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Interactions between main central American and the us stock markets with financial contagion: An application of FIEC-FIAPGARCH-DCC model

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Abstract: Given the limited empirical evidence on the relationship between the main Central American stock markets (Costa Rica and Panama) and the U.S. stock market, this study aims to empirically examine volatility spillovers, long memory characteristics, asymmetries, and dynamic conditional correlation (DCC) effects among these markets. In addition, we investigate the contagion effects of major financial events across the three stock markets using a multivariate Fractionally Integrated Error Correction (FIEC) model and a Fractionally Integrated Asymmetric Power GARCH (FIAPGARCH) model within DCC framework. Structural breaks associated with the subprime mortgage and Global Financial Crisis (GFC), as well as European Debt Crisis (EDC), are explicitly incorporated into the analysis. Our findings reveal significant cross-market effects, evidence of long-term volatility dependence, and asymmetric volatility responses to positive and negative shocks. Moreover, estimating the power term parameters allows for a nuanced understanding of variance heterogeneity across markets. The observed market interactions stem not only from fundamental co-movement but also from contagion effects triggered by systemic events such as the GFC and EDC. These insights provide valuable implications for financial risk managers, regulators and international investors, offering guidance for portfolio diversification, risk assessment, and strategic asset allocation decisions in an increasingly interconnected global market environment.

Keywords: *Asymmetric volatility, Contagion effects, FIEC- FIAPGARCH-DCC Model, Long memory, Volatility spillovers.*

1. Introduction

This study attempts to investigate the interrelationship among Costa Rica and Panama stock markets in Central America with the US stock market and contagion effects of the global financial crises via a multivariate FIEC-FIAPGARCH-DCC models. These empirical models not only measure returns and return volatility spillover, long memory and asymmetric effects among these three stock markets but also distinguish the contagion effect(direct crisis impact) from spillover effect(general market influence).

Over the past two decades, financial liberalization and global market integration have heightened interest in cross-border capital flows and diversification. This study examines the interdependence between Central American and US stock markets, particularly during major events like the global financial crisis. As emerging markets attract more investment post-crisis, understanding these linkages is vital for informed portfolio diversification and risk management. Market liberalization has boosted international investment and capital flows, making information transmission key to stock market co-movement. The globalization of finance and faster information spread have heightened the risk of financial crises, as a crisis in one country can quickly spread globally, as seen with the GFC and EDC. These crises show that stock market integration involves both regional co-movement and contagion from special events. This study focuses on how the GFC and EDC may influence stock returns and

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volatility in Central America, where trade and investment between Costa Rica, Panama, the U.S., and Europe have grown significantly over the past two decades. The goal is to assess whether these markets are impacted by the contagion effects of these global financial events.

Contagion can be studied by observing the increased co-movement during a crisis compared to tranquil periods. In this study, we define contagion as a significant rise in cross-market linkages following a crisis in one country or group of countries. We aim to explore whether contagion affects the stock markets of Costa Rica and Panama. Specifically, we focus on how stock market volatility in developed economies, like the U.S. and Europe, spreads to these Central American markets. Using multivariate co-integration techniques, we analyze the correlation between these emerging markets and U.S. stock prices. Costa Rica and Panama were chosen due to their rapid development, increased market liberalization, and growing financial integration, which have attracted international investors seeking diversification and hedging opportunities.

Information shocks drive market volatility, passing return and volatility effects across markets. This study explores information transmission and spillovers in market interactions, including long-memory effects and asymmetric volatility triggered more by negative price changes. Using dynamic conditional correlation (DCC) models, we analyze these effects across the Costa Rican, Panamanian, and US stock markets. Volatility spillovers, long-memory effects, and asymmetries with models like FIAPGARCH-DCC showing how financial data behaves under these phenomena. Previous studies, for example, Aloui [1], Dimitriou, et al. [2], Karanasos, et al. [3], Wajdi, et al. [4], Abed, et al. [5] and Goudarzi, et al. [6] highlight the limitations of GARCH models in explaining long memory and volatility asymmetries.

This study moreover focuses on slow-decaying autocorrelation and mean reversion in stock markets, combining the FIAPGARCH-DCC and fractional error correction (FIEC) models [7] to explore market interactions. By integrating these models as FIEC-FIAPGARCH-DCC, we analyze relationships between Central American markets, like Costa Rica and Panama, and the US stock market. The findings offer valuable insights into portfolio diversification and hedging, highlighting Central America's growing appeal to international investors. The empirical results aim to guide better asset management, allocation, and risk decisions for investors and policymakers.

The remainder of this study is structured as follows. In section 2 we detail the methodology framework related to this study. Section 3 presents the empirical results and analyses. While section 4 reports the concluding remarks.

2. Methodology Framework

2.1. Long Memory Properties and Fractionally Integrated Process

This study uses a fractionally integrated process to analyze the long-memory behaviors of financial variables. Cheung [8] suggested that if the unit root test's null hypothesis isn't rejected, the time series is not I (0) stationary, but it may still exhibit long-term, slow mean reversion. Long memory reflects persistent autocorrelation and gradual decay over time [9-13]. It indicates the lasting influence of past periods on the present. In the ARFIMA (p, d, q) model, co-integrated variables are represented by Z_t, and when d=0, it simplifies to a VAR model. For 0<d<1, the process is described by a fractionally integrated error correction (FIEC) model process, $[(1-L)^d-(1-L)]Z_t$ [14-16].

2.2. The APGARCH and FIAPGARCH Model

The Asymmetric Power GARCH (APGARCH) model by Ding, et al. [17] captures the conditional variance's response to past volatility. It accounts for both fractional integration and the skewed, leptokurtic distribution of innovations, providing highly accurate one-day-ahead volatility forecasts [18]. The general APGARCH (p, q) model is expressed by the following conditional variance equation:

$$\sigma_t^\delta = \omega + \sum_{i=1}^{q} \alpha_i \left(|\varepsilon_{t-i}| - \gamma_i\, \varepsilon_{t-i}\right)^\delta + \sum_{j+1}^{p} \beta_j\, \sigma_{t-j}^\delta \qquad (1)$$

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With an exponent $\delta > 0$ and asymmetry coefficient $-1 < \gamma_i < 1$ (for i=1, q). When $\gamma i > 0$ (<0), negative (positive) shocks give rise to higher volatility than positive (negative) shocks.

Under $(p = 1, q = 1)$ the APGARCH (1, 1) models is represented as follow:

$$\sigma_t^\delta = \eta + \alpha(L)(|\varepsilon_t| - \gamma\varepsilon_t)^\delta + \beta(L)\,\sigma_t^\delta \quad (2)$$

Where $\alpha(L) = \sum_{i=1}^{q} \alpha_i$, $\beta(L) = \sum_{j+1}^{p} \beta_j$ and L denotes the lag operator

The power term coefficient (δ) measures the degree of heterogeneous variance, while γ captures the effects of asymmetric shocks. α and β represent the standard ARCH and GARCH parameters, γ is the leverage parameter, and δ is the power term parameter. A positive (negative) γ indicates that negative (positive) past shocks affect current volatility more than positive (negative) shocks. The model applies a Box and Cox [19] transformation to both the conditional standard deviation and asymmetric absolute innovations. In the APGARCH model, good news (ε_{t-1}>0) and bad news (ε_{t-1}<0) affect future volatility differently, as the conditional variance depends on both the magnitude and sign of ε_{t-1}.

The FIAPGARCH model combines the long memory properties of the FIGARCH model [20] with the asymmetric power GARCH (APGARCH) model proposed by Ding, et al. [17] to extend the FIGARCH model to account for different degree of heterogeneous variance and asymmetric dynamics [2-5]. Accordingly, the fractionally integrated with APGARCH (1, 1), the FIAPGARCH (1, 1) model can be shown as follows:

$$(1 - \beta L)\,\sigma_t^\delta = \omega + \left((1 - \beta L) - (1 - \alpha L)\,(1 - L)^d\right)(|\varepsilon_t| - \gamma\varepsilon_t)^\delta \quad (3)$$

Alternatively,

$$\sigma_t^\delta = \omega + \beta\sigma_{t-1}^\delta + \left((1 - \beta L) - (1 - \alpha L)\,(1 - L)^d\right)(|\varepsilon_t| - \gamma\varepsilon_t)^\delta \quad (4)$$

If $\beta = 0$ we obtain FIAPARCH (1) as follows:

$$\sigma_t^\delta = \omega + \left(1 - (1 - \alpha L)\,(1 - L)^d\right)(|\varepsilon_t| - \gamma\varepsilon_t)^\delta \quad (5)$$

Where L denotes the lag operator, d is the $0 \le d \le 1$ functional differencing parameter, β denotes the autoregressive parameters, α represents the moving average parameters of the conditional variance equation, δ represents the optimal power transformation, γ represents the asymmetry parameter and $\gamma < 1$ ensures that positive and negative innovations of the same size can have asymmetric effects on the conditional variance.

2.3. Dynamic Conditional Correlation (DCC)-GARCH Model

Engle [21] introduced the Dynamic Conditional Correlation (DCC) model, a multivariate model that can be viewed as a nonlinear combination of univariate GARCH models. The DCC is a generalized version of Bollerslev [22] Constant Conditional Correlation (CCC) model.

