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An asymmetric bounce scenario to the dark energy ERA in $f(Q,T)$ gravity

Metoka Narzary[1], Mukunda Dewri[2*]
[1,2]Department of Mathematical Sciences, Bodoland University, Kokrajhar, 783370, Assam, India; metokanarzary@gmail.com (M.N.) dewri11@gmail.com (M.D.).

Abstract: We explore the bouncing scenario within the $f(Q,T)$ gravity framework in a Bianchi Type-VI backgrounds, utilizing a perfect fluid as the matter content. We take into account the idea of the bouncing model as bouncing cosmology circumvents the initial singularity, preventing the universe from collapsing into a singular point. To elucidate the bouncing and late-time cosmological scenarios, we initially sought the exact solution to the field equations by assuming that the shear scalar (σ) is proportional to the expansion scalar (θ). To demonstrate the bouncing cosmology, we consider a bouncing scale factor given by $\mathcal{R}(t) = [1 + a_0(t/t_0)^2]^{\beta} e^{\left[\frac{1}{(\alpha-1)}\left(\frac{t_s-t}{t_0}\right)^{1-\alpha}\right]}$. We also consider specific forms of $f(Q,T)$ gravity expressed as $f(Q,T) = aQ^k + bT$, where Q and T denotes the non-metricity scalar and trace of the energy-momentum tensor respectively. In this investigation, the Hubble parameter begins with a negative value, transitions through $H = 0$, and then exhibits a positive behavior, aligning with the outlined bouncing cosmology. Near the bouncing point, it is observed that the Equation of State parameter crosses the phantom divide line $(\omega = -1)$. Then, we discuss the energy conditions, noting that both null and strong energy conditions are violated in the vicinity of the bouncing point. The outcomes of this study may enhance our understanding of bouncing cosmological scenarios within the context of $f(Q,T)$ gravity.

Keywords: *Bouncing solution, Dark energy, Energy conditions, EoS parameter, Hubble parameter, $f(Q,T)$ gravity.*

1. Introduction

The modified gravity theory has been proposed as an alternative to General Relativity (GR) to address early and late time acceleration. A plethora of theories have been proposed in literature to explain this cosmic acceleration. The General Relativity (GR) has provided the most accurate predictions for describing cosmological phenomena by decades of experimentation. The GR assumes a Levi-Civita connection and implies zero torsion and non-metricity. One of the geometrical modifications to GR is known as the teleparallel equivalent of general relativity (TEGR), which utilizes the Weitzenbock connection and elicits vanishing curvature and non-metricity. There also exists another possibility, to adopt a connection with vanishing curvature and torsion, which provides another equivalent formulation of GR known as symmetric teleparallel equivalent of GR (STEGR) [1-4]. In TEGR, the gravitational interaction is defined by the torsion, while in STEGR, non-metricity tensor Q describes the gravitational interaction.

In this paper, we consider the extension of $f(Q)$ theory which is known as $f(Q,T)$ gravity. It is based on the coupling of non-metricity Q, and the trace of energy-momentum tensor T, which was presented by Xu, et al. [5]. The coupling between the non-metricity Q and the trace of energy-momentum tensor T leads to the non-conservation of the energy-momentum tensor. Many researchers have studied the $f(Q,T)$ gravity theory in different contexts: Godani and Samanta [6] considered Friedmann-Lemaître-Robertson-Walker (FLRW) model and explored the evolution of the universe.

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* Correspondence: dewri11@gmail.com

Pati, et al. [7] studied the Rip cosmological model in $f(Q,T)$ gravity. Shiravand, et al. [8] studied cosmological inflation in $f(Q,T)$ gravity, and the results indicate that the proposed model provides appropriate predictions that are consistent with the observational data. Koussour, et al. [9] studied the existence of bulk viscous FLRW cosmological models in a $f(Q,T)$ gravity. Loo, et al. [10] investigated an anisotropic cosmology in the modified $f(Q,T)$ gravity theory. Narawade, et al. [11] presented an accelerating cosmological model of the universe in $f(Q,T)$ gravity. They also performed a dynamic system analysis to validate the stability of the model. Pati, et al. [12] assumed the hyperbolic scale factor and constructed the model, and studied its evolutionary behaviour. Narzary and Dewri [13] studied the evolutionary behaviour of the $f(Q,T)$ gravity theory within the context of Bianchi type VI metric. Bekkhozhayev, et al. [14] present a model depicting the LRS Bianchi type-I spacetime filled with a viscous fluid, examining the effects of viscosity on cosmic expansion within the $f(Q,T)$ gravity framework. Additionally, Das and Mandal [15]; Mishra, et al. [16] and Kausar, et al. [17] investigate cosmic evolution using $f(Q,T)$ gravity in different contexts.

In accordance with the principles of Big Bang cosmology, our universe began from a singularity but had several cosmological issues, such as the flatness problem, horizon problem, entropy problem, and singularity problem. To confront these issues, Inflationary cosmology was introduced by Guth [18] where the universe expanded exponentially for an extremely short period of time, about 10^{-30} seconds immediately after the Big Bang. Inflationary models can resolve all of the aforementioned problems. However, the singularity problem remains unsolved, and therefore, as an alternative to the inflationary model, the concept of the bouncing model has been considered. In the context of bouncing cosmology, the universe contracts until a minimal radius is attained and then expands. Therefore, the initial singularity is avoided because the universe never collapses to a single point. The bouncing cosmological scenario has been investigated in the context of several modified gravity theories. The classical bouncing behaviour in the framework of $f(R,T)$ gravity theories has been studied by introducing an effective fluid through defining effective energy density and pressure and obtained cosmological scenarios exhibiting a non-singular bounce before and after which the universe lies within a de-Sitter phase Shabani and Ziaie [19]. Singh, et al. [20] constructed the bouncing cosmological model with a specific form of the Hubble parameter within the context of $f(R,T)$ gravity. Skugoreva and Toporensky [21] explored the bouncing solution in $f(T)$ gravity and provided a global analysis of the corresponding cosmological dynamics in the cases when bounces and static configurations exist by constructing phase diagrams. Sahoo, et al. [22] present a matter bounce scenario in the framework of $f(R,T)$ gravity where $f(R,T) = R + 2\lambda T$, and it is found that the present model is highly unstable at the bounce but the perturbations decay out rapidly away from the bounce safeguarding its stability at late times. Caruana, et al. [23] investigated the possibility of reproducing some important bouncing cosmology scenario, namely symmetric bounce, super bounce, oscillatory cosmology, matter bounce, and Type I–IV singularity cases within the framework of $f(T,B)$ gravity. Ilyas and Rahman [24] studied bouncing cosmology under consideration of different viable models in $f(R)$ gravity theory that can resolve the difficulty of singularity in standard Big-Bang cosmology, and the stability of the model is analyzed with the help of sound speed feature, which illustrates late-time stability. Ahmad, et al. [25] explore the possibility of some bouncing Universe in Gauss-Bonnet cosmological model with logarithmic trace term. Shamir [26] explored the bouncing cosmological model with the logarithmic term in the context of $f(G,T)$ gravity, and all the results of the proposed $f(G,T)$ gravity model provide good bouncing solutions. Odintsov, et al. [27] proposed a unified cosmological scenario of an asymmetric bounce to the dark energy in the context of Chern-Simons $F(R)$ gravity. Nojiri, et al. [28] investigated a non-singular cosmological scenario in a ghost-free $f(R,G)$ model where the universe contracts through an ekpyrotic bounce, and it smoothly connected to the dark energy era. Agrawal, et al. [29] presented a bouncing cosmological model of the universe in an extended theory of gravity. The geometrical parameters show the singularity at the bouncing epoch, and the coupling parameter has a significant role in avoiding the singularity of the EoS (equation of state) parameter at the bouncing epoch. Singh, et al. [30] investigate the bounce realization in the context of $f(R,T)$ gravity by performing a detailed analysis of the cosmological parameters to explain the contraction phase, the bounce phase, and the expansion phase and also using linear perturbations in the Hubble parameter as well as the energy density discusses the stability of the model. Zubair and Farooq [31] examined the various bouncing cosmological models: symmetric bounce, matter bounce, super bounce, and oscillatory bounce in a 4D Einstein Gauss-Bonnet gravity and analyzed that the Gauss-Bonnet coupling parameter has a lesser contribution to the dynamics of modified gravity while the bouncing parameter has noticeable effects. Singh, et al. [20] explored the bouncing scenario in FLRW space-time by using the reconstruction technique for the power-law parametrization of the Hubble parameter in a modified gravity theory with higher-order curvature terms, and it is found that extremal acceleration occurs at the bouncing point. Agrawal, et al. [32] have shown the matter bounce scenario of the universe in the $f(Q)$ gravity through the phase space analysis, where both the stable and unstable nodes are obtained. Malik, et al. [33] studied a bouncing universe considering FLRW space-time to find the solutions of field equations within the framework of $f(R,T)$ gravity. Gul, et al. [34] explore the feasibility of bouncing cosmological models by using various scale factor forms alongside a perfect matter configuration within the context of $f(Q,T)$ gravity. Sharif, et al. [35] examine non-singular cosmic bounce models related to Bianchi type-I spacetime within the framework of $f(Q)$ theory.

