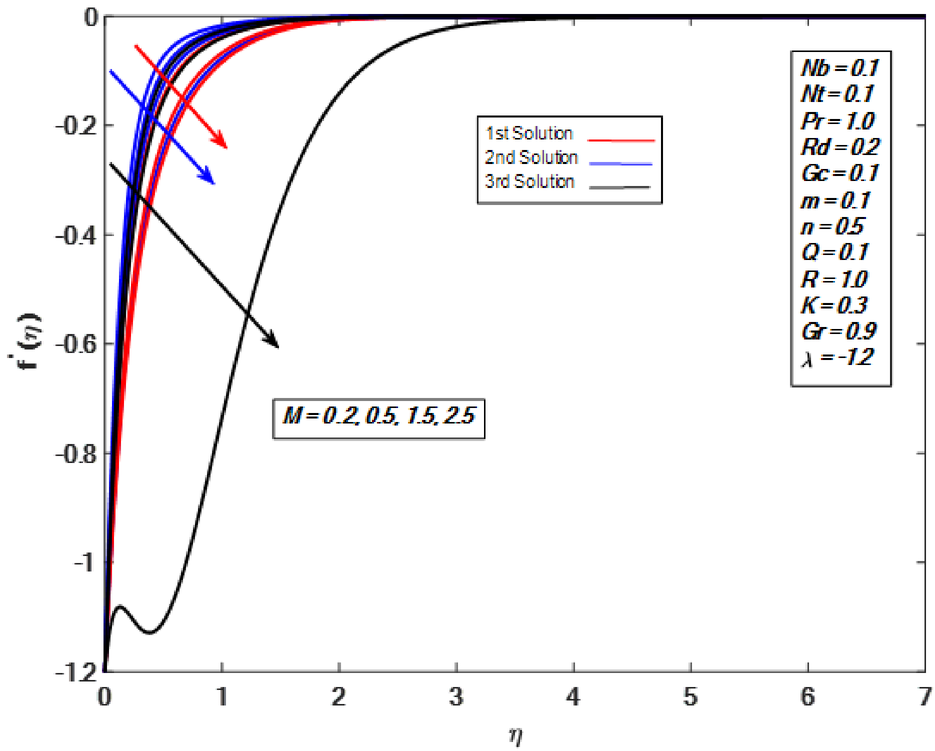

Figure 2.
The impact of different values of M on the velocity distribution.

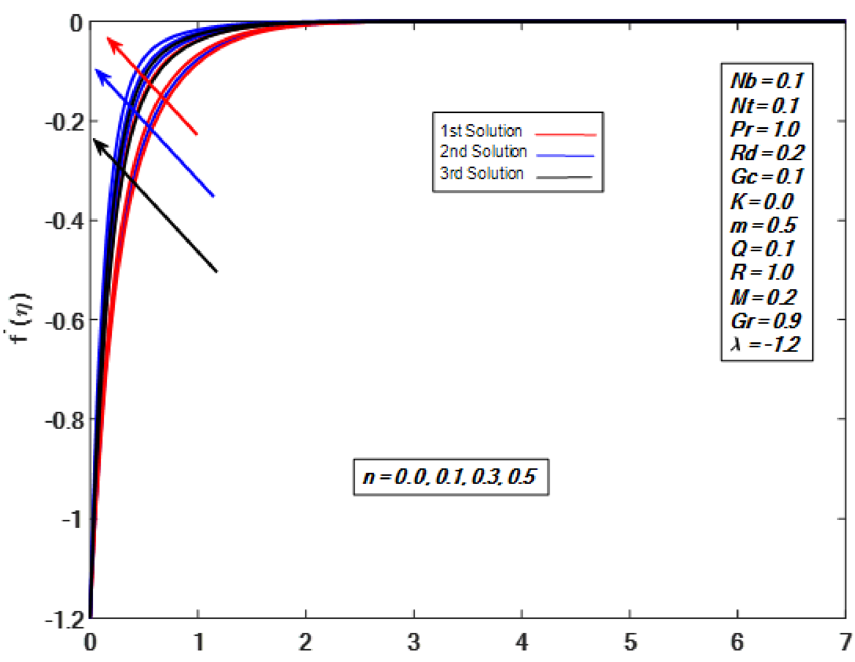

Figure 3.
Different values of n on the velocity distribution.