Utilizing the conditional correlation coefficients and variances of the *i* and *j* stock returns to parameterize the stock return covariance matrix H_t, we specify a multivariate conditional variance:

$$H_t = D_t R_t D_t \quad (6)$$

Where R_t is the (n x n) time-varying correlation matrix; D_t is the (n x n) diagonal matrix of time-varying standard deviations from univariate GARCH models:

$$D_t = \mathrm{diag}(h_{11t}^{1/2} \ldots h_{nnt}^{1/2}) \quad (7)$$

With $h_{iit}^{1/2}$ ith diagonal, $i = 1,2,\dots,n;$ h_{iit} can be any univariate GARCH model

The evolution of the correlation in the DCC model is given by:

$$Q_t = (1-\alpha-\beta)\bar{Q} + \alpha u_{t-1}u'_{t-1} + \beta Q_{t-1} \qquad (8)$$

Where $u_{i,t} = \frac{\varepsilon_{i,1}}{\sqrt{h_{ii,t}}}$ and $Q_t = (q_{ij,1})$, the n x n time-varying covariance matrix of is $u_{t,}$ $\bar{Q} = E[u_1 u'_t]$ is the n x n unconditional variance matrix of $u_{t,}$ and $\alpha\ and\ \beta$ are nonnegative scalar parameters satisfying$(\alpha+\beta) < 1$. Since Q_t does not generally have ones on the diagonal, we scale it to obtain a proper correlation matrix R_t. Thus,

$$R_t = (diag(Q_t))^{-1/2} Q_t = (diag(Q_t))^{-1/2} \qquad (9)$$

Where $(diag(Q_t))^{-1/2} = diag(1/\sqrt{q_{11,t}}, \dots, 1/\sqrt{q_{nn,t}})$

Now R_t is a correlation matrix with ones on the diagonal and off-diagonal elements less than one in absolute value, as long as Q_t is positive definite. A typical element of Q_t is given by:

$$q_{ij,t} = (1-\alpha-\beta)\bar{\rho}_{ij} + \alpha u_{i,t-1}u_{j,t-1} + \beta q_{ij\ t-1} \qquad (10)$$

Where $\bar{\rho}_{ij}$ is the unconditional correlation of $u_{i,t}\ and\ u_{j,t}$. $(q_{ij,t})$is the n x n time-varying covariance matrix of u_t, $u_{i,t-1}u_{j,t-1}$ is the cross-product of time-varying standardized residuals matrix. α and β are non-negative scalars that $\alpha + \beta < 1$.

Engle [21] defines conditional correlations as a weighted sum of past correlations, modeling Q_t as a GARCH process and transforming it into a correlation matrix. He proposed a two-step estimation procedure for the DCC model. In the first step,the conditional variance $H_t = D_t R_t D_t$ separates volatility and correlation. Replacing R_t with the identity matrix simplifies the likelihood function to a sum of N univariate models' log-likelihoods. In the second step, R_t parameters are estimated. This method provides consistent but inefficient estimators. The log-likelihood of the two-step procedure can be compared with the one-step method and other models.

Now R_t is a correlation matrix with ones on the diagonal and off-diagonal elements less than one in absolute value, as long as $Q_{12,t}$ if two markets 1 and 2 is positive definite. A typical element of R_t is obtained from Q_t of the form as:

$$q_{11,t} = (1-\alpha-\beta)\bar{\rho}_{11} + \alpha u^2_{1,t-1} + \beta q_{11\ t-1} \qquad (11)$$

$$q_{22,t} = (1-\alpha-\beta)\bar{\rho}_{22} + \alpha u^2_{1,t-1} + \beta q_{22\ t-1} \qquad (12)$$

$$q_{12,t} = (1-\alpha-\beta)\bar{\rho}_{12} + a u_{1,t-1}u_{2,t-1} + \beta q_{12\ t-1} \qquad (13)$$

Under DCC we can obtain the conditional correlation coefficient for any two markets 1 and 2 as follows:

$$\rho_{12,t} = \frac{q_{12,t}}{\sqrt{q_{11,t}q_{22,t}}} = \frac{h_{12,t}}{\sqrt{h_{11,t}h_{22,t}}} \qquad (14)$$

Expressing the correlation coefficient in a bivariate case, we have:

$$\rho_{12,t} = \frac{[\bar{\rho}_{12}(1-a-\beta)+\alpha\,(u_{1,t-1}u_{2,t-1})+\beta(q_{12,t-1})]}{\sqrt{\bar{\rho}_{11}\,(1-\alpha-\beta)+\alpha(u_{1,t-1}^2)+\beta(q_{11,t-1})}\sqrt{\bar{\rho}_{22}\,(1-\alpha-\beta)+\alpha(u_{2,t-1}^2)+\beta(q_{22,t-1})}} \quad (15)$$

Based on the (7) equation we derive the conditional covariance equation as follows as equation (9):

$$h_{12,t} = \rho_{12,t} \times \sqrt{h_{11,t}h_{22,t}} = \frac{q_{12,t}}{\sqrt{q_{11,t}q_{22,t}}} \times \sqrt{h_{11,t}h_{22,t}} \quad (16)$$

$$h_{12,t} = \frac{[\bar{\rho}_{12}+\alpha\,(u_{1,t-1}u_{2,t-1}-\bar{\rho}_{12})+\beta(q_{12,t-1}-\bar{\rho}_{12})]\sqrt{h_{11,t}h_{22,t}}}{\sqrt{\bar{\rho}_{11}+\alpha\,(u_{1,t-1}^2-\bar{\rho}_{11})+\beta(q_{11,t-1}-\bar{\rho}_{11})}\sqrt{\bar{\rho}_{22}+\alpha\,(u_{2,t-1}^2-\bar{\rho}_{22})+\beta(q_{22,t-1}-\bar{\rho}_{22})}} \quad (17)$$

or

$$h_{12,t} = \frac{[\bar{\rho}_{12}(1-a-\beta)+\alpha\,(u_{1,t-1}u_{2,t-1})+\beta(q_{12,t-1})]\sqrt{h_{11,t}h_{22,t}}}{\sqrt{\bar{\rho}_{11}(1-a-\beta)+\alpha\,(u_{1,t-1}^2)+\beta(q_{11,t-1})}\sqrt{\bar{\rho}_{22}(1-a-\beta)+\alpha\,(u_{2,t-1}^2)+\beta(q_{22,t-1})}} \quad (18)$$

When $\alpha+\beta=0$, the model reduces to Bollerslev [22]. The DCC model integrates two GARCH (1,1) processes for stock returns and disturbances. Consequently, the log-likelihood function accounts for stock return fluctuations and correlations.

$$i(\theta,\varphi) = \left[-\frac{1}{2}\sum_t(nlog(2\pi) + \log|D_t|^2 + e_t^1 D_t^{-2} e_t)\right] + \left[-\frac{1}{2}\sum_t(\log\,|R_t| + e_t^1 R_t^{-1} e_t)\right] \quad (19)$$

The first part of the likelihood function represents volatility, as the sum of individual GARCH likelihoods. In the first stage, we maximize the log-likelihood over the parameters in D_t. In the second stage, we estimate correlation coefficients. To obtain more accurate coefficient estimates, we apply the BHHH method to fulfill [23].

3. Empirical Results and Analysis

3.1. Data and Preliminary Analyses

We obtained weekly stock price index data from Bloomberg database for Costa Rica, Panama, and the US (S&P500) from 1st week of January 1994 to 4th week of December 2024. Their respective price indices are Costa Rica Stock Exchange index (CRSMBCT), Panama Stock Exchange Index (BVPS) and one of the United States major Stock Exchange Index (S&P500).The data is expressed as stock returns, calculated as the logarithmic change in closing prices from one week to the next: $R_{i,t} = \left(lnP_{i,t} - lnP_{i,t-1}\right) X\ 100$. Where $R_{i,t}$ is the weekly return for stock market i in week t, and $P_{i,t}$ and $P_{i,t-1}$ are the closing prices for that week and the previous week, respectively. This transformation converts the closing prices into weekly stock price returns.

This section provides descriptive statistics for the logarithms of stock prices and stock price returns, including mean, median, maximum, minimum, standard deviation, skewness, kurtosis, Jarque-Bera normality test, and Ljung-Box serial correlation test. An ARCH effect is observed in the statistics for the logarithms and returns of Costa Rica, Panama, and the US, as shown in Table 1. Costa Rica has the highest mean for both the logarithmic stock price (9.1837) and stock return (0.002856), while Panama has the lowest logarithmic mean (4.9718), and the US has the lowest return mean (0.001309). Regarding stock risk, Costa Rica shows the highest standard deviations for both the logarithmic stock price (0.9263) and stock return (0.02223), while the US has the lowest standard deviation for the logarithmic stock price (0.3681), and Panama has the lowest for stock returns (0.01793). The skewness measures indicate that logarithmic stock prices in all three markets are left-skewed (less than 0) and platykurtic less than 3 except for the US which is leptokurtic (3.3447) relative to the normal distribution with logarithm. For stock price returns, the US shows a negative skew (-1.6435), indicating a left-skewed distribution, while Costa Rica (0.8353) and Panama (0.5775) are positively skewed. All stock return variables exhibit leptokurtic distributions, suggesting the presence of fat-tail phenomena. In addition, Jarque-Bera tests reject the normality of stock prices and returns for Costa Rica, Panama, and the US. The Ljung-Box Q-statistic shows autocorrelation in the stock prices and returns for these markets. Significant second-order autocorrelation suggests conditional heteroskedasticity, making the ARCH/GARCH process suitable for modeling their time series behavior.

Table 1.
Descriptive Statistics of Logarithmic Stock Prices and Stock Price Returns.

	logarithm of Stock Prices			**Stock Price Returns**		
	Costa Rica	Panama	US	Costa Rica	Panama	US
Mean	10.2175	4.4201	7.2843	0.0031	0.0024	0.0018
Median	9.6104	4.7281	7.1104	0.0008	0.0012	0.0045
Maximum	11.2458	6.0612	7.7821	0.2201	0.1415	0.0915
Minimum	6.8015	2.8011	5.955	-0.1025	-0.1135	-0.2025
Std. Dev.	0.9416	0.8125	0.4105	0.0310	0.0192	0.0205
Skewness	-0.9921	-0.4515	-0.7625	0.9102	0.5825	-1.7352
Kurtosis	2.9105	2.4150	3.6152	23.0041	28.1142	20.1150
Jarque-Bera	186.2251***	54.1102***	115.3978***	19201.96***	11201.28***	13882.01***
Probability	0.0000	0.0000	0.000000	0.000000	0.000000	0.000000
First-Order Serial Correlation						
$Q(12)$	124.25***	68.910***	22.152***	226.75***	480.76***	250.26***
$Q(18)$	141.25***	83.125***	32.115***	252.91***	515.66***	261.35***
$Q(24)$	157.66***	96.871***	36.671***	252.94***	541.30***	271.30 ***
Second-Order Serial Correlation						
$Q^2(12)$	241.35***	492.15***	268.12***	440.15***	801.35***	531.25***
$Q^2(18)$	261.45***	515.88***	274.20***	460.25***	826.25***	526.56***
$Q^2(24)$	262.85***	538.40***	283.15***	465.83***	866.75***	554.28***

Note: ***, **, and * denote statistical significance at the 1%, 5% and 10% levels, respectively. The software used by Eviews12.