In the present work, we investigate the possibility of bouncing solutions within the framework of $f(Q,T)$ gravity. We consider the Bianchi type VI geometry in which the particular forms of bouncing frameworks emerge through the scale factor. We first introduce the basic formalism of $f(Q,T)$ gravity in section 2. In section 3, we solve the $f(Q,T)$ field equations for Bianchi type-VI metric in the presence of perfect fluid and obtain the exact solutions of the field equations in section 4. In sections 5 and 6, the dynamical parameters and energy conditions of the cosmological model have been analyzed, respectively. The stability of the model is also analyzed in the section 7. The conclusion of our results is summarized in the last section 8.

2. Basic Formalism in f(Q, T) Gravity

The $f(Q,T)$ gravity is constrained with the curvature and torsion-free assumptions, i.e., $R^{\rho}{}_{\sigma\mu\nu}=0$ and $T^{\rho}{}_{\mu\nu}=0$. The general action for $f(Q,T)$ gravity [4] is given as.

$$S=\int\sqrt{(-g)}\left(\frac{1}{16\pi}f(Q,T)+\mathcal{L}_m\right)d^4x \qquad (1)$$

Where Q stands for the non-metricity scalar, T for the trace of the stress-energy momentum tensor, $\mathcal{L}_m$ for the matter lagrangian and $g\equiv det(g_{\mu\nu})$. Here, the energy-momentum tensor can be defined as.

$$T_{\mu\nu}=\frac{2}{\sqrt{g}}\frac{\delta(\sqrt{g\mathcal{L}_m})}{\delta g^{\mu\nu}} \qquad (2)$$

The variation of the energy-momentum tensor with respect to the metric tensor becomes

$$\frac{\delta g^{\mu\nu}T_{\mu\nu}}{\delta g_{\alpha\beta}}=T_{\alpha\beta}+\Theta_{\alpha\beta}$$

$$\text{and } \Theta_{\mu\nu}=g^{\alpha\beta}\frac{\delta T_{\alpha\beta}}{\delta g^{\mu\nu}}$$

Further, the non-metricity scalar is defined as.

$$Q\equiv -g^{\mu\nu}\left(L^{\alpha}{}_{\beta\mu}L^{\beta}{}_{\nu\alpha}-L^{\alpha}{}_{\beta\alpha}L^{\beta}{}_{\mu\nu}\right) \qquad (3)$$

Where $L^{\alpha}{}_{\beta\mu}$ is the disformation tensor written as.

$$L^{\alpha}{}_{\beta\mu}=-\frac{1}{2}g^{\alpha\lambda}\left(\nabla_{\mu}g_{\beta\lambda}+\nabla_{\beta}g_{\lambda\mu}-\nabla_{\lambda}g_{\mu\beta}\right)==\frac{1}{2}g^{\alpha\lambda}\left(Q_{\mu\beta\lambda}+Q_{\beta\mu\lambda}-Q_{\alpha\beta\mu}\right) \qquad (4)$$

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As for the non-metricity tensor $Q_{\gamma\mu\nu}$ is expressed as.

$$Q_{\gamma\mu\nu} \equiv \nabla_\gamma g_{\mu\nu} = -\frac{\partial g_{\mu\nu}}{\partial x^\gamma} + g_{\nu\sigma}\widetilde{\Gamma^\sigma}{}_{\mu\gamma} + g_{\sigma\mu}\widetilde{\Gamma^\sigma}{}_{\nu\gamma} \qquad (5)$$

Where $\widetilde{\Gamma^\sigma}{}_{\mu\gamma}$ is known as weyl-cartan connection defined as $\widetilde{\Gamma^\gamma}{}_{\mu\nu} = L^\gamma{}_{\mu\nu} + \Gamma^\gamma{}_{\mu\nu}$

The non-metricity tensor and its traces are obtained as $Q_{\mu\nu\alpha} \equiv \nabla_\mu g_{\nu\alpha}$ and $Q_\alpha \equiv Q_\alpha{}^\beta{}_\beta$, $\widetilde{Q_\alpha} = Q^\beta{}_{\alpha\beta}$ respectively.

The superpotential tensor, known as non-metricity conjugate, can be expressed by

$$P^\alpha{}_{\mu\nu} = \frac{1}{4}\left[-Q^\alpha_{\mu\nu} + 2Q_{(\mu}{}^\alpha{}_{\nu)} + Q^\alpha g_{\mu\nu} - \widetilde{Q^\alpha} g_{\mu\nu} - \delta^\alpha_{(\mu}Q_{\nu)}\right]$$
$$= -\frac{1}{2}L^\alpha_{\mu\nu} + \frac{1}{4}\left(Q^\alpha - \widetilde{Q^\alpha}\right)g_{\mu\nu} - \frac{1}{4}\delta^\alpha_{(\mu}Q_{\nu)} \qquad (6)$$

Regarding this super potential, the non-metric scalar can be defined as

$$Q = -Q_{\alpha\beta\gamma}P^{\alpha\beta\gamma} = -\frac{1}{4}\left(-Q^{\alpha\gamma\rho}Q_{\alpha\gamma\rho} + 2Q^{\alpha\gamma\rho}Q_{\rho\alpha\gamma} - 2Q^\rho\widetilde{Q_\rho} + Q^\rho Q_\rho\right)$$

Now, varying the gravitational action (1) w.r.t the metric tensor $g_{\mu\nu}$ the corresponding field equations of $f(Q,T)$ gravity is obtained as.

$$-\frac{2}{\sqrt{-g}}\nabla_\alpha\left(f_{Q\sqrt{-g}}P^\alpha_{\mu\nu}\right) - \frac{1}{2}fg_{\mu\nu} + f_T\left(T_{\mu\nu} + \Theta_{\mu\nu}\right) - f_Q\left(P_{\mu\alpha\beta}Q^{\alpha\beta}_\nu - 2Q^{\alpha\beta}{}_\mu P_{\alpha\beta\nu}\right) = 8\pi T_{\mu\nu} \qquad (7)$$

Where $f_Q \equiv \frac{\partial f}{\partial Q}$, $f_T \equiv \frac{\partial f}{\partial T}$.

