Edelweiss Applied Science and Technology
ISSN: 2576-8484
Vol. 9, No. 8, 586-606
2025
Publisher: Learning Gate
DOI: 10.55214/2576-8484.v9i8.9367

Assessing the efficiency of African stock markets through multifractal cross-correlation insights

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Abstract: This paper investigates the informational efficiency and cross-correlations among the five largest African stock markets—Johannesburg, Casablanca, Botswana, Nigerian, and Egyptian—using Multifractal Detrended Cross-Correlation Analysis (MF-DCCA). Spanning the period from January 30, 2012, to August 8, 2024, with nearly 3,050 observations, the study explores the multifractal characteristics and complex interdependencies among these markets. Initial results from the Cross-Correlation Significance Test indicate statistically significant relationships across most index pairs. Further analysis using MF-DCCA components—Generalized Hurst exponents, Rényi exponents, and the Hölder Singularity Spectrum—reveals persistent long-range cross-correlations and strong multifractal behavior. The application of surrogate and shuffling procedures confirms that both long-memory effects and heavy-tailed distributions contribute to the observed multifractality. These findings suggest the presence of informational inefficiencies within and between African stock markets, as evidenced by deviations from random-walk behavior. The study provides new insights into market dynamics in emerging economies, with practical implications for investors, portfolio managers, and policymakers.

Keywords: *Cross-correlation, Efficiency, Generalized hurst exponents, Hölder singularity spectrum, Multifractality, Rényi exponents.*

1. Introduction

In today's increasingly interconnected and volatile global financial system, understanding the intricate relationships between financial markets is not just a theoretical exercise, it is a practical necessity. Market shocks in one region can quickly transmit across borders, magnifying systemic risks and affecting investment outcomes worldwide. For investors, risk managers, and policymakers, this growing complexity demands analytical tools that can capture the full range of market behaviors, including non-linear dependencies, structural breaks, and time-varying dynamics. Traditional linear correlation models, while useful in simple settings, often fail to reflect the true interconnectedness of modern financial systems. In contrast, multifractal methods, particularly Multifractal Detrended Cross-Correlation Analysis (MF-DCCA), offer a more robust framework to uncover subtle and dynamic patterns of co-movement between markets.

The MF-DCCA approach is an advanced analytical tool that extends beyond traditional methods by incorporating multifractality, which captures diverse scaling behaviors within financial time series. This method, introduced by Zhou [1] enables a detailed analysis of cross-correlation patterns across different time scales, effectively addressing both short-term fluctuations and long-term dependencies between markets. These features are particularly relevant in the context of informational efficiency, as defined by the Efficient Market Hypothesis (EMH). According to the EMH, asset prices fully reflect all available information, and price changes should follow a random walk with no predictable structure. Persistent

History: Received: 9 June 2025; Revised: 14 July 2025; Accepted: 17 July 2025; Published: 9 August 2025
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long-range dependencies and multifractal patterns, however, suggest deviations from this ideal, indicating that past price information may still contain predictive power, a hallmark of informational inefficiency.

Multifractality in financial markets arises from the heterogeneous scaling behavior of asset returns, phenomena often linked to volatility clustering, extreme events, and long memory effects. Unlike classical monofractal models, which assume uniform behavior across scales, multifractal models detect a wide spectrum of scaling exponents that characterize both regular and erratic patterns in market dynamics. When applied in a cross-correlation setting, these techniques can identify complex and persistent relationships between financial markets that evolve over time and vary across frequencies. Such findings are significant in testing the degree of informational efficiency both within and across markets, especially when the persistence of multifractal cross-correlations challenges the assumption of market independence and instantaneous information diffusion. This is especially critical during periods of financial turbulence, when traditional correlation measures tend to break down. Thus, multifractal cross-correlation analysis provides unique value by offering a multi-scale, non-linear lens through which market integration, contagion, and informational inefficiencies can be understood and managed.