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Figure 4 exhibits the velocity distribution for the growth of K. The first solution shows the velocity distribution decreases while portrays an increasing behavior in the 2^{nd} and third solution in short we noticed dual behavior of the solution. Physically, the boundary thickness increases due to the reduction of drag force. Figure 5 displays various values of Grashof number Gr on the behaviour of velocity profile. The first, second and third solutions shows the velocity distribution for the first solution increases while decreases for others two solutions because first solution is feasible while the remaining two solutions are not stable. Physically enhancing the (Grashof number) Gr diminishes the viscous effects which increases the velocity of the nanoliquid due to which velocity distribution shows increasing behaviour in Figure 5.

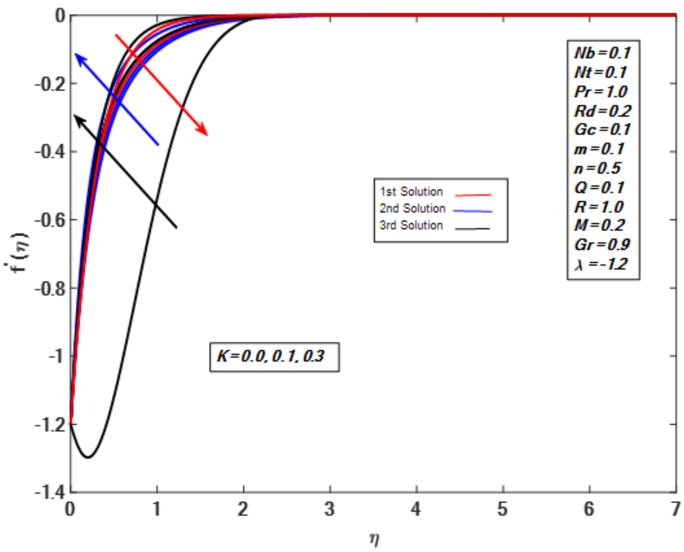

Figure 4.
Different values of factor K on the velocity distribution.

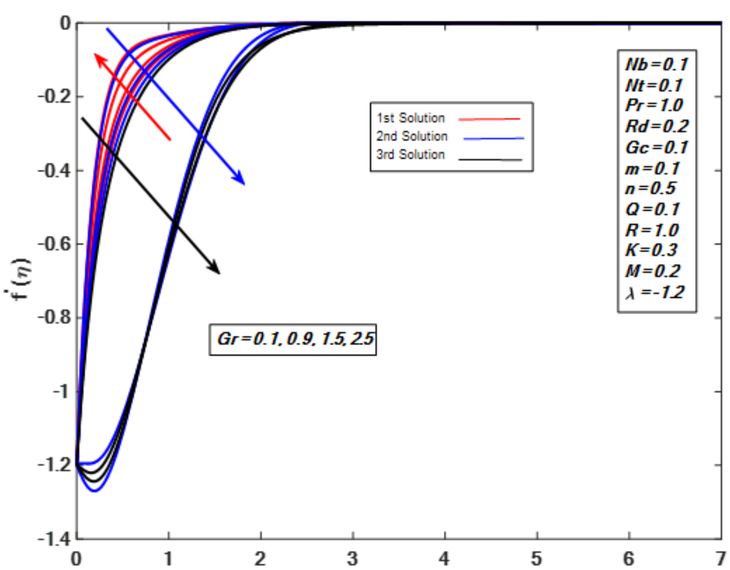

Figure 5.
Impacts of Garshoof number Gr on the velocity distribution.