3. Bianchi Type VI Universe in $\mathbf{f(Q,T)}$ Gravity

We consider the universe described by spatially homogeneous, anisotropic, and Bianchi type VI space-time as.

$$ds^2 = -dt^2 + A^2dx^2 + B^2e^{-2mx}dy^2 + C^2e^{2mx}dz^2 \qquad (8)$$

Where the scale factors A, B and C are the functions of cosmic time t and m is a non-zero constant.

The non-metricity scalar for Bianchi type-VI space-time becomes.

$$Q = 2\left[\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} + \frac{m^2}{A^2}\right] \qquad (9)$$

Here, we consider the energy-momentum tensor $T_{\mu\nu}$ in the form of a perfect fluid, which can be parametrized as.

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu} \qquad (10)$$

Where ρ and p are the energy density and pressure of the matter content. The four-velocity vector u^μ is presumed to satisfy $u^\mu u_\mu = -1$.

By the definition of $T_{\mu\nu}$, the $\Theta_{\mu\nu}$ can be expressed.

$$\Theta_{\mu\nu} = \mathcal{L}_m g_{\mu\nu} - 2T_{\mu\nu} \qquad (11)$$

Furthermore, we take the matter Lagrangian as $\mathcal{L}_m = p$ and hence.

$$\Theta^\mu_\nu = p\delta^\mu_\nu - 2T^\mu_\nu \qquad (12)$$

By using the equations (8) and (10) in equation (7), the field equation of $f(Q,T)$ gravity (7) in the Bianchi type-VI space-time can be obtained as.

$$\frac{f(Q,T)}{2} - f_Q\left[\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + \frac{2\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} + \frac{2m^2}{A^2}\right] - \dot{f}_Q\left[\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right] = -8\pi p \qquad (13)$$

$$\frac{f(Q,T)}{2} - f_Q\left[\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{2\dot{A}\dot{C}}{AC}\right] - \dot{f}_Q\left[\frac{\dot{A}}{A} + \frac{\dot{C}}{C}\right] = -8\pi p \qquad (14)$$

$$\frac{f(Q,T)}{2} - f_Q\left[\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{2\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC}\right] - \dot{f}_Q\left[\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right] = -8\pi p \qquad (15)$$

$$\frac{f(Q,T)}{2} - 2f_Q\left[\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC}\right] = 8\pi\rho + 8\pi G(\rho + p) \qquad (16)$$

$$mf_Q\left[\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\right]=0 \qquad (17)$$

The overhead dot represents the derivative with respect to cosmic t. In this case, f_Q and $8\pi G \equiv f_T$ represent differentiation with respect to Q and T respectively.

We define the following physical parameters, which are important in solving field equations and cosmological observations.

The spatial volume V and average scale factor $\mathcal{R}$ are defined as.

$$V=\mathcal{R}^3=ABC \qquad (18)$$

The Hubble parameter is defined as.

$$H=\frac{\dot{\mathcal{R}}}{\mathcal{R}}=\frac{1}{3}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right) \qquad (19)$$

The deceleration parameter, expansion scalar θ and shear scalar σ^2 are defined as.

$$q=-\frac{\mathcal{R}\ddot{\mathcal{R}}}{\dot{\mathcal{R}}^2}=-1+\frac{d}{dt}\left(\frac{1}{H}\right) \qquad (20)$$

$$\theta=3H \qquad (21)$$

$$\sigma^2=\frac{1}{2}\left(\sum_{i=1}^{3}H_i^2-3H^2\right) \qquad (22)$$

The anisotropy parameter A_m of the expansion is characterized by the directional Hubble parameters, and the mean Hubble parameter is given as.

$$A_m=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i-H}{H}\right)^2=\frac{2\sigma^2}{3H^2} \qquad (23)$$

WhereH_1, H_2 and H_3 are directional Hubble parameters in the direction of x, y and $z-axis$, respectively, and $H_1=\frac{\dot{A}}{A}$, $H_2=\frac{\dot{B}}{B}$ and $H_3=\frac{\dot{C}}{C}$

4. Solutions of Bianchi Type VI Model

The solution of Eq. (17) yields.

$$C=c_1B \qquad (24)$$

Where $c_1>0$ is the constant of integration. Without loss of generality, we take $c_1=1$ for the sake of simplicity. Using the value of C in the above equations (13)-(16), we obtain.

$$\frac{f(Q,T)}{2}-f_Q\left[\frac{2\ddot{B}}{B}+\frac{2\dot{A}\dot{B}}{AB}+\frac{2\dot{B}^2}{B^2}+\frac{2m^2}{A^2}\right]-2\dot{f}_Q\frac{\dot{B}}{B}=-8\pi p \qquad (25)$$

$$\frac{f(Q,T)}{2}-f_Q\left[\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{3\dot{A}\dot{B}}{AB}+\frac{\dot{B}^2}{B^2}\right]-\dot{f}_Q\left[\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right]=-8\pi p \qquad (26)$$

$$\frac{f(Q,T)}{2}-2f_Q\left[\frac{2\dot{A}\dot{B}}{AB}+\frac{\dot{B}^2}{B^2}\right]=8\pi\rho+8\pi G(\rho+p) \qquad (27)$$

From Eq. (25) and (26) we get.

$$\frac{f(Q,T)}{2}-\frac{f_Q}{2}\left[\frac{\ddot{A}}{A}+\frac{3\ddot{B}}{B}+\frac{5\dot{A}\dot{B}}{AB}+\frac{3\dot{B}^2}{B^2}+\frac{2m^2}{A^2}\right]-\frac{\dot{f}_Q}{2}\left[\frac{3\dot{B}}{B}+\frac{\dot{A}}{A}\right]=-8\pi p \qquad (28)$$

Therefore, Eq. (27) becomes.

$$\frac{f(Q,T)}{2}-\frac{2f_Q}{1+G}\left[\frac{2\dot{A}\dot{B}}{AB}+\frac{\dot{B}^2}{B^2}\right]-\frac{f_QG}{2(1+G)}\left[\frac{\ddot{A}}{A}+\frac{3\ddot{B}}{B}+\frac{5\dot{A}\dot{B}}{AB}+\frac{3\dot{B}^2}{B^2}+\frac{2m^2}{A^2}\right]$$
$$-\frac{\dot{f}_QG}{2(1+G)}\left[\frac{3\dot{B}}{B}+\frac{\dot{A}}{A}\right]=8\pi\rho \qquad (29)$$

We assume that the scalar expansion is proportional to the shear scalar. i.e., $\theta \propto \sigma$, which leads to a relation between the metric functions as follows.

$$A=B^n \qquad (30)$$

Where $n\neq 1$ is a positive constant. Using this relation in Eqs. (28) and (29), it follows that.