3.2. Unit Root Tests

Table 2 presents the results of unit root tests on the natural logarithms and return series of the weekly indices for Costa Rica, Panama and the US. Both the Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests were applied to the levels and first differences of each series under three regression models: without constant and trend, with constant and trend, and with constant but no trend. The results from both tests reject the null hypothesis of a unit root, indicating that the first differences of each series are stationary. Therefore, all variables are integrated of order one, I(1).

Table 2.
Results of Unit Root Tests

	Country	With Intercept Term	With Intercept and Trend Term	Without Intercept and Trend Term
Augmented Dickey-Fuller Test				
Logarithm	Costa Rica	-4.9101 [4]	-4.9951 [4]	-4.9101 [4]
Stock Price	Panama	-5.4715[3]	-5.4251 [3]	-5.6157 [3]
Series	US	-5.2262 [2]	-5.3215 [2]	-5.1126 [2]
Stock Price	Costa Rica	13.2811*** [4]	13.4625*** [4]	15.3015*** [4]
Return	Panama	4.5120*** [3]	4.6254*** [3]	5.1425*** [3]
Series	US	10.1102*** [2]	9.3781*** [2]	10.2021*** [2]
Phillips-Perron Test				
Logarithm	Costa Rica	-4.9101 [5]	-4.9011 [5]	-4.9946 [5]
Stock Price	Panama	-5.4387 [7]	-5.4271 [7]	-5.3890 [7]
Series	US	-5.0125 [3]	-5.0125 [3]	-5.1128 [3]
Stock Price	Costa Rica	14.6231*** [4]	15.6710*** [4]	14.6712*** [4]
Return	Panama	4.6512*** [6]	4.6266*** [6]	4.6358*** [6]
Series	US	9.3457*** [2]	9.3578*** [2]	9.3225*** [2]

Note: The values in [.] are the most fitting lags determined by the SBC criterion. ***, **, and * denote statistical significance at the 1%, 5% and 10% levels, respectively.

3.3. Testing Results for Structural Breaks

To examine how shocks of events influence stock price co-movement or interaction effects in Costa Rica, Panama and the US, the iterated cumulative sums of squares (ICSS) algorithm presented by Incln and Tiao [24] is used to detect variance breaks (volatility shifts) in stock returns. The results suggest that the subprime mortgage crisis and the Global Financial Crisis (GFC) affected the three markets from the 1st week of July 2007 to the 3rd week of April 2009; the European Debt Crisis (EDC) occurred from the 4th week of April 2009 to the 4th week of March 2012; the most intense US-China trade war from the 3rd week of March, 2018 to the 4th week of December 2019 and the COVID-19 pandemic period from the 1st week of March 2020 to the 4th week of December 2020, based on our estimates.

3.4. Johansen Co-integration Tests

Based on the unit root test results, it is known that the stock prices of Costa Rica, Panama, and the United States are integrated I(1). When series share the same integration order, a long-term co-integration relationship may exist. To test for this relationship and avoid spurious regression, the Johansen and Juselius [25] method is applied. The Trace and Max-eigenvalue tests identify the number of co-integration vectors, and the parameters are estimated using maximum likelihood. The results, presented in Table 3, show that after taking the natural logarithms of the stock price indices, a long-term co-integrating relationship exists among the three markets. This suggests one statistically significant co-integrating vector in the model, indicating a long-run relationship and co-movement among these three stock markets.

Table 3.
Johansen Co-integration Tests.

Unrestricted Co-integration Rank Test (Trace)				
Trace Test				
Hypothesized		**Trace**	**0.05**	**Prob.**
No. of CE(s)	Eigenvalue	Statistic	Critical Value	
None	0.0481	72.3588	35.4015	0.0000
At most 1	0.0224	18.4925	21.2271	0.0815
At most 2	0.0060	4.9011	10.9012	0.4125
Unrestricted Co-integration Rank Test (Maximum Eigenvalue)				

Table 3.
Continue…

Hypothesized		**Max-Eigen**	**0.05**	**Prob.**
No. of CE(s)	**Eigenvalue**	**Statistic**	**Critical Value**	
None	0.0625	54.0012	23.1010	0.000
At most 1	0.0321	14.2121	16.0012	0.1815
At most 2	0.0068	5.0128	10.0254	0.4015
Co-integrating Vector Estimates				

Panama	**Costa Rica**	**US**	**C**
1	-1.8252***	1.2021***	4.6256
	-0.2628	-0.7128	-2.8104

Note: ( ) denotes standard deviation. *** show statistically significant at 1% level.

According to the results of Table 3, the error correction term Z_{t-1} can be determined as follows:

$$Z_{t-1} = PN_{t-1}\text{-}1.8252\mathrm{CR}_{t-1}\text{-}1.2021\mathrm{US}_{t-1} + 4.6256 \quad (20)$$

Where PN_{t-1}, CR_{t-1} and US_{t-1} are natural logarithms of the stock price index for Panama, Costa Rica and US, respectively.

Furthermore, the error correction term (Z_t) from the co-integration test is used to estimate the autocorrelation function (ACF) shown in Figure 1. The slow decline in autocorrelation coefficient as lags increase suggests that the stock indices require a longer time to achieve co-integration, reflecting long-term memory. Based on fractionally integrated process, the conditional mean equation which contains long memory effects, the co-integration process represented by $\left[(1-L)^d-(1-L)\right]Z_t$ should be incorporated into the model system, i.e., the fractionally integrated error correction (FIEC) model is considered.

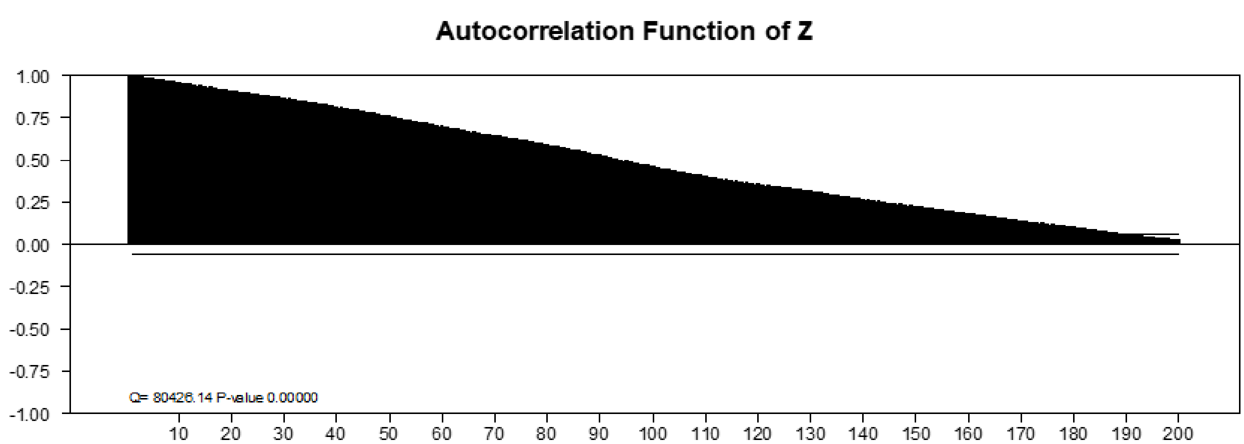


Figure 1.
ACF of the Error Correction Term Z_t

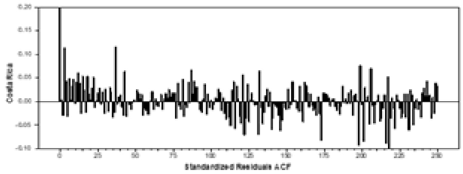
Table 4.
Serial Correlation, ARCH and Asymmetric Effect Tests on Standardized Residuals Terms for Estimated FIEC Models.

	Costa Rica	**Panama**	**US**
	$Z_{COR,t} = \varepsilon_{COR,t}/\sqrt{var\ (\varepsilon_{COR,t})}$	$Z_{PAN,t} = \varepsilon_{PAN,t}/\sqrt{var\ (\varepsilon_{PAN,t})}$	$Z_{US,t} = \varepsilon_{US,t}/\sqrt{var\ (\varepsilon_{US,t})}$
$Q(12)$	33.412	33.675	8.210
$Q(18)$	44.295	42.857	16.684
$Q(24)$	49.212	54.625	21.308
	${Z_{COR}}_t^2$	${Z_{PN}}_t^2$	${Z_{US}}_t^2$
$Q(12)$	269.301***	215.045**	97.254***
$Q(18)$	262.045***	274.695***	110.254***
$Q(24)$	261.384***	326.455***	128.301***
	$Z_{COR,t}\ Z_{PAN,t}$	$Z_{COR,t}\ Z_{US,t}$	$Z_{PAN,t}\ Z_{US,t}$
$Q(12)$	91.254***	25.501***	48.265***
$Q(18)$	98.301***	44.886***	56.021***
$Q(24)$	110.246***	110.200***	64.245***
Asymmetric Effects Test for FIEC Model			
	Costa Rica	Panama	US
SBT	-2.6015***	-0.9925***	3.6114***
NSBT	-3.9124***	-4.4825***	-0.5025***
PSBT	12.1140***	4.5301***	-0.8015***
JT	162.201***	45.2301***	22.001***

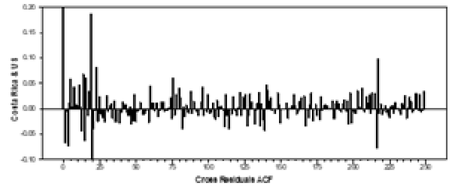
Note: $Z_{i,t} = \varepsilon_{i,t}/\sqrt{var\ (\varepsilon_{i,t})}$, ${Z_i}_t^2$ and $Z_{i,t}\ Z_{j,t}$ represent standardized residuals, squared standardized residuals and cross-product of standardized residuals, respectively. ** and *** denote statistically significant at 5% and 1% level, respectively.