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Figure 6 demonstrates the effect of micropolar parameter K on the microrotation profile. In the first, second and third solutions, boundary layer thickness and microrotation profiles rise as factor K grows.

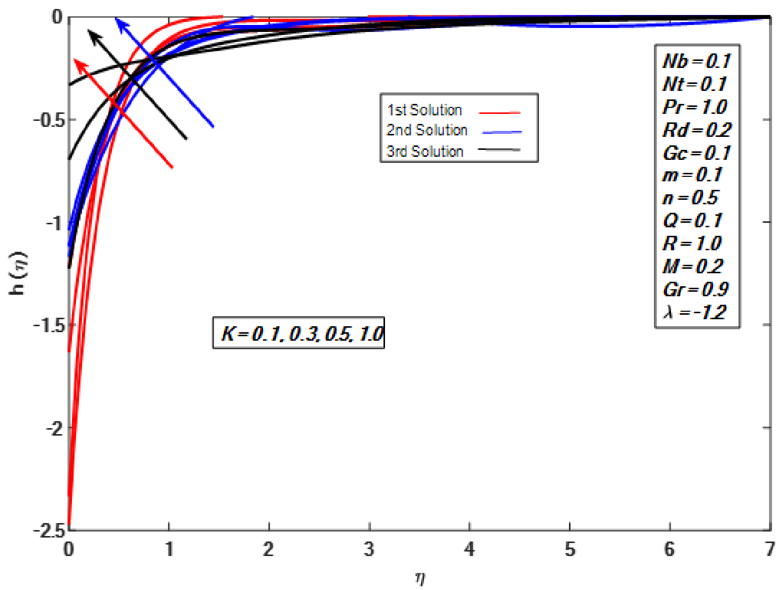

Figure 6.
Various values of K on microrotation distribution.

Figure 7, 8 indicates the impact of different values of the thermal radiation Rd and heat generation/absorption factor Q on the temperature profile. Figure 7 illustrates how thermal radiation affects the temperature profile. The rate of heat transfer and the thermal boundary layer are decreased in the first, second, and third solutions by increased thermal radiation. On the other hand, In Figure 8, When Q grows positively, it is clear that heat generation/absorption occurs at the thermal boundary layer. All solutions show that the thermal boundary layer rises in response to an increase in heat generation or absorption.

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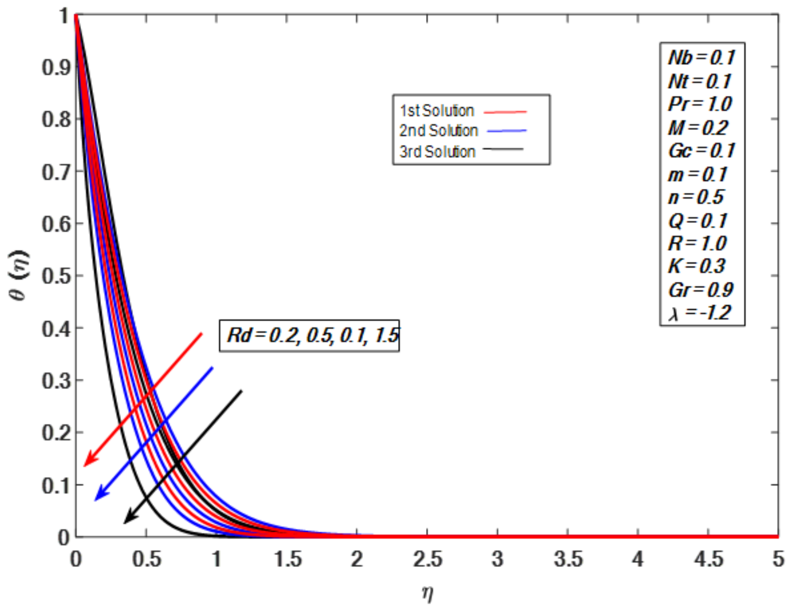

Figure 7.
The impact of thermal radiation on temperature profile.