$$f(Q,T)-f_Q\left[\frac{\dot{B}^2}{B^2}(n^2+4n+3)+\frac{\ddot{B}}{B}(n+3)+\frac{2m^2}{B^{2n}}\right]-\frac{\dot{f}_Q\dot{B}}{B}(n+3)=-16\pi p \qquad (31)$$

$$f(Q,T) - \frac{2f_Q\dot{B}^2}{(1+G)B^2}(4n+2) - \frac{f_Q G}{(1+G)}\left[\frac{\dot{B}^2}{B^2}(n^2+4n+3) + \frac{\ddot{B}}{B}(n+3) + \frac{2m^2}{B^{2n}}\right]$$
$$-\frac{f_Q\dot{B}G}{(1+G)B}(n+3) = 16\pi\rho \quad (32)$$

The scale factor present in the Bianchi type-VI metric is considered as [27,28].

$$\mathcal{R}(t) = [1 + a_0(t/t_0)^2]^{\beta} exp\left[\frac{1}{(\alpha-1)}\left(\frac{t_s-t}{t_0}\right)^{1-\alpha}\right] \quad (33)$$

Where β, α are dimensionless parameters, while t_s and t_0 have the dimesions of time. The parameter t_o is a fiducial time taken to make the above expression dimensionally correct, and we take $t_0 = 1BY$ (Billion Years). We also assume that $t \leq t_s$. In effect, the scale factor can be re-written as.

$$\mathcal{R}(t) = [1 + a_0 t^2]^{\beta}\, exp\left[\frac{1}{(\alpha-1)}(t_s - t)^{1-\alpha}\right] \quad (34)$$

The above scale factor can be written as a product of $\mathcal{R}_1(t)$ and $\mathcal{R}_2(t)$ respectively, where $\mathcal{R}_1(t) = [1 + a_0 t^2]^{\beta}$ and $\mathcal{R}_2(t) = exp\left[\frac{1}{(\alpha-1)}(t_s - t)^{1-\alpha}\right]$. One may get non-singular ekpyrotic bounce with $\beta < 1/6$ for $R_1(t)$, however for the large positive cosmic time, $\mathcal{R}_1(t)$ goes as $t^{2\beta}$ which is incompatible in getting viable dark energy model. The goal of the scale factor $\mathcal{R}_2(t)$ is to obtain a feasible dark energy epoch at a later period. Because of exponential behavior, $\mathcal{R}_2(t)$ plays essentially no part in the universe's contracting stage; hence, the bouncing behavior is solely governed by $\mathcal{R}_1(t)$. However, the time of bounce is slightly altered by the presence of $\mathcal{R}_2(t)$. The scale factor $\mathcal{R}(t)$ of Eq. (34) appears to emphasize, in particular, the unification of an ekpyrotic bounce to a feasible dark energy era, with an intermediate deceleration period in between the bounce and followed by late-time acceleration [27, 28]. Using Eq. (30) and (34) in Eq. (18) we obtained the expressions.

$$A(t) = \left[\{1 + a_0 t^2\}^{\beta} exp\frac{1}{(\alpha-1)}(t_s - t)^{1-\alpha}\right]^{\frac{3n}{n+2}} \quad (35)$$

$$B(t) = \left[\{1 + a_0 t^2\}^{\beta} exp\frac{1}{(\alpha-1)}(t_s - t)^{1-\alpha}\right]^{\frac{3}{n+2}} \quad (36)$$

$$C(t) = \left[\{1 + a_0 t^2\}^{\beta} exp\frac{1}{(\alpha-1)}(t_s - t)^{1-\alpha}\right]^{\frac{3}{n+2}} \quad (37)$$

Now, using Eqs. (35), (36) and (37) in equation (8), we can write the Bianchi type-VI model in the present case as.

$$ds^2 = -dt^2 + [1 + a_0(t/t_0)^2]^{\beta} exp\left[\frac{1}{(\alpha-1)}\left(\frac{t_s-t}{t_0}\right)^{1-\alpha}\right]^{\frac{6n}{n+2}} dx^2 + \left[1 + a_0\left(\frac{t}{t_0}\right)^2\right]^{\beta} exp\left[\frac{1}{(\alpha-1)}\left(\frac{t_s-t}{t_0}\right)^{1-\alpha}\right]^{\frac{6}{n+2}} (e^{-2mx}dy^2 + e^{2mx}dz^2) \quad (38)$$

5. Bouncing Behaviour in $f(Q,T)$ Gravity

Now, using eq. (34), the Hubble parameter can be obtained as.

$$H = \frac{2\beta a_0 t}{1+a_0 t^2} + \frac{1}{(t_s-t)^{\alpha}} \quad (39)$$

In a general bouncing cosmology, the evolution of the universe consists of two eras: the era of contraction, where the Hubble parameter is negative, and an era of expansion, having a positive Hubble parameter. In particular, the bounce phenomena are defined by the conditions $H = 0$ and $\dot{H} > 0$ at the bouncing epoch. The evolutionary behavior of the Hubble parameter is shown in the fig. (1). The Hubble parameter satisfies the conditions for the bouncing phenomena as it comes from the negative value passing through zero at $t < 0$, and then it has a positive behavior.

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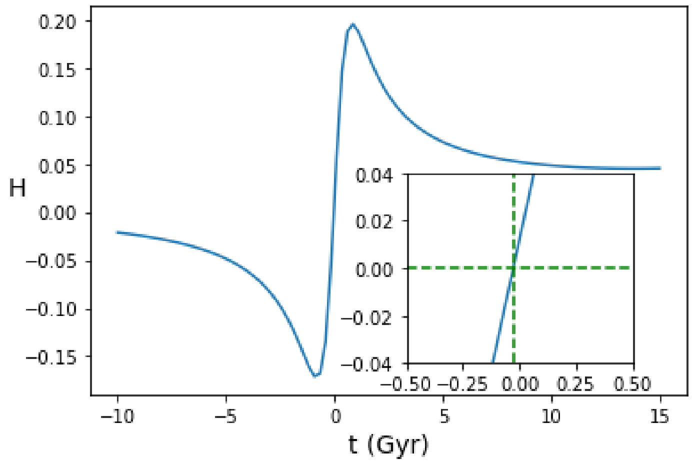

Figure 1.
Time evolution of Hubble Parameter $\boldsymbol{H(t)}$. The inner insert shows the zoomed-in version of $\boldsymbol{H(t)}$ near the bounce $(\boldsymbol{a_0 = 1.5, t_s = 33, \alpha = 1.27, \beta = 0.15})$.

Table 1.
Evolution of Hubble Parameter.

Time (Gyr)	**Nature of *H***
$-\infty < t < -0.026$	Contracting
$t \approx -0.026$	$H \approx 0$
$-0.026 < t < \infty$	Expanding

The deceleration parameter q in cosmology indicates that the decelerating and accelerating aspect of the expansion of the universe is an essential cosmological variable. The deceleration parameter for this model can be expressed as:

$$q = -1 - \frac{(t_s - t)^{\alpha-1}\left[\alpha\left(1 + a_0 t^2\right)^2 - 2a_0\beta\left(a_0 t^2 - 1\right)(t_s - t)^{\alpha+1}\right]}{[a_0 + t(2\beta(t_s - t)^{\alpha+1} + 1]^2} \quad (40)$$

The positive range of $q(t)$ predicts the accelerated universe, and the negative range of $q(t)$ indicates the accelerated universe. From the graphical representation of q which is shown in fig (2), it can be seen that for the small values of cosmic time, $q(t)$ ranges at $-1 < q < 0$, it values gradually decreases to large negative values near the bouncing point. For the large values of cosmic time, $q(t)$ ranges at $-1 < q < 0$. Therefore, it can be stated that the present model predicts the accelerated phase of the universe.