Based on the estimated FIEC model, we calculate the standardized residuals, squared standardized residuals, and their cross-products. The Ljung-Box Q tests (Table 4) show no autocorrelation in the standardized residuals ($Z_{COR,t}, Z_{PAN,t}\ and\ Z_{US,t}$), but serial correlation exists in the squared residuals (${Z_{COR}}_t^2,\quad {Z_{PN}}_t^2\ and\ {Z_{US}}_t^2$) and cross-product of the two standardized residuals ($Z_{COR,t}\ Z_{PAN,t}, Z_{COR,t}\ Z_{US,t}\ and\ Z_{PAN,t}\ Z_{US,t}$), indicating the presences of ARCH effects in the residuals in our model system. The results of testing for asymmetric effects by the sign bias test (SBT), negative sign bias test (NSBT), positive sign bias test (PSBT), and joint test (JT) are statistically significant for the squared standardized residuals, revealing volatility asymmetry in these three stock markets. Consequently, the AP-GARCH model is applied.

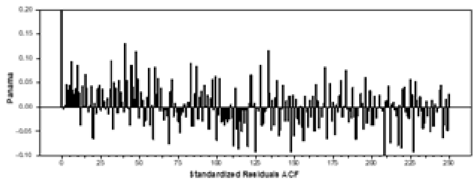
Moreover, Figure 2 display the autocorrelation function (ACF) of standardized squared residuals, squared standardized residuals, and the cross-product of standardized residuals in the estimated FIEC model for each stock market. The autocorrelation coefficients exhibit slow decay and persistence, indicating long memory properties in return volatilities and co-variances. Some of the researches have proven that the fractionally integrated EC (FIEC) model is empirically important for long-memory in returns, the following results may hold true when modeling long-range dependence in volatility on conditional mean, variance and co-variance equations with dynamic interaction effects, i.e., the FIAPGARCH-DCC model capturing long-memory, asymmetric volatility and DCC effects will also be applying in the model. Thus, the FIEC-FIAPGARCH-DCC is built up for analyzing the interrelationships among these three stock markets.



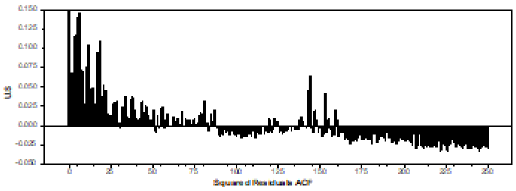
ACF of the Standardized Residuals Returns in the FIEC Model of Costa Rica



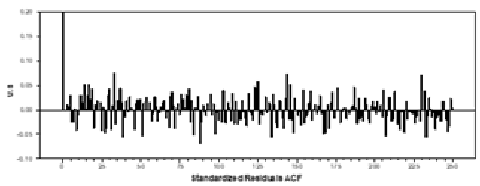
ACF of Squared Standardized Residuals in the FIEC Model of Panama



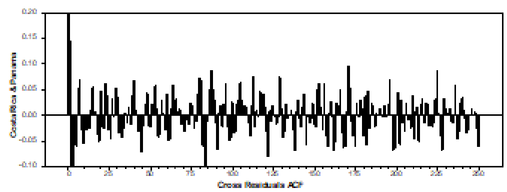
ACF of the Standardized Residuals Returns in the FIEC Model of Panama



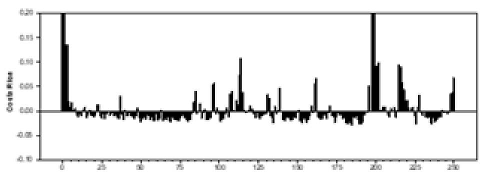
ACF of Squared Standardized Residuals in the FIEC Model of the US



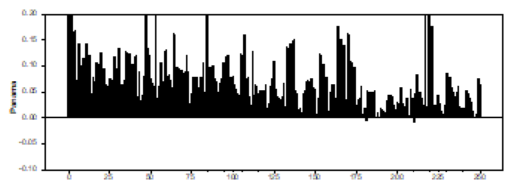
ACF of the Standardized Residuals Returns in the FIEC Model of US



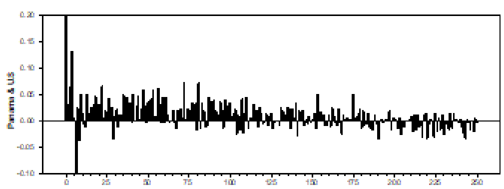
ACF Cross Residuals of Panama and Costa Rica in the FIEC Model



ACF of Squared Standardized Residuals in the FIEC Model of Costa Rica



ACF of Cross Residuals of Panama and the US in the FIEC Model



ACF of Cross Residuals of Costa Rica and the US in the FIEC Model

Figure 2.
ACF of the First-Power, Squared and Cross Standardized Residuals in the FIEC Model.

3.5. Set-Up of FIEC-FIAPGARCH-DCC Model

The FIEC-FIAPGARCH-DCC model of the stock market indexes for Costa Rica, Panama and the US are built up as follows:

3.5.1. Conditional Mean Equations

$$\Delta COR_t = u_1 + \lambda_1[(1-L)^d - (1-L)]Z_t + \sum_{i=1}^{4} a_{1i}\,\Delta COR_{,t-i} + \sum_{i=1}^{4} a_{2i}\,\Delta PAN_{,t-i} + \sum_{i=1}^{4} a_{3i}\,\Delta US_{,t-i} + k_1D_1 + k_2D_2 + k_3D_3 + k_4D_4 + \varepsilon_{COR,t}$$

$$\Delta PAN_t = u_2 + \lambda_2[(1-L)^d - (1-L)]Z_t + \sum_{i=1}^{4} b_{1i}\,\Delta PAN_{,t-i} + \sum_{i=1}^{4} b_{2i}\,\Delta COR_{,t-i} + \sum_{i=1}^{4} b_{3i}\,\Delta US_{,t-i} + k_5D_1 + k_6D_2 + k_7D_3 + k_8D_4 + \varepsilon_{PAN,t}$$

$$\Delta US_t = u_3 + \lambda_3[(1-L)^d - (1-L)]Z_t + \sum_{i=1}^{4} c_{1i}\,\Delta US_{,t-i} + \sum_{i=1}^{4} c_{2i}\,\Delta PAN_{,t-i} + \sum_{i=1}^{4} c_{3i}\,\Delta COR_{,t-i} + k_9D_1 + k_{10}D_2 + k_{11}D_3 + k_{12}D_4 + \varepsilon_{US,t} \quad (21)$$

3.5.2. Conditional Variance Equations

$$h_{COR,t}^{\delta_1/2} = \omega_1 + \beta_1 h_{COR,t-1}^{\delta_1/2} + [(1-\beta_1 L) - (1-\phi_1 L)(1-L)^{d1}]\,(|\varepsilon_{COR,t}| - \gamma_1\,\varepsilon_{COR,t})^{\delta_1} + \beta_{12}h_{PAN,t-1}^{\delta_2/2} + \beta_{13}h_{US,t-1}^{\delta_3/2} + g_1D_1 + g_2D_2 + g_3D_3 + g_4D_4$$

$$h_{PAN,t}^{\delta_2/2} = \omega_2 + \beta_2 h_{PAN,t-1}^{\delta_2/2} + [(1-\beta_2 L) - (1-\phi_2 L)(1-L)^{d2}](|\varepsilon_{PAN,t}| - \gamma_2\,\varepsilon_{PAN,t})^{\delta_2} + \beta_{21}h_{COR,t-1}^{\delta_1/2} + \beta_{23}h_{US,t-1}^{\delta_3/2} + g_5D_1 + g_6D_2 + g_7D_3 + g_8D_4$$

$$h_{US,t}^{\delta_3/2} = \omega_3 + \beta_3 h_{US,t-1}^{\delta_3/2} + [(1-\beta_3 L) - (1-\phi_3 L)(1-L)^{d3}](|\varepsilon_{US,t}| - \gamma_3\,\varepsilon_{US,t})^{\delta_3} + \beta_{31}h_{COR,t-1}^{\delta_1/2} + \beta_{32}h_{PAN,t-1}^{\delta_2/2} + g_9D_1 + g_{10}D_2 + g_{11}D_3 + g_{12}D_4 \quad (22)$$

3.5.3. Conditional Covariance Equations

Under the sense and evolution of DCC-GARCH Model, we set up conditional covariance equations as follows:

$$q_{PANCOR,t} = (1-\alpha_{PANCOR,t} - \beta_{PANCOR,t})\bar{\rho}_{PARCOR} + \alpha_{PANCOR,t}u_{PAN,t-1} + \beta_{PANCOR,t}q_{PANCOR,t-1}$$
$$q_{PANUS,t} = (1-\alpha_{PANUS,t} - \beta_{PANUS,t})\bar{\rho}_{PANUS} + \alpha_{PANUS,t}u_{PAN,t-1}u_{US,t-1} + \beta_{PANUS,t}q_{PANUS,t-1}$$
$$q_{CORUS,t} = (1-\alpha_{CORUS,t} - \beta_{CORUS,t})\bar{\rho}_{CORUS} + \alpha_{CORUS,t}u_{COR,t-1}u_{US,t-1} + \beta_{CORUS,t}q_{CORUS,t-1}$$
$$q_{PANPAN,t} = (1-\alpha_{PAN,t} - \beta_{PAN,t})\bar{\rho}_{PAN} + \alpha_{PAN,t}u_{PAN,t-1}^2 + \beta_{PANPAN,t}q_{PANPAN,t-1}$$
$$q_{CORCOR,t} = (1-\alpha_{COR,t} - \beta_{COR,t})\bar{\rho}_{COR} + \alpha_{COR,t}u_{COR,t-1}^2 + \beta_{CORCOR,t}q_{CORCOR,t-1}$$
$$q_{USUS,t} = (1-\alpha_{US,t} - \beta_{US,t})\bar{\rho}_{US} + \alpha_{US,t}u_{US,t-1}^2 + \beta_{USUS,t}q_{USUS,t-1}$$

And

$h_{PANCOR,t} =$

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$$\frac{[(1-\alpha_{PANCOR}-\beta_{PANCOR})\bar{\rho}_{PANCOR}+\alpha_{PANCOR}(u_{PAN,t-1}u_{COR,t-1})+\beta_{PANCOR}(q_{PANCOR,t-1})]X\,\delta_1\sqrt{(h_{PANPAN,t}{}^{1/2})^{\delta_1}}\,\delta_2\sqrt{(h_{CORCOR,t}{}^{1/2})^{\delta_2},}}{\sqrt{(1-\alpha_{PANCOR}-\beta_{PANCOR})\bar{\rho}_{PANPAN}+\alpha_{PANCOR}(u^2_{PAN,t-1})+\beta_{PANCOR}(q_{PANPAN,t-1})}\sqrt{(1-\alpha_{PANCOR}-\beta_{PANCOR})\bar{\rho}_{CORCOR}+\alpha_{PANCOR}(u^2_{COR,t-1})+\beta_{PANCOR}(q_{CORCOR,t-1})}}$$