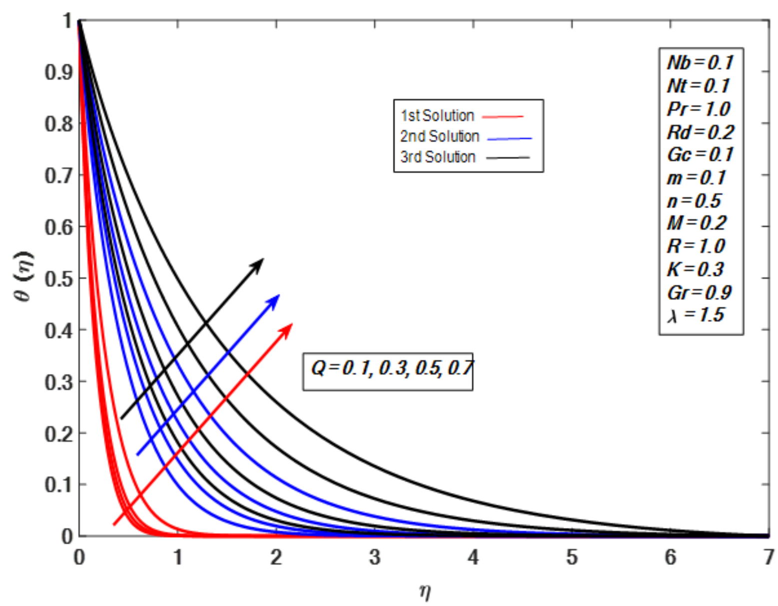

Figure 8.
Impacts of heat generation/absorption parameter Q on the temperature profile.

Figure 9 and 10, shows the effects of N_b and N_t on temperature distributions. The impact of Brownian motion on the temperature distribution is showed in Figure 9. As the Brownian motion factor N_b rises, the temperature of micropolar nanofluid decreases in all solutions. In contrast, the width of thermal boundary layer and the temperature distribution are increased in the first, second and third solutions, however an increase in the thermophoresis parameter N_t.

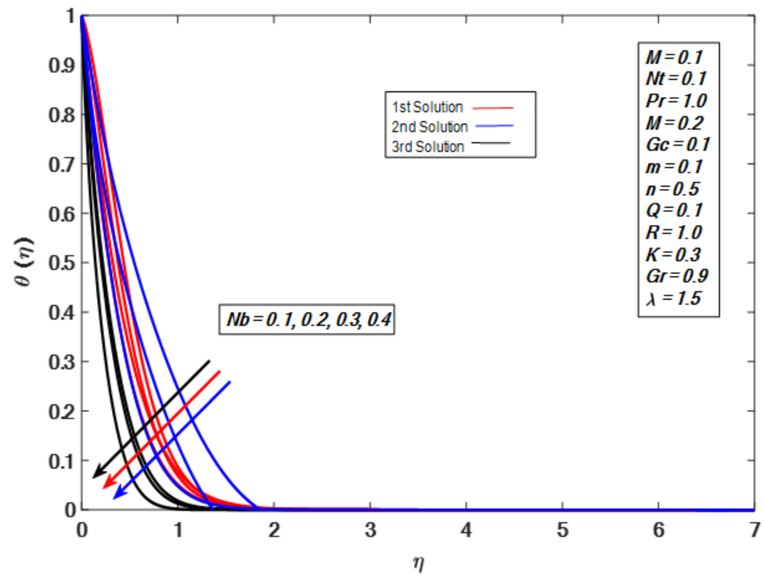

Figure 9.
Various values of N_b on temperature profile.