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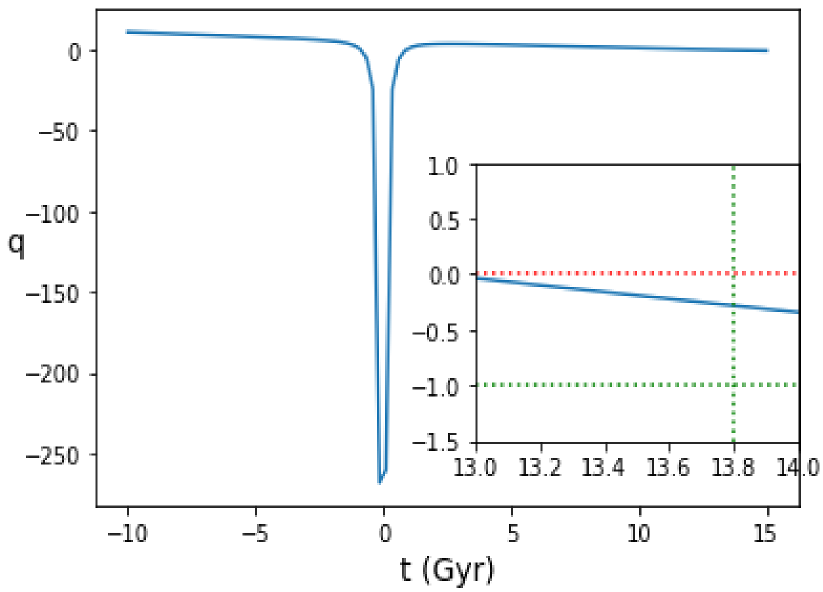

Figure 2.
Time evolution of Deceleration Parameter q(t). The inner insert shows the zoomed-in version of q(t) around the present cosmic time. ($\boldsymbol{a_0 = 1.5, t_s = 33, \alpha = 1.27, \beta = 0.15}$).

Table 2.
Deceleration Parameter at different epoch.

Epoch	**Time (Gyr)**	$\boldsymbol{q}$	**Nature of** $\boldsymbol{q}$
At Bounce	$t = -0.026$	$q < -1$	Super exponential expansion
Early Phase	$t = 3$	3.255 9	Decelerating expansion
At Present	$t = 13.8$	-0.28	Exponential expansion
Late Phase	$t = 16$	-0.904	Exponential expansion

Also, the physical quantities such as Spatial volume of scale factor (V), scalar of expansion (θ), Shear scalar (σ), and mean anisotropy parameter (A_m) are derived as:

$$V = \left[(1 + a_0 t^2)^{\beta} exp \frac{1}{(\alpha-1)} (t_s - t)^{1-\alpha}\right]^3 \quad (41)$$

$$\theta = 3\left[\frac{2\beta a_0 t}{1+a_0 t^2} + \frac{1}{(t_s-t)^{\alpha}}\right] \quad (42)$$

$$\sigma^2 = 3\left[\frac{2\beta a_0 t}{1+a_0 t^2} + \frac{1}{(t_s-t)^{\alpha}}\right]^2 \left(\frac{n-1}{n+2}\right)^2 \quad (43)$$

$$A_m = 2\left(\frac{n-1}{n+2}\right)^2 \quad (44)$$

The graphical representation of Volume V, Shear scalar σ, and expansion scalar θ with time is shown in the fig (3). The spatial volume is finite at $t = 0$, and it increases to a large value with an increase in time. The expansion scalar θ is negative at small values of cosmic time passing through zero and takes the positive values as $t \to \infty$. The Shear Scalar σ decreases positively at the bounce point, after which it increases to its maximal value near the bounce. However, it rapidly decreases and ends up vanishingly small with an increase in time. It can also be observed that the average anisotropy parameter associated with the expansion is constant; hence, the model is anisotropic. For $n = 1$, it becomes isotropic and shear-free.

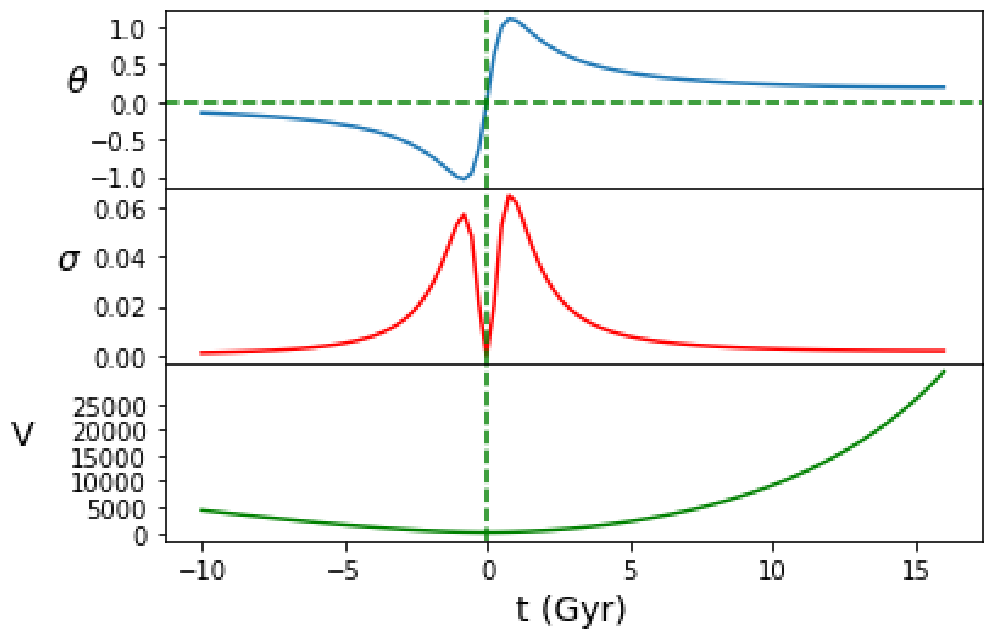

Figure 3.
Time evolution of expansion scalar θ (upper panel), Shear Scalar σ (middle panel) and Volume V (lower panel) $(a_0 = 1.5, t_s = 33, \alpha = 1.27, \beta = 0.15, n = 3)$.

To investigate a viable cosmological scenario in the framework of $f(Q,T)$ gravity theory, a certain assumed form of the functional $f(Q,T)$ must be considered. In this paper, to get the viable cosmological model, we consider the non-linear functional form of $f(Q,T)$ gravity as $f(Q,T) = aQ^k + bT$ where a and b are the free parameters.