$h_{PANUS,t} =$

$$\frac{[(1-\alpha_{PANUS}-\beta_{PANUS})\bar{\rho}_{PANUS}+\alpha_{PANUS}(u_{PAN,t-1}u_{US,t-1})+\beta_{PANUS}(q_{PANUS,t-1})]X\,\delta_1\sqrt{(h_{PANPAN,t}{}^{1/2})^{\delta_1}}\,\delta_3\sqrt{(h_{USUS,t}{}^{1/2})^{\delta_3}}}{\sqrt{(1-\alpha_{PANUS}-\beta_{PANUS})\bar{\rho}_{PANPAN}+\alpha_{PANUS}(u^2_{PAN,t-1})+\beta_{PANUS}(q_{PANPAN,t-1})}\sqrt{(1-\alpha_{PANUS}-\beta_{PANUS})\bar{\rho}_{USUS}+\alpha_{PANUS}(u^2_{US,t-1})+\beta_{PANUS}(q_{USUS,t-1})}}$$

$h_{CORUS,t} =$

$$\frac{[(1-\alpha_{CORUS}-\beta_{CORUS})\bar{\rho}_{CORUS}+\alpha_{CORUS}(u_{COR,t-1}u_{US,t-1})+\beta_{CORUS}(q_{CORUS,t-1})]X\,\delta_2\sqrt{(h_{CORCOR,t}{}^{1/2})^{\delta_2}}\,\delta_3\sqrt{(h_{USUS,t}{}^{1/2})^{\delta_3}}}{\sqrt{(1-\alpha_{CORUS}-\beta_{CORUS})\bar{\rho}_{CORCOR}+\alpha_{CORUS}(u^2_{COR,t-1})+\beta_{CORUS}(q_{CORCOR,t-1})}\sqrt{(1-\alpha_{CORUS}-\beta_{CORUS})\bar{\rho}_{USUS}+\alpha_{US}(u^2_{US,t-1})+\beta_{CORUS}(q_{USUS,t-1})}}$$

$$u_t = \begin{bmatrix} u_{PAN,t} \\ u_{COR,t} \\ u_{PAN,t} \end{bmatrix} \quad u_t \sim N(0, H_t) \quad H_t = \begin{bmatrix} h^2_{PANPAN,t} & h_{PANCOR,t} & h_{PANUS,t} \\ h_{CORPAN,t} & h^2_{CORCOR,t} & h_{CORUS,t} \\ h_{USPAN,t} & h_{USCOR,t} & h^2_{USUS,t} \end{bmatrix}$$

$u_{ij,t}$ Represent the standardized residuals $u_{ij,t} = \frac{\varepsilon_{ij,t}}{\sqrt{h_{ij,t}}}$ $i, j =$ Panama, Costa Rica and the US (23)

Variables and Parameters can be defined as follow:

3.5.4. Variable Definition

$\Delta COR_t, \Delta PAN_t, \Delta US_t$**:** The stock index returns of Costa Rica, Panama and the US at time t, respectively.

$\varepsilon_{COR,t}$, $\varepsilon_{PAN,t}$, $\varepsilon_{US,t}$: The residual term of the stock index returns of Costa Rica, Panama and the US at time t in conditional mean equation.

$\varepsilon_{COR\ t-1}, \varepsilon_{PAN\ t-1}, \varepsilon_{US\ t-1}$: The residual term of the stock index returns of Costa Rica, Panama and the US at time $t-1$.

Z_t: The error correction term at time t.

L: Lag or backshift operator

$h^{\delta_1/2}_{COR,t}, h^{\delta_2/2}_{PAN,t}, h^{\delta_3/2}_{US,t}$: Represent conditional variance of Costa Rica, Panama and the US stock markets return at time t, respectively.

$h^{\delta_1/2}_{COR,t-1}, h^{\delta_2/2}_{PAN,t-1}, h^{\delta_3/2}_{US,t-1}$: Represent conditional variance of Costa Rica, Panama and the United States stock markets return at time $t-1$, respectively.

$h_{PANCOR_{,t}}$: The conditional covariance of the stock index returns of Panama and Costa Rica at time t.

$h_{PANUS_{,t}}$: The conditional covariance of the stock index returns of Panama and the United States at time t.

$h_{CORUS_{,t}}$: The conditional covariance of the stock index returns of Costa Rica and the United States at time t.

$\rho_{PANCOR_{,t}}$: A pair-wise conditional correlation coefficient between Panama and Costa Rica at time t.

$\rho_{PANUS_{,t}}$: A pair-wise conditional correlation coefficient between Panama and the United States at time t.

$\rho_{CORUS_{,t}}$: A pair-wise conditional correlation coefficient between Costa Rica and the US at time t.

$q_{PANCOR,t}$: The n x n time-varying covariance matrix of u_t at time t for Panama and Costa Rica stock index.
$q_{PANUS,t}$: The n x n time-varying covariance matrix of u_t at time t for Panama and the US stock index.
$q_{CORUS,t}$: The n x n time-varying covariance matrix of u_t at time t for Costa Rica and the US stock index.
$q_{PANPAN,t}$: The time-varying variance of u_t at time t of Panama stock index.
$q_{CORCOR,t}$: The time-varying variance of u_t at time t of Costa Rica Stock index.
$q_{USUS,t}$: The time-varying variance of u_t at time t of the US stock index.
$D_{1,t}$: Dummy variable for the subprime mortgage & global financial crisis (GFC), the 1st week of July 2007 ~ the 3rd week of April 2009
$D_{2,t}$: Dummy variable for the European debt crisis (EDC), the 4th week of April 2009 ~ the 4th week of March 2012.
$D_{3,t}$: Dummy variable for the most intense US-China trade war (UCT) from the 3rd week of March 2018 ~ the 4th week of December, 2019
$D_{4,t}$: Dummy variable for the COVID-19 pandemic period (COV) from the 1st week of March 2020 ~ the 4th week of December 2020

3.5.5. Parameter Definition
u_1, u_2, u_3: Intercept term to estimate whether these two countries stock index returns have common long-term trend or not.
$\lambda_1, \lambda_2, \lambda_3$: Measuring the adjustment of speed of the stock index of Costa Rica, Panama and US.
a_{1i}: Estimating the spillover effects for Panama's stock index return.
b_{1i}: Estimating the spillover effects for Costa Rica's stock index return.
c_{1i}: Estimating the spillover effects for the US stock index return.
$\omega_1, \omega_2, \omega_3$: Intercept term to estimate whether the conditional variances of the stock index market returns have long-term dependencies or not.
$\beta_1, \beta_2, \beta_3$: Estimate the own-mean spillover effects of the stock return of Panama, Costa Rica and the US.
ϕ_1, ϕ_2 and ϕ_3: Estimating the ARCH effects of Panama, Costa Rica and the US stock index prices.
β_{12}: Estimating the volatility spillover effect of Costa Rica stock index returns on Panama stock index return.
β_{13}: Estimating the volatility spillover effect of the US stock index returns on Panama stock index return.
β_{21}: Estimating the volatility spillover effect of Panama stock index returns on Costa Rica stock index return.
β_{23}: Estimating the volatility spillover effect of the US stock index returns on Costa Rica stock index return.
$\gamma_1, \gamma_2, \gamma_3$: Measures the asymmetric effects in Costa Rica, Panama and the US stock markets.
d : Long-Memory coefficients in the conditional mean equations for Costa Rica, Panama and the US.
d_1, d_2, d_3: Long-Memory coefficients in the conditional variance equations for Panama, Costa Rica and the US.
$\bar{\rho}_{PANCOR}$: The n x n unconditional correlation matrix of u_t between Panama and Costa Rica.
$\bar{\rho}_{PANUS}$: The n x n unconditional correlation matrix of u_t between Panama and the US.
$\bar{\rho}_{CORUS}$: The n x n unconditional correlation matrix of u_t between Costa Rica and the US.
$\bar{\rho}_{PANPAN}$: The n x n unconditional correlation matrix of u_t of Panama stock index.

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$\bar{\rho}_{CORCOR}$: The n x n unconditional correlation matrix of u_t of Costa Rica stock index.
$\bar{\rho}_{USUS}$: The n x n unconditional correlation matrix of u_t of the US stock index.
$u_{i,t-1}$: The standardized disturbance matrix at period $t-1$ for Panama, Costa Rica and the US.
α_{PANCOR} and β_{PANCOR}: Nonnegative scalar parameters for Panama and Costa Rica satisfying ($\alpha + \beta < 1$) in DCC model.
α_{PANUS} and β_{PANUS}: Nonnegative scalar parameters for Panama and the US satisfying ($\alpha + \beta < 1$) in DCC model.
α_{CORUS} and β_{CORUS}: Nonnegative scalar parameters for Costa Rica and the US satisfying ($\alpha + \beta < 1$) in DCC model.
k_1, k_5, k_9: Measuring the effect of the subprime mortgage and global financial crisis (GFC) in the mean equations for each stock markets.
k_2, k_6, k_{10}: Measuring the effect of the European debt crisis (EDC) in the mean equations for each stock markets.
k_3, k_7, k_{11}: Measuring the effect of the US-China trade war in the mean equations for each stock markets.
k_4, k_8, k_{12}: Measuring the effect of the COVID-19 pandemic in the mean equations for each stock markets.
g_1, g_5, g_9: Measuring the effect of the subprime mortgage and global financial crisis (GFC) in the variance equations for each stock markets.
g_2, g_6, g_{10}: Measuring the effect of the European debt crisis (EDC) in the variance equations for each stock markets.
g_3, g_7, g_{11}: Measuring the effect of the US-China trade war (UCT) in the variance equations for each stock markets.
g_4, g_8, g_{12}: Measuring the effect of the COVID-19 pandemic (COV) in the variance equations for each stock markets.