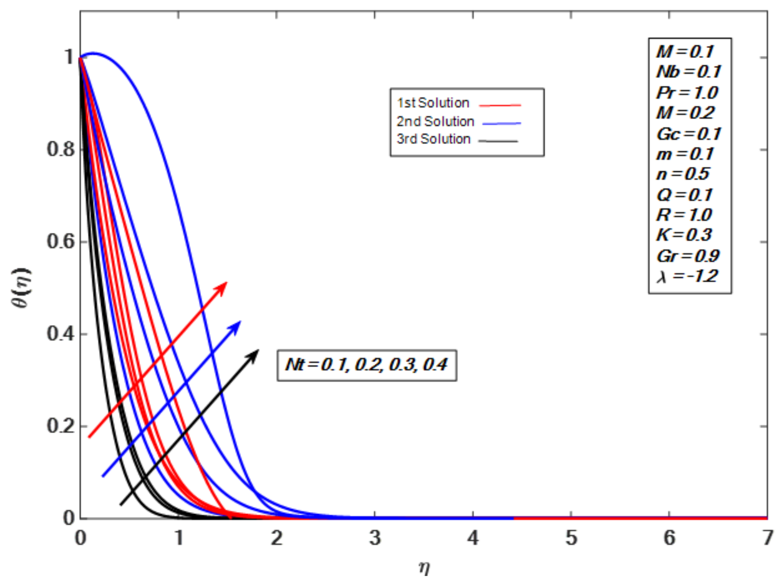

Figure 10.
Various values of N_t on temperature distribution.

Figure 11 represents the behavior of the temperature distribution for various values of the Prandtl number Pr. This figure's first, second, and third solutions display a rise in Pr lowers temperature profiles, and that effect causes the thickness of the thermal boundary layer to decrease.

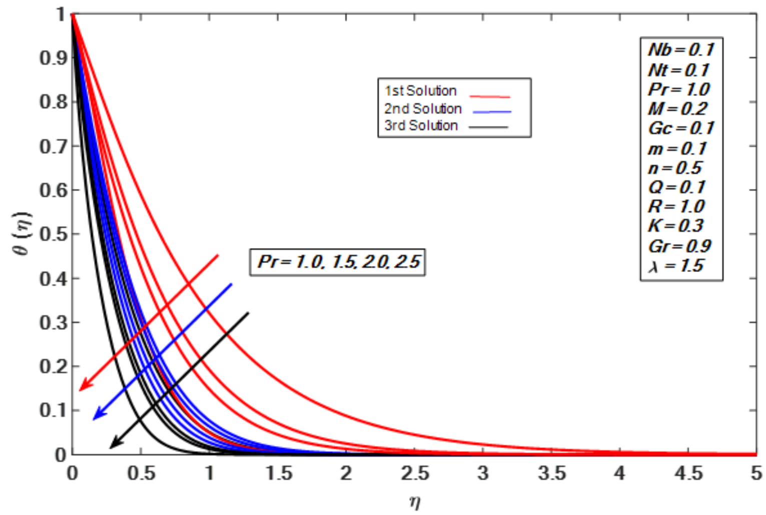

Figure 11.
Impacts of Prandtl number Pr on temperature distribution.

Figures 12 and 13 indicate how the concentration profile is affected by the Brownian motion and thermophoresis factor. In figure 12, the concentration distribution and its boundary thickness in the first, second and third solutions are visibly reduced with increase in every value of N_b. Conversely, when the thermophoresis parameter N_t is increased, Figure 13 displays the concentration distribution, and its boundary thickness increase in all solutions.