5.1. Cosmological Model with $f(Q,T) = aQ^k + bT$

We consider the linear form of the functional in the form $f(Q,T) = aQ^k + bT$ such that $f_Q = akQ^{k-1}$, $\dot{f}_Q = (k-1)\frac{F}{Q}\frac{dQ}{dt}$, $G = \frac{b}{8\pi}$, $G_1 = \frac{G}{1+G}$ and $Q = 2\{\frac{3(2n+1)}{(n+2)}\left[\frac{2\beta a_0 t}{1+a_0t^2} + \frac{1}{(t_s-t)^{\alpha}}\right]^2 + m^2\left[\{1+a_0t^2\}^{\beta} exp\{\frac{1}{(\alpha-1)}(t_s - t)^{1-\alpha}\}\right]^{\frac{-6n}{n+2}}\}$

So, the pressure and energy density can be obtained as.

$$p = \frac{1}{8\pi(4+8G)}\left[-2aQ^k + (2+G-GG_1)\{(n^2+4n+3)f_Q\frac{\dot{B}^2}{B^2} + (n+3)\left(\frac{f_Q\ddot{B}}{B} + \frac{\dot{f}_Q\dot{B}}{B}\right) + \frac{2m^2 f_Q}{B^{2n}}\} + 2G_1(4n+2)f_Q\frac{\dot{B}^2}{B^2}\right] \tag{45}$$

$$\rho = \frac{1}{8\pi(4+8G)}\left[2aQ^k + (3G-(2+3G)G_1)\{(n^2+4n+3)f_Q\frac{\dot{B}^2}{B^2} + (n+3)\left(\frac{f_Q\ddot{B}}{B} + \frac{\dot{f}_Q\dot{B}}{B}\right) + \frac{2m^2 f_Q}{B^{2n}}\} - 2(4n+2)\left(\frac{2}{1+G} + 3G_1\right)f_Q\frac{\dot{B}^2}{B^2}\right] \tag{46}$$

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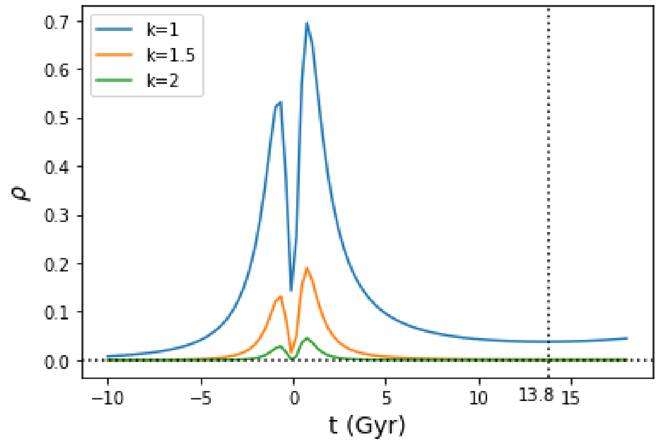

Figure 4.
Time evolution of energy density ρ.

$$(a_0 = 1.5, t_s = 33, \alpha = 1.27, \beta = 0.15, n = 3, a = -80, b = -2, m = 0.42)$$

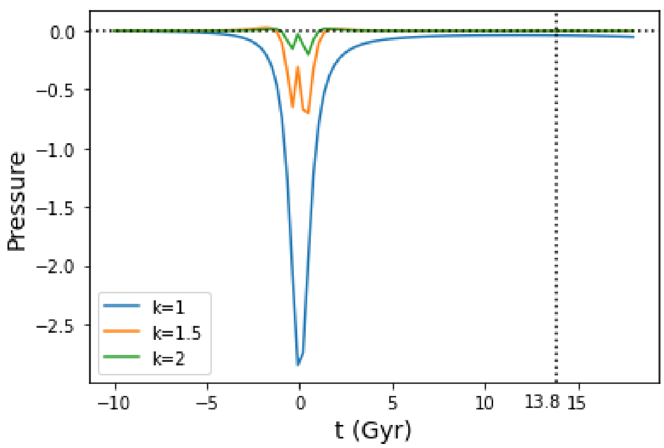

Figure 5.
Time evolution of pressure p.

$$(a_0 = 1.5, t_s = 33, \alpha = 1.27, \beta = 0.15, n = 3, a = -80, b = -2, m = 0.42)$$

The Equation of state parameter can be obtained as $\omega = \frac{p}{\rho}$,

$$\omega = \left[-2aQ^k + (2 + G - GG_1)\{(n^2 + 4n + 3)f_Q \frac{\dot{B}^2}{B^2} + (n + 3)\left(\frac{f_Q \ddot{B}}{B} + \frac{\dot{f_Q}\dot{B}}{B}\right) + \frac{2m^2 f_Q}{B^{2n}}\} + 2G(n + 2)f_Q \frac{\dot{B}^2}{B^2}\right] / \left[2aQ^k + (3G(2 + 3G)G_1)\{(n^2 + 4n + 3)f_Q \frac{\dot{B}^2}{B^2} + (n + 3)\left(\frac{f_Q \ddot{B}}{B} + \frac{\dot{f_Q}\dot{B}}{B}\right) + \frac{2m^2 f_Q}{B^{2n}}\} - 2(4n + 2)\left(\frac{2}{1+G} + 3G_1\right) f_Q \frac{\dot{B}^2}{B^2}\right] \quad (47)$$

We may substitute in equations (45)-(47) as $\frac{\dot{B}}{B} = \frac{3}{n+2}\left[\frac{2a_0\beta t}{1+a_0t^2} + \frac{1}{(t_s-t)^\alpha}\right]$ and $\frac{\ddot{B}}{B} = \frac{3}{n+2}\left[\frac{3}{n+2}\left(\frac{2a_0\beta t}{1+a_0t^2} + \frac{1}{(t_s-t)^\alpha}\right)^2 + \frac{2a_0\beta}{1+a_0t^2}\left(1 - \frac{2}{1+a_0t^2}\right) + \frac{1}{(t_s-t)^{\alpha+1}}\right]$ to get the respective expressions for pressure, energy density and EoS parameter.

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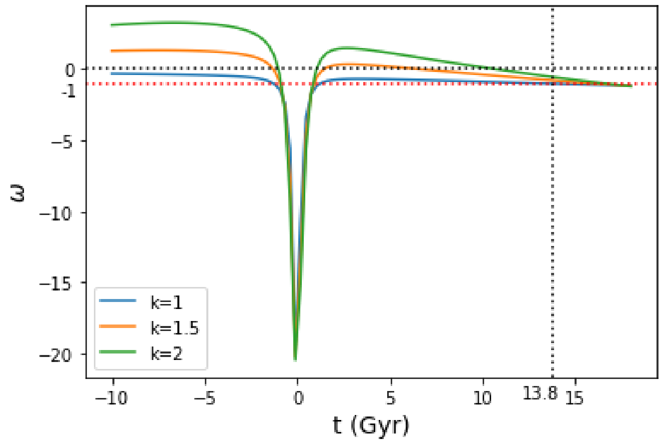

Figure 6.
Time evolution of EoS parameter ω.

$$(a_0 = 1.5, t_s = 33, \alpha = 1.27, \beta = 0.15, n = 3, a = -80, b = -2, m = 0.42)$$

Table 3.
Variation of EoS Parameter (ω) at different Epoch for different values of k.

Epoch	**Time (Gyr)**	$\boldsymbol{\omega(k=1)}$	$\boldsymbol{\omega(k=1.5)}$	$\boldsymbol{\omega(k=2)}$
At Bounce	$t = -0.026$	-22.7555	-22.5472	-22.4445
Early Phase	$t = 3$	-0.6795	0.1839	1.1375
At Present	$t = 13.8$	-1.0157	-0.7838	-0.5489
Late Phase	$t = 16$	-1.2113	-1.3355	-1.4582

The graphical behaviour of energy density and pressure are shown in fig (4) and fig (5), respectively, for different values of k. From fig (4), it can be seen that the energy density of the model is positive throughout the evolution of the universe for the values of k. The behaviour of the pressure for the present model shows the negative values throughout the cosmic evolution for $k = 1$. In contrast, for $k = 1.5$ and $k = 2$, pressure takes the positive values at the pre-bounce and post-bounce epoch for some cosmic time and then varies to negative values in the present and late phases of the universe. The negative values of the pressure might be an indication of accelerated expansion of the universe. Also, the graphical behaviour of the EoS (Equation of State) parameter is represented in fig (6). It is shown that at the bouncing point, the EoS parameter $\omega < -1$ satisfies the phantom-dominated phase. The EoS parameter shows the quintessence phase near the bounce for $k = 1$ while for $k = 1.5$ and $k = 2$, it takes the positive values for some time during the decelerating era. The EoS parameter ω at different epochs can be seen in the table (3).