3.6. Empirical Evidences and Discussions

3.6.1. Results and Analyses of Conditional Mean Equations

We use the BHHH method to estimate the coefficients. Table 5 shows that the fractional differencing coefficient (d) of FIEC is 0.92439, significant at the 1% level. With d between 0 and 1, it suggests these stock markets exhibit a long memory effect, where past behavior influences current performance. This implies that historical data can predict future returns, and even delayed news continues to affect today's stock prices.

The estimated results of mean return own-spillover (Table 5) shows that Costa Rica's stock return is negatively influenced by its third lag and positively by its second and fourth lags. Panama's stock return is negatively affected by its second and third lags, and positively by its fourth lag. US stock return is positively influenced by its second lag and negatively by its third lag. The US market is more efficient in responding to market information than Costa Rica and Panama, suggesting that these markets are inefficient and rely on past data to predict future returns. Additionally, the empirical results of the mean return cross-spillover effects of Costa Rica, Panama and the US stock market indicate bidirectional(two-way) spillovers between Costa Rica and Panama stock markets, and unidirectional(one-way) spillovers from the US to both. The US stock market plays a crucial role in price discovery across these three markets.

Table 5.
Estimates the Parameters of the Conditional Mean Equations for Costa Rica, Panama and the US Stock Markets.

d = 0.92439* (t-value = 7.9043)**								
Costa Rica	**Coeff**	**t-value**	**Panama**	**Coeff**	**t-value**	**US**	**Coeff**	**t-value**
u_1	0.1924	1.8716	u_2	0.2028***	20.884	u_3	0.2626	1.1321
λ_1	-0.6156***	-4.0842	λ_2	-0.4150***	-2.9152	λ_3	-0.3010***	-3.0250
a_{11}	0.5921	1.0855	b_{11}	0.3115	1.0005	c_{11}	-0.7495	-1.2025
a_{12}	0.8824***	16.4151	b_{12}	-0.5051***	-10.2712	c_{12}	0.1625***	7.5656
a_{13}	-5.5014***	-20.3651	b_{13}	-0.6421***	-15.1012	c_{13}	-0.8915***	-10.9981
a_{14}	2.2628***	11.4571	b_{14}	0.0779***	20.0008	c_{14}	0.5625	1.0001
a_{21}	0.5141***	10.0011	b_{21}	0.1225***	14.1144	c_{21}	0.5115	1.0025
a_{22}	-0.6125***	-4.8754	b_{22}	-0.8261***	-29.0041	c_{22}	-0.2225	-1.0005
a_{23}	0.3130***	5.2154	b_{23}	0.6925***	13.5591	c_{23}	0.0190	1.5425
a_{24}	-0.6154***	-7.2152	b_{24}	-0.7025***	-15.2827	c_{24}	-0.2614	-0.0421
a_{31}	-0.6867***	-12.2161	b_{31}	-0.3031***	-10.0101	c_{31}	-0.2889	-1.6826
a_{32}	0.8025***	20.0518	b_{32}	0.6721***	18.9081	c_{32}	0.1725	1.4826
a_{33}	-0.4118***	-12.0121	b_{33}	-0.0181***	-16.2025	c_{33}	-0.1339	-1.8057
a_{34}	0.1329***	18.0705	b_{34}	-0.1151***	-21.0051	c_{34}	0.1055	1.2826
k_1	-0.2256***	-3.1101	k_5	-0.2312***	-3.2115	k_9	-0.2569***	-4.0315
k_2	-0.2025***	-2.9052	k_6	-0.2125***	-2.7005	k_{10}	-0.2320***	-3.0025
k_3	-0.1727***	-3.6248	k_7	-0.1804***	-3.7748	k_{11}	-0.2167***	-4.6456
k_4	-0.1614***	-2.8716	k_8	-0.1696***	-2.6389	k_{12}	-0.2006***	-2.9726

Note: ***, ** and * denote rejection of the hypothesis at the 1%, 5% and 10% level, respectively.

The error correction term (z_{t-1}) for Costa Rica, Panama and the US stock markets is negative and significant at the 1% level (estimated coefficients are $\lambda_1 = -0.6156$, $\lambda_2 = -0.4150$ and $\lambda_3 = -0.3010)$ (Table 5). The US has the lowest λ value, indicating a faster adjustment speed to long-run equilibrium compared to Costa Rica and Panama, showing that the US market leads in returning to equilibrium. This supports the notion that the US stock market plays a leading or dominant role in these three markets. The estimated dummy variables in the conditional mean equation indicate that major global events exerted statistically significant negative effects on stock returns in Costa Rica, Panama, and the United States. During the Global Financial Crisis (GFC) indicated in dummy variable (D_1), the impact coefficients were $k_1 = -0.2256$ for Costa Rica, $k_5 = -0.2312$ for Panama, and $k_9 = -0.2569$ for the United States, all significant at the 1% level, indicating that U.S. stock returns were the most adversely affected. Similarly, the European Debt Crisis (EDC) dummy variable (D_2) yielded significant negative coefficients: $k_2 = -0.2025$ (Costa Rica), $k_6 = -0.2125$ (Panama) and $k_{10} = -0.2320$ (United States). Again, the United States experienced the largest decline in stock returns. The effects of the U.S.–China Trade War (UCT), captured by the D_3 dummy variable, were also negative and statistically significant at the 1% level: $k_3 = -0.1727$ (Costa Rica), $k_7 = -0.1804$ (Panama) and $k_{11} = -0.2167$ (United States). Finally, the COVID-19 pandemic (COV) dummy variable showed substantial and statistically significant negative effects on returns, with estimated coefficients of $k_4 = -0.1614$, $k_8 = -0.1696$ and$k_{12} = -0.2006$ for Costa Rica, Panama and the United States, respectively.

Overall, these findings suggest that all four events led to significant reductions in stock returns across the three markets, with the United States consistently exhibiting the most pronounced declines. Notably, financial crisis events, namely the subprime mortgage crisis and the European debt crisis, exerted considerably stronger negative effects on stock performance than non-financial shocks such as the US-China Trade War and the COVID-19 pandemic. This distinction highlights the more disruptive nature of systemic financial disturbances compared to geopolitical or public health-related events [26].

3.6.2. Results and Analysis of Conditional Variance Equations

Table 6 presents the estimated parameters of the conditional variance equations in the FIEC-FIAPGARCH-DCC framework. The subsequent discussions and analysis focuses on the ARCH and GARCH effects, volatility spillover, long memory, and asymmetric effects.

Table 6.
Estimates Parameters of the Conditional Variance Equations for Costa Rica, Panama and the US Stock Markets.

Costa Rica	Coeff.	t-value	Panama	Coeff	t-value	US	Coeff.	t-value
ω_1	-0.7721	-1.5150	ω_2	-0.1921	-1.2051	ω_3	-0.2125	-1.7123
β_1	0.6502***	3.1848	β_2	0.6238***	4.2455	β_3	0.6989***	4.1142
ϕ_1	0.3328***	3.2628	ϕ_2	0.3415***	3.4125	ϕ_3	0.2626***	4.2628
d_1	0.2625***	3.3754	d_2	0.2125***	3.3614	d_3	0.1969***	3.2536
β_{12}	0.3914***	3.4415	β_{21}	0.3054***	3.5458	β_{31}	0.4562	1.1814
β_{13}	0.4518***	7.9596	β_{23}	0.3868***	8.2315	β_{32}	0.3233	0.8025
δ_1	1.3826***	5.6124	δ_2	1.5021***	11.9491	δ_3	1.6821***	6.2997
γ_1	0.2024***	7.2425	γ_2	0.2114***	9.2028	γ_3	0.2736***	8.4314
g_1	0.1628***	2.2125	g_5	0.6826***	4.0596	g_9	0.5935***	4.1528
g_2	0.1469**	2.0521	g_6	0.3544**	2.2021	g_{10}	0.1274**	2.2874
g_3	0.1121***	2.8054	g_7	0.3036***	4.3415	g_{11}	0.2216***	4.0366
g_4	0.1069***	3.0112	g_8	0.2201***	3.8871	g_{12}	0.1826***	3.0125

Note: ***, ** and * denote rejection of the hypothesis at the 1%, 5% and 10% level, respectively.

As indicated in Table 6, the estimated coefficients of lagged variance ($\beta_1 = 0.6502, \beta_2 =$0.6238 and $\beta_3 = 0.6989)$ in the conditional variance equations are positive and significant at the 1% level for Costa Rica, Panama and the US. This indicates strong GARCH effects, meaning that current volatility can be predicted by past volatility. Additionally, the coefficients of lagged conditional residuals ($\phi_1 = 0.3328$, $\phi_2 = 0.3415\ and\ \phi_3 = 0.2626)$ are also significantly positive at the 1% level, showing that exogenous shocks (unexpected news) significantly cause volatility impacts in all three markets.

Regarding inter-market volatility effects, the effects (β_{12} and β_{13}) of the volatility impacts from Panama and US on Costa Rica stock market volatility are positive and significant at the 5% level (Table 6). Meaning that there exists volatility spillovers measured by $h_{PAN,t-1}^{\delta_2/2}, h_{US,t-1}^{\delta_3/2}$ from Panama and US to Costa Rica stock market with US having a greater impact $(\beta_{13}, 0.4518 > \beta_{12}, 0.3914)$. The effects ($\beta_{21}$and β_{23}) of the volatility impacts from Costa Rica and US on Panama stock market volatility are positive and significant at the 1% level. Meaning that there exists volatility spillovers measured by $h_{COR,t-1}^{\delta_1/2}, h_{US,t-1}^{\delta_3/2}$ from Costa Rica and US to Panama stock market with US having a greater impact $(\beta_{23}, 0.3868 > \beta_{21}, 0.3054)$. However, the effects ($\beta_{21}$and β_{23}) of the volatility impacts from both Costa Rica and Panama on US stock market volatility are not significant. There is a two-way volatility spillover between Costa Rica and Panama's stock markets, while the US only impacts Costa Rica and Panama through one-way spillovers. International investors should account for both local market volatility and the spillover risks from the US. This might lead investors to use arbitrage strategies or adjust their asset allocation in response to flight-to-quality or flight-to-safety effects.