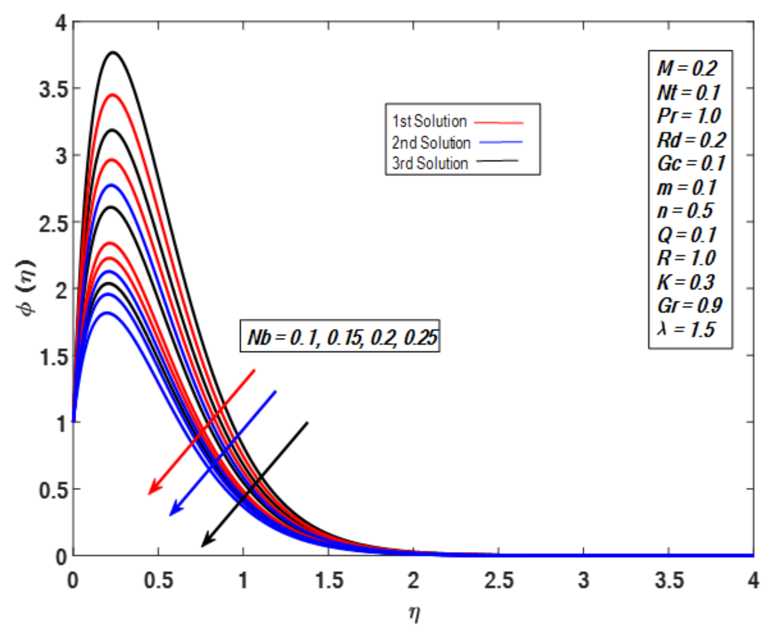

Figure 12.
Different values of N_b on concentration distribution.

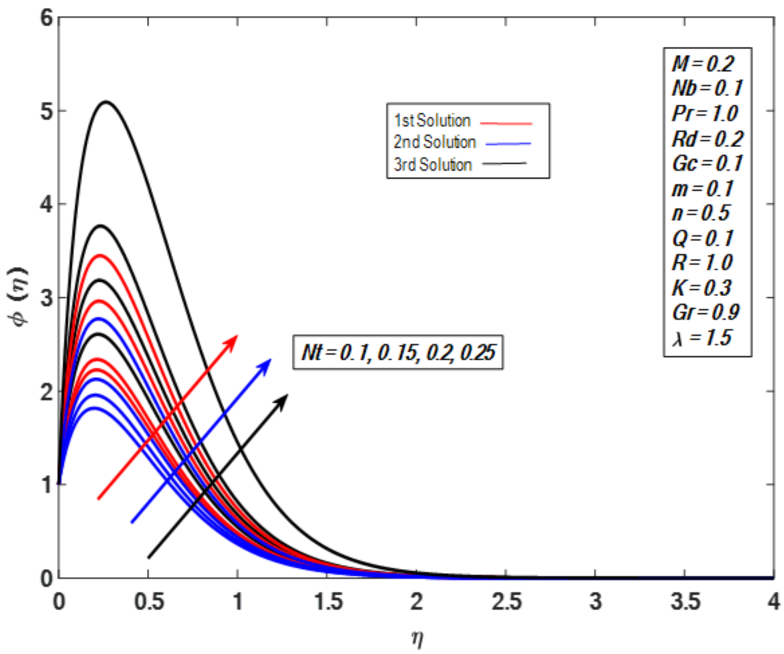

Figure 13.
Different values of N_t on concentration distribution.

5. Stability Analysis

The stability analysis is to discover the solution that is both physically and mathematically possible. It also helps to identify the solution that is stable. The first step in solving the stability problem is to add a new time dependent variable τ, to the governing equations (7) – (10) to change them into unstable form.

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=(\mu+k)\frac{\partial^2 u}{\partial y^2}+k\frac{\partial N}{\partial y}-\sigma_m B^2 u cos\gamma+\rho g[\,\beta_T(T-T_\infty)+\ \beta_C(C-C_\infty)], \tag{22}$$

$$\left(\frac{\partial N}{\partial t}+u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}\right)=\frac{1}{\rho_j}\left[\gamma\frac{\partial^2 N}{\partial y^2}-k\left(2N+\frac{\partial u}{\partial y}\right)\right], \tag{23}$$

$$\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\left(\alpha+\frac{16\sigma^* {T_\infty}^3}{3k^*\rho C_p}\right)\frac{\partial^2 T}{\partial y^2}+\sigma_m B^2 u^2 cos\gamma+\ \tau_w\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right]+\frac{Q_0}{\rho C_p}(T-T_\infty), \tag{24}$$

$$\left(\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}\right)=\ D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2}-\ R^*(C-C_\infty). \tag{25}$$