6. Energy Conditions

In this work, we consider energy conditions to test the validity of the models in the context of cosmic acceleration. There are several forms of energy conditions, such as null energy conditions (NEC), weak energy conditions (WEC), strong energy conditions (SEC), and dominant energy conditions (DEC) are given for the content of the universe in the form of a viscous fluid in $f(Q, T)$ gravity as follows [36]:

A. Null energy conditions (NEC) ⇔ $\rho + p \geq 0$.
B. Weak energy conditions (WEC) ⇔ $\rho + p \geq$and $\rho \geq 0$.
C. Strong energy conditions (SEC) ⇔ $\rho + 3p \geq 0$.
D. Dominant energy conditions (DEC) ⇔ $\rho - |\{p\}| \geq 0$ and $\rho \geq 0$.

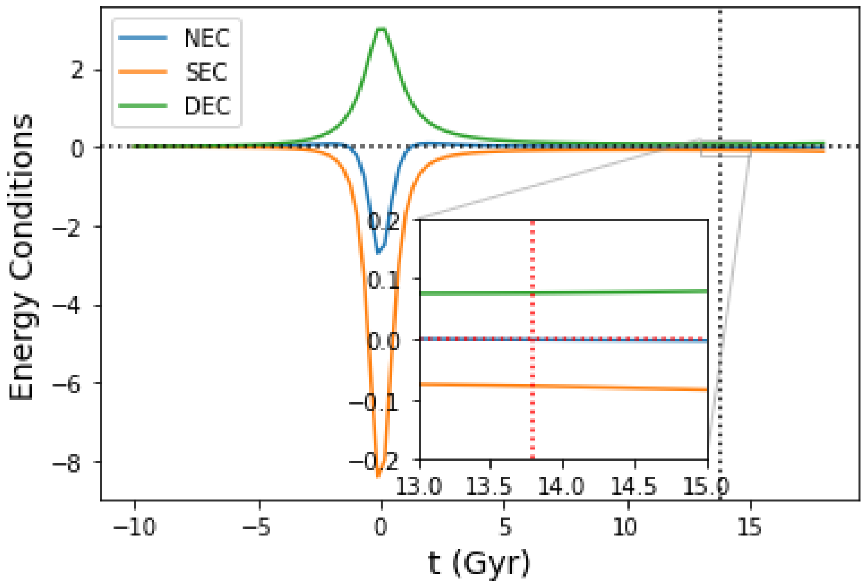

Figure 7.
Time evolution of energy conditions for $k = 1$. The inner insert shows the zoomed-in version around the present cosmic time.

$$(a_0 = 1.5, t_s = 33, \alpha = 1.27, \beta = 0.15, n = 3, a = -80, b = -2, m = 0.42)$$

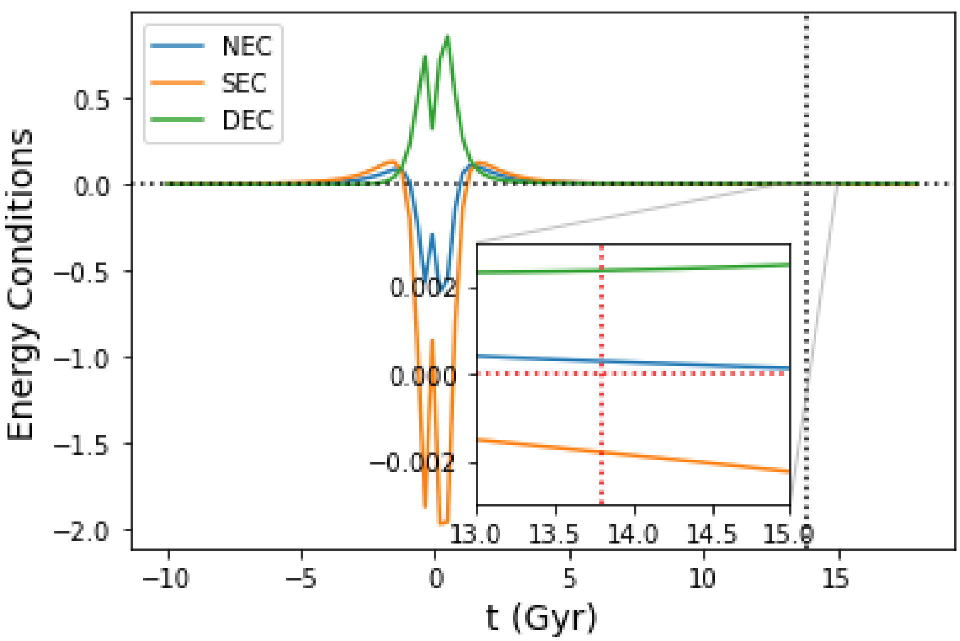

Figure 8.
Time evolution of energy conditions for $k = 1.5$. The inner insert shows the zoomed-in version around the present cosmic time.

$$(a_0 = 1.5, t_s = 33, \alpha = 1.27, \beta = 0.15, n = 3, a = -80, b = -2, m = 0.42)$$

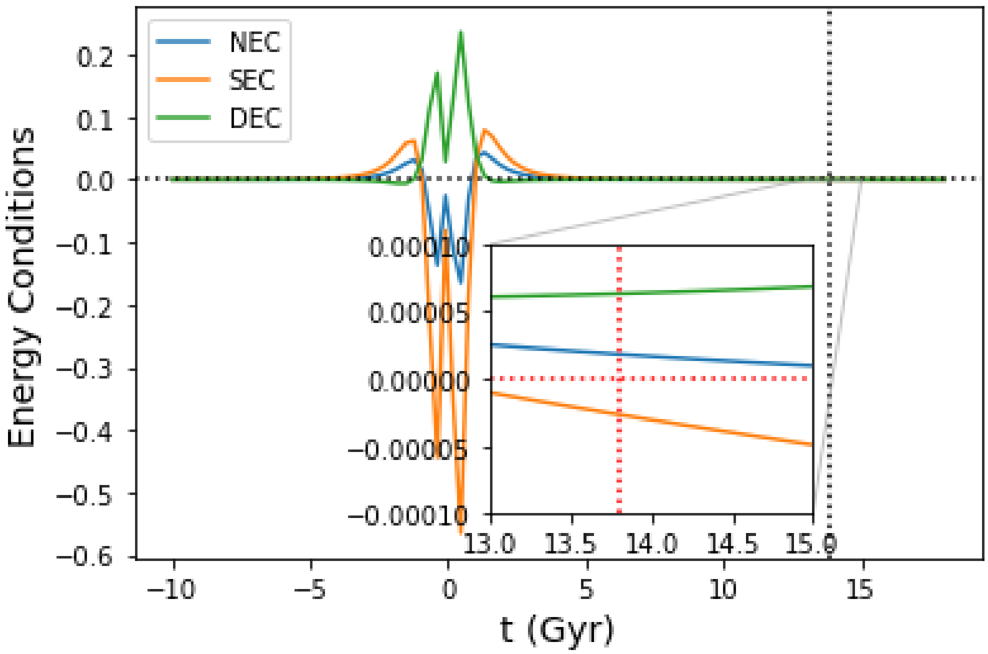

Figure 9.
Time evolution of energy conditions for $k = 2$. The inner insert shows the zoomed-in version around the present cosmic time.