The fractional differencing coefficients (d_1, $d_2\ and\ d_3$) measure the long memory effect for each stock market. All coefficients are positive and significant at the 1% level (Table 6), indicating that past information significantly influences future return volatilities. Costa Rica has the highest coefficient (d_1=0.2625), suggesting its return volatility takes longer to adjust, followed by Panama (d_2=0.2125). The US has the lowest coefficient (d_3=0.1969) due to higher market transparency, enabling faster

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response to new information. Costa Rica and Panama, being less mature and less transparent, exhibit longer-lasting volatility effects.

The asymmetric impacts of good and bad news were estimated and are significant at the 1% level for Costa Rica (γ_1= 0.2024), Panama (γ_2= 0.2114), and the US (γ_3= 0.2736) (Table 6). All coefficients are positive, indicating that negative shocks (bad news) have a greater impact on return volatility than positive shocks (good news). The US shows the highest coefficient (γ_3), reflecting a stronger leverage effect, meaning that the US market is more sensitive to negative news. This is likely due to its higher market dynamism and faster information flow compared to Costa Rica and Panama.

The power term coefficient (δ) measures variance heterogeneity across stock markets. Table 6 shows that the coefficients for Costa Rica, Panama, and the US are statistically significant at the 1% level. These coefficients are different from 1 and 2, supporting the asymmetric power fractionally integrated model. This suggests that when the series follows a non-normal distribution, power transformations (other than squared terms ($\delta = 2$)) are more appropriate. Research by Ding, et al. [17] and Ding and Granger [27] reinforce that power transformations do not require simple squared shocks in the conditional variance equation. Therefore, the FIAPGARCH model is appropriate for modeling conditional variance of stock market returns. The estimated power term coefficient of the US market (δ_3=1.6821) is greater than Costa Rica's (δ_1=1.3826) and Panama's (δ_2=1.5021), likely due to the trading activities and/or frequencies of Central American (Costa Rica and Panama) stock markets are not higher than the US stock market.

Table 6 presents the estimated impacts of the Global Financial Crisis (GFC) and European Debt Crisis (EDC), represented by dummy variables D_1 and D_2, on stock return volatility in Costa Rica, Panama, and the United States stock markets. The coefficients associated with the GFC ($g_1 = 0.1628, g_5 = 0.6826$ and $g_9 = 0.5935$) and EDC event ($g_2 = 0.1469, g_6 = 0.3544$ and $g_{10} = 0.1274$) are all positive and statistically significant at the 1% and/or 5% significance levels. These results indicate that both financial crises exerted a significant amplifying effect on return volatility across all three markets. Notably, Panama exhibited the highest sensitivity to both financial crisis events, particularly with respect to the GFC. In contrast, the United States displayed greater responsiveness to the GFC than to the EDC. The non-financial shocks, U.S.–China Trade War (UCT) and COVID-19 pandemic (COV), are captured by dummy variablesD_3 and D_4, respectively. The effects of the UCT event ($g_3 = 0.1121, g_7 = 0.3036$ and $g_{11} = 0.2216$) and COV event ($g_4 = 0.1069, g_8 = 0.2201$ and $g_{12} = 0.1826$) on return volatility are likewise positive and statistically significant at conventional levels. These findings suggest that non-financial events also contributed to heightened return volatility in the three markets, although their impact was less pronounced than that of the financial crises [28]. Panama demonstrates the highest sensitivity to such events, with g_7 and g_8 exceeding the corresponding coefficients for Costa Rica and the United States. The U.S. stock market, meanwhile, shows a greater volatility response to the UCT than to the COVID-19 shock.

3.6.3. Diagnostic Checking for Goodness-of-fit of the Estimated Model

To assess model adequacy, we used the Ljung-Box Q test on the estimated standardized residuals ($Z_{COR,t}, Z_{PAN,t}$ and $Z_{US,t}$), the estimated squared residuals($Z_{COR}{}_t^2, Z_{PN}{}_t^2$ and $Z_{US}{}_t^2$), and their cross-products ($Z_{COR,t}\, Z_{PAN,t}, Z_{COR,t}\, Z_{US,t}$ and $Z_{PAN,t}\, Z_{US,t}$). The results (Q (12), Q (18), Q (24)) in Table 7 fail to reject the null hypothesis, indicating no autocorrelation or ARCH effects in the residuals. Additionally, tests for the presence of asymmetric behavior of volatility (SBT, NSBT, PSBT and JT) on the estimated standardized residuals show no significant effects. Therefore, these tests confirm that the FIEC-FIAPGARCH-DCC model is appropriate for the three stock markets, and the interpretations of empirical results are valid and reliable.

Table 7.
Goodness-of-Fit Test and Diagnostic Checking.

	Costa Rica	**Panama**	**US**
	$Z_{COR} = \varepsilon_{COR,t}/\sqrt{h_{COR,t}}$	$Z_{PAN} = \varepsilon_{PAN,t}/\sqrt{h_{PAN,t}}$	$Z_{US} = \varepsilon_{US,t}/\sqrt{h_{US,t}}$
$Q(12)$	21.243	32.331	16.054
$Q(18)$	22.089	33.571	26.714
$Q(24)$	26.178	35.605	34.518
	z_{COR}^2	z_{PAN}^2	z_{US}^2
$Q(12)$	19.312	30.248	20.369
$Q(18)$	27.010	33.501	30.115
$Q(24)$	30.336	36.771	32.465
	Z_{COR} x Z_{PAN}	Z_{US} x Z_{COR}	Z_{US} x Z_{PAN}
$Q(12)$	27.115	26.551	31.287
$Q(18)$	32.089	31.812	33.456
$Q(24)$	38.323	35.805	38.715
Diagonal Test for Asymmetric Effect of the Estimated Standardized Residuals			
	$Z_{COR,t}/\sqrt{h_{COR,t}}$	$Z_{PAN,t}/\sqrt{h_{PAN,t}}$	$Z_{US,t}/\sqrt{h_{US,t}}$
SBT	-0.9015	-0.7258	0.3278
NSBT	-0.9728	-0.1894	0.5859
PSBT	1.9928	0.6245	0.9498
JT	4.3101	4.2254	2.9134

Note: $Z_{i,t} = \varepsilon_{i,t}/\sqrt{var\ (\varepsilon_{i,t})}$, $Z_{i\,t}^{\,2}$ and $Z_{i,t}\,Z_{j,t}$ represent standardized residuals, squared standardized residuals and cross-product of standardized residuals, respectively. ** and *** denote statistically significant at 5% and 1% level, respectively.

3.7. Results of DCC Estimation and Contagion Effect

This section examines the dynamic impacts of conditional correlations in stock market returns for pairs of Costa Rica, Panama, and the US, using the multivariate DCC framework linked to the FIEC-FIAPGARCH model.

3.7.1. Estimated Coefficients of Conditional Correlations

In the DCC model, the parameters α and β quantify the influence of past market behaviors (shocks and correlations) on current relationships between stock markets. Our results show that $\alpha + \beta$ is close to 1, signifying high correlation between market pairs, suggests that these markets are highly interdependent, reacting to each other's shocks cross-market shocks. Table 8 shows $\alpha + \beta = 0.9671$ for Costa Rica-Panama, 0.9693 for Costa Rica-US, and 0.9704 for Panama-US, all statistically significant at the 1% level, confirming time-varying correlations. The non-negative values of α and β support the validity of FIEC-FIAPGARCH-DCC model. When $\alpha = 0$ and $\beta = 0$, we get the CCC model [22]. Table 8 summarizes the DCC parameter estimates, showing significant positive values that satisfy $\alpha + \beta < 1$ for all market pairs.

The statistical significance of α and β in the DCC model shows strong time-varying co-movement, indicating persistent conditional correlations. The sum of these parameters is close to 1, reflecting high volatility persistence. Since $\alpha + \beta < 1$, the dynamic correlations tend to stabilize around a constant, showing a mean-reverting process. The multivariate FIEC-FIAPARCH-DCC model is essential due to its advantages: capturing long-range dependence, providing pairwise conditional correlations, analyzing behavior during major crises (like subprime mortgage and financial tsunami and European debt crises), and testing for long-memory ARCH effects in these stock prices.

Table 8.
DCC Parameters Estimates.

	Costa Rica-Panama		Costa Rica-US		Panama-US	
	Coeff	t-stat	Coeff	t-stat	Coeff	t-stat
$\hat{\alpha}$	0.2130***	9.9241	0.2105***	10.0156	0.2114***	11.4546
$\hat{\beta}$	0.7541***	26.4578	0.7588***	25.1316	0.7590***	26.3987
Persistence	0.9671		0.9693		0.9704	

Note: ***, ** and * denote rejection of the hypothesis at the 1%, 5% and 10% level, respectively.

Now, the dynamic conditional correlations between Costa Rica-Panama, Costa Rica-US, and Panama-US stock markets are discussed. Table 9 presents average correlation coefficients, considering financial events, with two financial dummies incorporated into the conditional mean and variance equations. Using the ICSS algorithm to detect variance (volatilities) breaks, the data was divided into four periods: chaos and confusion (the 1st week of January 1994 to the 4th week of December 1999), tranquil times (the 1st week of July 2007 to the 3rd week of April 2009), subprime mortgage & global financial crisis (1st week of July 2007 to the 3rd week of April 2009), and the European debt crisis (the 4th week of April 2009 to the 4th week of March 2012). The study examines if conditional correlations increased during these periods compared to those without financial events. The average correlation values for Costa Rica- Panama during these periods are: 0.8146 (chaos and confusion), 0.6501 (tranquil), 0.8810 (GFC), and 0.8525 (EDC). For Costa Rica-US, the averages are: 0.8325 (chaos and confusion), 0.6921 (tranquil), 0.8829 (GFC), and 0.8635 (EDC).For Panama-US, the averages are: 0.8025 (chaos and confusion), 0.6371 (tranquil), 0.9088 (GFC), and 0.9015 (EDC). These results suggest that average correlation values increased during the GFC and EDC periods, indicating contagion effects across these stock markets during these crises. This hypothesis will be further tested using regression analysis of event dummies to detect stock market contagion during these crises.

Table 9.
Estimated Average Correlation Coefficients during the Periods of Two Events Occurred.