The following are new similarity variables:

$$\eta=y\sqrt{\frac{(m+1)ax^{m-1}}{2\vartheta}},\ \tau=ax^{m-1}t,\ N=\ ax^m\sqrt{\frac{a(m+1)x^{m-1}}{2\vartheta}}\ h(\eta,\ \tau)\,,\theta(\eta,\ \tau)=\ \frac{(T-T_\infty)}{(T_w-T_\infty)}\,,\phi\,(\eta,\ \tau)=\frac{(C-C_\infty)}{(C_w-C_\infty)}, \tag{26}$$

By using equation (21) in equations (17) - (20), it is obtained

$$(1+K)\frac{\partial^3 f}{\partial\eta^3}+K\frac{\partial h}{\partial\eta}+f\frac{\partial^2 f}{\partial\eta^2}+\frac{2}{m+1}(Gr\theta+Gc\phi)-\frac{2}{m+1}\frac{\partial^2 f}{\partial\eta\partial\tau}-\frac{2m}{m+1}\left(\frac{\partial f}{\partial\eta}\right)^2-\frac{2M}{m+1}\frac{\partial f}{\partial\eta}cos\gamma=0 \tag{27}$$

$$\left(1+\frac{K}{2}\right)\frac{\partial^2 h}{\partial\eta^2}+f\frac{\partial h}{\partial\tau}-\left(\frac{3m-1}{m+1}\right)h\frac{\partial f}{\partial\eta}-\frac{2K}{m+1}\left(2h+\frac{\partial^2 f}{\partial\eta^2}\right)-\frac{2}{m+1}\frac{\partial h}{\partial\tau}=0, \tag{28}$$

$$\frac{1}{P_r}\left(1+\frac{4}{3}\,Rd\right)\frac{\partial^2\theta}{\partial\eta^2}+N_b\frac{\partial\theta}{\partial\eta}\frac{\partial\phi}{\partial\eta}+N_t\left(\frac{\partial\theta}{\partial\eta}\right)^2+EcM\frac{\partial^2 f}{\partial\eta\partial\tau}cos\gamma+Q\theta-\frac{\partial\theta}{\partial\tau}+f\frac{\partial\theta}{\partial\eta}=0, \tag{29}$$

$$\frac{\partial^2\phi}{\partial\eta^2}+\frac{\partial\phi}{\partial\eta}f\,Sc+\frac{N_t}{N_b}\frac{\partial^2\theta}{\partial\eta^2}-RSc\phi-\frac{\partial\phi}{\partial\tau}Sc=0. \tag{30}$$

Subjected to boundary conditions

$$f(0,\tau)=S,\ \frac{\partial f(0,\tau)}{\partial\eta}=\lambda,\ h(0,\tau)=-n\frac{\partial^2 f(0,\tau)}{\partial\eta^2},\ \theta(0,\tau)=1,\ \phi(0,\tau)=1,$$
$$\frac{\partial f(\eta,\tau)}{\partial\eta}\to 0\,, h(\eta,\tau)\to 0\,, \theta(\eta,\tau)\to 0\,, \phi(\eta,\tau)\to 0 \qquad as\ \eta\to\infty. \tag{31}$$

In view of Zainal, et al. [56] and Verma, et al. [57] to depict the stability of the steady flow $f(\eta)=f_0(\eta), h(\eta)=h_0(\eta), \theta(\eta)=\theta_0(\eta), \phi(\eta)=\phi_0(\eta)$ following disturbances are need to be employed for the numerical outcomes

$$\begin{aligned} f(\eta,\tau) &= f_0(\eta)+e^{-\varepsilon\tau}F(\eta)\\ h(\eta,\tau) &= h_0(\eta)+e^{-\varepsilon\tau}H(\eta)\\ \theta(\eta,\tau) &= \theta_0(\eta)+e^{-\varepsilon\tau}G(\eta)\\ \phi(\eta,\tau) &= \phi_0(\eta)+e^{-\varepsilon\tau}S(\eta) \end{aligned} \tag{32}$$