While multifractal methods have gained increasing attention in studies of advanced and some emerging markets, they remain largely absent from research on African stock markets. Most empirical studies on African exchanges rely on linear or semi-linear models, such as ARIMA, GARCH, or cointegration tests; which are ill-suited for capturing the intricate, long-range, and non-stationary dependencies typical of these markets. This has also limited our understanding of the informational efficiency of African stock markets, where market frictions, low liquidity, and structural imbalances may prevent the rapid incorporation of new information into asset prices. This is a critical oversight, as African markets operate under distinct economic and structural conditions: they often experience lower liquidity, are heavily influenced by commodity prices, face regulatory fragmentation, and are more vulnerable to global shocks. At the same time, African economies are experiencing rapid transformation, increasing financial openness, and growing foreign investment inflows. As a result, there is a pressing need to revisit and reassess inter-market relationships using more advanced tools that can reflect the real nature of these evolving financial systems.

2. Literature Review

Multifractal Detrended Cross-Correlation Analysis (MF-DCCA) has become a key tool in analyzing long-range, nonlinear dependencies between time series. Unlike traditional linear models, MF-DCCA detects complex, scale-dependent relationships that are critical in systems characterized by volatility, irregularity, and structural change. This section compares findings across various applications to contextualize this study's focus on African financial markets.

MF-DCCA has found broad use in environmental and geophysical sciences, where complex interdependencies are common. For example, Shadkhoo and Jafari [2] applied it to earthquake data and found scale-dependent multifractal structures, showing the method's sensitivity to natural randomness. Similarly, Wan, et al. [3] examined geochemical element concentrations and observed that both individual and paired series exhibited multifractality—consistent with Yao, et al. [4] who found streamflow and sediment data to display strong long-range correlations, with sediment showing greater multifractality. These findings align in showing the utility of MF-DCCA for uncovering hidden, persistent relationships in natural systems. However, the strength of multifractality varied across studies and datasets. For instance, Yao et al. [4] reported stronger multifractality in sediment, whereas Shadkhoo and Jafari [2] emphasized differing behavior across small and large scales—highlighting that even within natural systems, multifractal properties are context-dependent. In atmospheric sciences, Marin, et al. [5] revealed consistent multifractal patterns in pollutant data across urban and regional sites, whereas in agriculture, Adarsh, et al. [6] showed agro-meteorological variables like wind speed and air pressure had lower multifractal exponents compared to evapotranspiration. This variability suggests MF-DCCA can distinguish between high and low complexity variables, making it useful in heterogeneous systems. In structural health monitoring, Ayisha and Adarsh [7] emphasized MF-DCCA's untapped potential, particularly in crack detection and pore structure analysis. However, their review remained mostly conceptual, lacking empirical validation, unlike other environmental studies that provided detailed statistical outputs. Taken together, these applications demonstrate MF-DCCA's effectiveness in diverse scientific settings but also underscore the importance of data selection, variable properties, and temporal scale when interpreting multifractal behavior.