Period without considering Subprime Mortgage & Global Financial Crisis or European Debt Crisis			**Subprime Mortgage & Global Financial Crisis (GFC)**	**European Debt Crisis (EDC)**
	Under Chaos & Confusion	**Tranquil Time**		
	From the 1st week of January 1994 to the 4th week of December 1999	From the 1st week of January 2000 to the 4th week of June 2007	From the 1st week of July 2007 to the 3rd week of April 2009	From the 4th week of April 2009 to the 4th week of March 2012
ρ_{CORPAN}	0.8146	0.6501	0.8810	0.8525
ρ_{CORUS}	0.8325	0.6921	0.8829	0.8635
ρ_{PANUS}	0.8025	0.6371	0.9088	0.9015

Note: DCC equations: $\rho_{ij,t} = \frac{q_{ij,t}}{\sqrt{q_{ii,t}q_{jj,t}}}$. With dummy variables for the global events (GFC & EDC).

3.7.2. Financial Contagion Effects

This section analyzes the increased correlation between stock markets after the GFC and EDC to understand how market linkages change during financial crises and detect contagion effects. According to Forbes and Rigobon [29] contagion is defined as a significant rise in cross-market linkages after a shock hits one country. If markets show high co-movement during both stable and crisis periods, it indicates interdependence, not contagion.

Here, we apply two dummy variables (D_1 and D_2) in the research period to analyze how the dynamic feature of the correlations change during different crisis phases. The regression model is presented with and without these dummy variables:

Excluding dummies:

$$\rho_{ij,t} = \psi_{ij} + \delta_1 \rho_{ij,t-1} + \varepsilon_{ij,t} \qquad (24)$$

Including dummies:

$$\rho_{ij,t} = \psi_{ij} + \delta_1 \rho_{ij,t-1} + \theta_1 D_{1,t} + \theta_2 D_{2,t} + \varepsilon_{ij,t} \qquad (25)$$

In the above auto-regression analysis, we estimate the DCCs ($\rho_{ij,t}$) between pairs of stock markets (Costa Rica, Panama, US) using an intercept term (ψ_{ij}) for each pair of stock markets, two crisis dummies (D_1 for GFC, D_2 for EDC) with coefficients $\theta_1$and θ_2 to capture their effects on correlations. We also include an AR (1) lag ($\rho_{ij,t-1}$) with coefficient δ_1 to account for past correlation effects. $\varepsilon_{ij,t}$ represents the error term with $iidN(0, \sigma^2)$.

Table 10 compares dynamic conditional correlations under two scenarios. Without the financial crisis dummies, the AR (1) coefficients are 0.8811 for Costa Rica-Panama, 0.8639 for Costa Rica-US, and 0.8296 for Panama-US. These high values indicate strongly persistent autocorrelation, reflecting interactions or interdependence between each pair of stock markets or among these three stock markets.

Considering the financial crisis dummies, Table 10 shows the auto-regression results for DCCs ($\rho_{ij,t}$) with event dummies (D_1 for GFC, D_2 for EDC). The GFC has a significant positive impact on Costa Rica-Panama (θ_1), Costa Rica-US (θ_5), and Panama-US (θ_3) dynamic conditional correlation coefficients, with significance at the 5% and 1% levels. Similarly, the EDC significantly impacts all pairs, Costa Rica-Panama (θ_2), Costa Rica-US (θ_4), and Panama-US (θ_6), at the 5% level. All parameters for the crisis event dummies show a positive impact on dynamic correlation coefficients, indicating that contagion occurred.

In addition, the auto-regression results in Table 10 show AR(1) coefficients of 0.8702 for Costa Rica-Panama, 0.8426 for Costa Rica-US, and 0.8025 for Panama-US based on prior-period dynamic conditional correlations. Compared to the case without considering special events, we observe a decrease in these coefficients: from 0.8811 to 0.8702 for Costa Rica-Panama, from 0.8639 to 0.8426 for Costa Rica-US, and from 0.8296 to 0.8025 for Panama-US. This indicates that market interactions are not only driven by co-movement but also by contagion effects from financial events like the GFC and EDC. The changes in correlation are due to external shocks, confirming the existence of contagion. We also find that the GFC had a stronger impact on market correlations than the EDC.

For model diagnostics, we still used the BHHH method to estimate the auto-regression coefficients, as shown in Table 10. The results indicate a good fit, with significant R^2 and F-test statistics for both models (with and without dummy variables). Both the Durbin-h and Ljung-Box Q-tests show no serial correlation in the errors, confirming a white noise process. Therefore, we conclude that our estimated auto-regression models are appropriate, and the interpretations of the above empirical findings are both valid and applicable.

Table 10.
Auto-regression Estimation for DCC ($\rho_{ij,t}$) without and with Two Event Dummies.

	Costa Rica-Panama		Costa Rica-US		Panama-US	
	Without Dummy	With Dummy	Without Dummy	With Dummy	Without Dummy	With Dummy
ψ_{ij}	0.0182*** (3.4159)	0.0121 (1.9685)	0.0084 (1.6578)	0.0075 (0.9967)	0.0502*** (6.8025)	0.0605*** (7.8452)
$\rho_{ij,t-1}(\delta_1)$	0.8811*** 27.5141	0.8702*** (61.3054)	0.8639*** (28.5021)	0.8426*** (53.2514)	0.8296*** (28.004)	0.8025*** (45.1146)
$\theta_1(D_1)$		0.0504** (3.0145)				
$\theta_2(D_2)$		0.0417** (2.9458)				
$\theta_3(D_1)$				0.0881*** (4.3572)		
$\theta_4(D_2)$				0.0621** (2.9015)		
$\theta_5(D_1)$						0.0521** (3.0012)
$\theta_6(D_2)$						0.0421** (2.0915)
R^2	0.6889	0.8025	0.6625	0.7624	0.7012	0.6024
F-Stat	580.4102	1514.8891	541.0012	1192.4151	646.5015	760.3057
MSE	0.1527	0.1428	0.1523	0.1805	0.1559	0.1775
D-h	0.0041	0.0046	0.0041	0.0036	0.0049	0.0029
$Q(6)$	2.6001	2.5415	9.5226	9.5712	7.8326	7.8501

Note: (1) Auto-regression estimation for DCC $\rho_{ij,t} = \psi_{ij} + \delta_1\rho_{ij,t-1} + \varepsilon_{ij,t}$ without dummies.
(2) Auto-regression estimation for DCC $\rho_{ij,t} = \psi_{ij} + \delta_1\rho_{ij,t-1} + \theta_1 D_{1,t} + \theta_2 D_{2,t} + \varepsilon_{ij,t}$ with dummies.
(3) D_1 is the dummy variable for GFC period; D_2 is the dummy variable for the EDC period.
(4) Durbin-h $= \hat{\rho}\sqrt{\frac{n}{1-n\hat{V}(\hat{a})}}$
(5) ***, ** and * denote rejection of the hypothesis at the 1%, 5% and 10% level, respectively.
(6) The numbers in parenthesis are the t-statistic values.

4. Concluding Remarks

This study examines the interdependence of main Central America (Costa Rica and Panama) and the US stock markets. It uses a dynamic conditional correlation (DCC) model within a multivariate FIEC-FIAPARCH framework, accounting for long memory, power effects, asymmetry (leverage) effects, and time-varying correlations, while considering financial crises like the GFC and EDC. The results show that the FIEC-FIAPARCH-DCC model better captures volatility and time-varying conditional correlations compared to simpler models, with persistent correlations that increase during financial crises. The GFC had a greater impact on cross-market correlations than the EDC.

The corresponding coefficient of the error correction term (z_{t-1}) is estimated to be negative and significant, indicating that the markets maintain long-term equilibrium and exhibit co-integration. This suggests that, following an external shock, the Costa Rica, Panama, and US stock markets can return to long-term equilibrium through dynamic adjustments. The US stock market adjusts faster than the Costa Rican and Panamanian markets, meaning it converges more quickly to equilibrium, supporting the notion that the US stock market leads these Central American markets. Thus, the US stock market holds a dominant position among the three stock markets. Additionally, the results show a bidirectional (two-way) mean return spillover effect between Costa Rica and Panama, while the US stock market influences both with a unidirectional (one-way) spillover effect. The US stock market plays a key role for price discovery in these three markets.

With regard to inter-market volatility effects, we find a two-way volatility spillover between Costa Rica and Panama stock markets, and a one-way spillover from the US to Costa Rica and Panama.

International investors should consider not only the volatility in their local markets but also the volatility spillover from the US. This can influence their asset allocation strategies, including arbitrage, flight-to-quality, or flight-to-safety strategies. In terms of long memory, the fractional differencing coefficients suggest that return volatilities for all three markets exhibit long memory, meaning past information can help forecast future volatility. Costa Rica has the highest fractional differencing coefficient, followed by Panama, while the US has the lowest, due to its greater market transparency and faster absorption of new information. Asymmetric effects are present in return volatility, meaning negative shocks have a stronger impact than positive shocks. The three estimated power coefficients (δ) are significantly different from unity also significantly different from two, indicating significant differences in the variance for each market. This supports the asymmetric power fractionally integrated model. Finally, Panama's stock return volatility is more influenced by the EDC event compared to Costa Rica and the US. The US stock market is less affected by the EDC event compared to the GFC event. The results also underscore that Panama's stock market return volatility is the most responsive to both financial and non-financial global shocks. The U.S. market, by comparison, reacts more strongly to the GFC and the UCT, while exhibiting a weaker response to the EDC and the COVID-19 pandemic. Costa Rica, on the other hand, generally exhibits moderate volatility responses across all four events.

Moreover, for financial contagion detection, in the DCC analysis with financial structural breaks, we incorporate financial crisis dummies (D_1 and D_2) to examine how correlation coefficients $(\rho_{ij,t})$ between the paired stock markets (Costa Rica-Panama, Costa Rica-US, and Panama-US) change. Using two event dummies for the GFC and EDC, we analyze the dynamics of correlation changes during these crises and test for contagion effects. The results show that the financial crisis dummies positively impact the conditional correlation coefficients for all stock market pairs, indicating the occurrence of financial contagion. It further indicates that the change in the correlation coefficient is caused by an external shock. This suggests that the changes in market interactions or interrelationships are driven not only by natural co-movement between the markets but also by contagion from external shocks like the GFC and EDC events.

Transparency:

The author confirms that the manuscript is an honest, accurate, and transparent account of the study; that no vital features of the study have been omitted; and that any discrepancies from the study as planned have been explained. This study followed all ethical practices during writing.

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