Here smallest eigenvalue is represented by ε while $F(\eta)\,,H(\eta,\ G(\eta),\ S(\eta)$ stands for dimensionless functions with all its derivatives are comparatively lesser compare with steady solutions which are presented via $f_0(\eta),\ h_0(\eta),\ \theta_0(\eta), \phi_0(\eta)$ respectively. Equations (27)-(30) by employing equation (32) with τ=0 adopted the following form

$$(1+K)F_0'''+KH_0'-\frac{2}{(m+1)}F_0'\left[M-\varepsilon-2mf_0'\right]+\frac{2}{(m+1)}\left[GrG_0+GcS_0\right]+f_0F_0''+Ff_0''=0, \tag{33}$$

$$\left(1+\frac{K}{2}\right)H_0''-\frac{2}{(m+1)}H_0(2K-\varepsilon)-\frac{2K}{(m+1)}F_0''+f_0H_0'+F_0h_0'-\left(\frac{3m-1}{m+1}\right)\left[H_0f_0'+h_0F_0'\right]=0, \tag{34}$$

$$\frac{1}{Pr}\left(1+\frac{4}{3}Rd\right)G_0''+N_b\theta_0'(S_0'+2G_0')+G_0'(N_b\phi_0'+f_0)+QG_0+\varepsilon G_0-F_0\theta_0'=0, \tag{35}$$

$$S_0''+\frac{N_t}{N_b}G_0''-R\,Sc\,S_0+S_0\,Sc\,\varepsilon+Sc\,f_0S_0'+Sc\,F_0\phi_0'=0. \tag{36}$$

With boundary conditions:

$$F_0(0)=0, F_0'(0)=\delta F_0''(0),\ H_0(0)=-nF_0''(0),\ G_0(0)=0,\ S_0(0)=0,$$
$$F_0(\eta)\to 0\,, H_0(\eta)\to 0\,, G_0(0)\to 0\,, S_0(0)\to 0 \qquad as\ \eta\to\infty. \tag{37}$$

As stated by Harris, et al. [58] variable, $H_0'=1$ is used in place of the boundary conditions $H_0(\eta)\to 0$ as $\eta\to\infty$ in order to make sure that the least nonzero eigenvalues are produced accordingly.

6. Conclusions

The combined impacts of heat generation/absorption, chemical reaction, radiation, and slip parameters, on the micropolar nanofluid flow over a nonlinear vertical stretching/shrinking surface in a permeable medium is numerically calculated. By using similar transformations, PDE's are converted into ODE's. The transformed equations are solved by bvp4c in the MATLAB software. Furthermore, the graphical results have been obtained in order portrayed Brownian motion, thermophoretic, and micropolar material parameter impacts on temperature, concentration and velocity distribution. The main points of this study are:

1. Triple solutions show that first solution is feasible and stable while the remaining two are unstable.
2. The velocity distribution diminishes with the growth of M while increases with increment in n.
3. Temperature of the liquid improves with the growth of heat generation/ absorption parameter.
4. By increasing the impact of thermal radiations temperature distribution shows decreasing behavior.
5. Concentration distribution shows increasing behavior against the enhancement of N_t.

This motivation behind the investigation of this research work is numerous application in engineering, nanotechnology and pharmaceutical industry. Further this work can be extended for hybrid nanoliquid for different geometries including stretching/shrinking disk, wedge, cylinder, sphere, cone etc.

Further, the limitations of this research work contain its focus on laminar and two-dimensional flow of non-Newtonian nanofluid flow for a nonlinear stretching/shrinking vertical surface by incorporating Buongiorno model. The findings of current research can be utilized to industrial processes for improving energy and mass transmission rates in various applications including cooling systems, heat exchangers, and polymer processing.

Transparency:

The authors confirm that the manuscript is an honest, accurate, and transparent account of the study; that no vital features of the study have been omitted; and that any discrepancies from the study as planned have been explained. This study followed all ethical practices during writing.

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