$$(a_0 = 1.5, t_s = 33, \alpha = 1.27, \beta = 0.15, n = 3, a = -80, b = -2, m = 0.42)$$

The energy conditions are shown in the fig (7), fig (8) and fig (9) for $k = 1, 1.5$ and 2 respectively. It shows that the dominant energy condition (DEC) is in the positive domain throughout the evolution of the universe for all the values of k. The null energy condition (NEC) has been in the negative domain for some time near the neighborhood of the bouncing point. The strong energy condition (SEC) is violated throughout the cosmic evolution for $k = 1$. At the same time, for $k = 1.5$ and $k = 2$, the SEC remains positive for some time during the decelerated era. Then, it varies to the negative domain at the present and late phases of cosmic evolution. The negative value of SEC also ensures the accelerating expansion of the universe, according to recent observational data.

7. Stability Analysis

The stability of $f(Q,T)$ gravity is studied in this section. The universe is considered to be filled with a perfect fluid for which we may introduce the adiabatic speed of sound $C_s^2 = \frac{\partial p}{\partial \rho}$. For thermodynamically or mechanically stable systems, the sound velocity $C_s^2 = \frac{\partial p}{\partial \rho}$ should remain positive. Also, in order to ensure a mechanical stability, $C_s^2 = \frac{\partial p}{\partial \rho}$ should be less than 1. Therefore, the region bounded by $0 \leq C_s^2 \leq 1$ provides stable solutions. The violation of NEC can lead to the formation of ghost fields that suggest dangerous instabilities at both classical and quantum levels [37].

In fig (10), the stability of the $f(Q,T)$ gravity is depicted. In the vicinity of the bouncing point, the stability condition is not satisfied. Furthermore, in the present time, the stability condition is satisfied for $k = 1.5$ and $k = 2$, while for $k = 1$, the stability condition is not satisfied as C_s^2 remains negative. In view of these findings, the model may exhibit some instabilities in the late stages of cosmic evolution.

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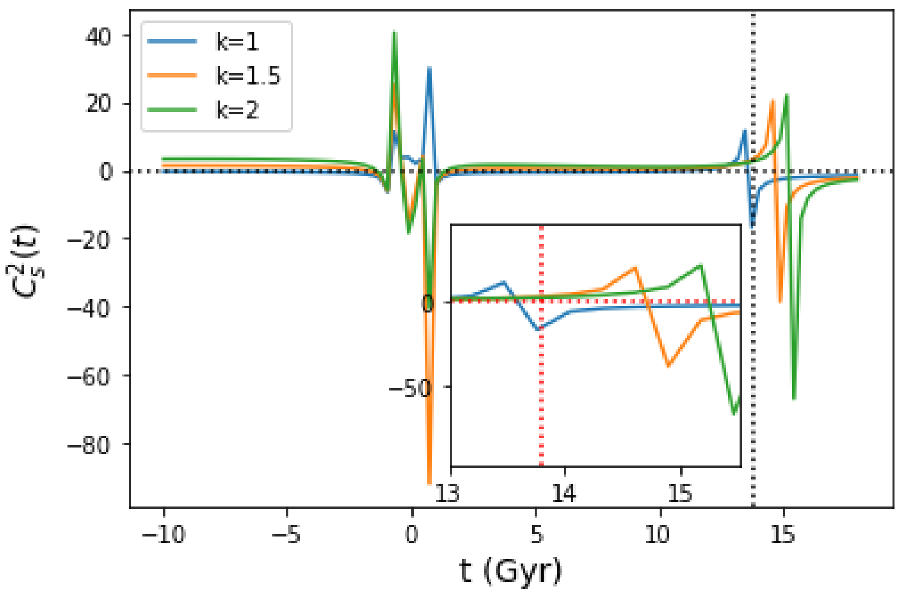

Figure 10.
The behaviour of sound velocity as functions of cosmic time. The inner insert shows the zoomed-in version of $C_s^2(t)$ around the present cosmic time.

$$(a_0 = 1.5, t_s = 33, \alpha = 1.27, \beta = 0.15, n = 3, a = -80, b = -2, m = 0.42)$$

8. Conclusion

In this study, we explored the universe's bouncing behaviour in the Bianchi type-VI metric within the framework of $f(Q,T)$ gravity. A generic functional form of $f(Q,T)$, as previously examined in the literature, has been taken into consideration. The functional form under consideration is $f(Q,T) = aQ^k + bT$, where a, b and k are model parameters. We investigated the possibility of a bouncing cosmological scenario using a well-known scale factor $\mathcal{R}(t) = [1 + a_0 t^2]^\beta exp\left[\frac{1}{(\alpha-1)}(t_s - t)^{1-\alpha}\right]$. The analysis of the bouncing cosmological model provides valuable insights into the dynamics and evolution of the universe under the context of $f(Q,T)$ gravity. The following briefly summarizes the main conclusions and findings:

- The behaviour of the scale factor and Hubble parameter exhibits the characteristics consistent with the prescribed bouncing cosmology as the Hubble parameter evolves from the negative value passing through $H = 0$, then it shows the positive behaviour. The deceleration parameter at the bouncing point shows the super-exponential nature. In the early universe, it became positive for quite some time, which shows the decelerating expansion of the universe. The deceleration parameter at present shows the exponential expansion of the universe.
- The spatial volume is finite at the bouncing point, and post bounce, the volume increases with an increase in time. The shear scalar and expansion scalar decrease positively during the bounce era with an increase in time. The anisotropy parameter associated with the expansion is constant, and for $n = 1$, the anisotropy parameter becomes zero, which indicates the isotropic and shear-free model.
- The model predicts that the energy density is positive throughout the cosmic evolution while the pressure profile is negative, which justifies the current cosmic expansion. Furthermore, the EoS parameter crosses the phantom divide line $(\omega = -1)$ in the vicinity of bouncing point. During the early universe, near the bouncing point, the model exhibits matter, and radiation dominated the

era for $k = 1.5$ and $k = 2$. The model shows the Quintessence behavior at the present cosmic time, and it exhibits the phantom behavior with increasing energy density during late time cosmic evolution.

- For the present model, the null energy condition is annihilated in the neighbourhood of the bouncing point. The intense energy condition was satisfied during the decelerating expansion era for the values of $k = 1.5$ and $k = 2$. During the accelerated expansion, the SEC is annihilated for all the values of k, which also justifies the current cosmic expansion.
- The stability of the present model shows that in the bouncing point, the sound velocity remains negative, which depicts the instability. The sound velocity gives the positive value in the present cosmic time for the certain value of k.

Based on the discussion mentioned above, the proposed $f(Q,T)$ gravity model offers a useful framework to comprehend the dynamics of the bouncing universe.

Transparency:

The authors confirm that the manuscript is an honest, accurate, and transparent account of the study; that no vital features of the study have been omitted; and that any discrepancies from the study as planned have been explained. This study followed all ethical practices during writing.

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