In financial markets, MF-DCCA has gained traction for analyzing asset interdependencies, especially during periods of market stress. Ling-Yun and Shu-Peng [8] found strong multifractal cross-correlations between US and Chinese agricultural futures, despite their geographic separation—echoing the findings of Ferreira, et al. [9] who noted region-specific correlation patterns between exchange rates and stock markets, with India showing more sensitivity than Europe. These results highlight how global integration influences multifractality, though the magnitude of dependence appears to be market specific. Burugupalli [10] examined gold and oil markets and found strong multifractality in the short term, which weakened over longer horizons, suggesting that correlation regimes shift with time scale. This behavior contrasts with Wang, et al. [11] who found long-memory correlations in both price and load data from electricity markets, where the structure appeared more stable. These findings indicate that asset class characteristics (e.g., energy vs. commodities) can shape the persistence of cross-correlations. Qiu and Ye [12] further refined this time-dependence, showing that multifractality between the S&P 500 and the Shanghai Composite intensified during the 2008 financial crisis. This is consistent with Zou and Zhang [13] who noted increased volatility and negative correlations between carbon and electricity markets, particularly during stress events. These studies underscore MF-DCCA's strength in capturing crisis-driven changes in inter-market relationships. Building on this, Chen, et al. [14] compared the multifractal features, correlation, complexity, and uncertainty of the CSI 300 (Shanghai and Shenzhen) and the S&P 500 using the MF-DCCA model. They found both indices to exhibit multifractal characteristics but with different degrees of long-term memory, complexity, and irregularity—demonstrating how market maturity and structure influence multifractal behavior. Comparatively, Wang, et al. [15] and Junjun, et al. [16] applied MF-DCCA to digital and alternative financial systems—China's P2P lending and Bitcoin, respectively. Both found strong multifractal cross-correlations with traditional financial indicators, though Junjun, et al. [16] emphasized greater sensitivity to external uncertainty (USEPU index), while Wang, et al. [15] focused on the potential stabilizing role of regulation. These results reveal the importance of institutional context in interpreting multifractal behavior. Recently, examined the multifractal properties and interconnections of six Islamic stock markets in the Pacific Asia region from 2011 to 2024, using the MF-DCCA model. Key metrics like the Generalized Hurst Exponent, Rényi Exponent, and Hölder Singularity Spectrum confirmed the presence of long-range dependence and multifractality. Surrogate and shuffled data tests indicated that this multifractality is driven by both heavy-tailed return distributions and long-term cross-market correlations.

In commodity and innovation-linked studies, Jia, et al. [17] confirmed persistent cross-correlations between soybean spot and futures prices, while Jinchuan, et al. [18] observed time-varying multifractality in the relationship between technological innovation and macroeconomic performance. The latter study adds a dynamic dimension often missing from earlier analyses, suggesting the importance of monitoring how interdependencies evolve.

In sustainability finance, Zeyi, et al. [19] revealed strong multifractality and low efficiency in China's new energy index—consistent with Acikgoz, et al. [20] who found long-range power-law correlations between green bonds and commodity prices. However, both studies assume relatively developed market mechanisms, which may not be present in frontier economies. Paulo Roberto, et al. [21] took a more visual approach by introducing MF-DCCA Heatmaps, linking political cycles to Brazilian economic indicators. While innovative, this approach needs broader application and comparison to standard metrics to assess robustness.

Edelweiss Applied Science and Technology
ISSN: 2576-8484
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Despite the growing body of literature on Multifractal Detrended Cross-Correlation Analysis Although the MF-DCCA method has been widely applied to various financial and non-financial systems, there remains a notable gap in research exploring dynamic cross-correlations within African stock markets. One of the few exceptions is the study by Marwane and Benbachir [22] which applied this method in the MENA region and uncovered multifractal correlations among markets in Morocco, Tunisia, Egypt, and Jordan. This gap underscores the need for further investigation into the multifractal cross-correlations and interdependencies among the largest African stock markets. Such analysis would enhance our understanding of their interconnected behavior and provide valuable insights for portfolio diversification and risk management across the region.

Building on previous studies that applied MF-DCCA to financial markets, particularly the work of this paper investigates the informational efficiency and multifractal cross-correlations among the five largest African stock markets: Johannesburg, Casablanca, Botswana, Nigeria, and Egypt, using MF-DCCA) method.

3. Data and Methodology

3.1. Data

The dataset comprises daily closing prices from the five largest African stock markets: the Johannesburg Stock Exchange (JSE), Casablanca Stock Exchange (MASI), Botswana Stock Exchange (BSE), Nigerian Exchange (NGX), and Egyptian Exchange (EGX). Together, these markets account for over 90% of the total market capitalization of African stock exchanges. The JSE, supported by South Africa's diversified economy and robust institutional framework, plays a central role in African financial markets and often serves as a regional benchmark. The MASI, one of North Africa's most active exchanges, reflects Morocco's growing economic diversification and increasing domestic and foreign investment. Though smaller in scale, the BSE is noted for its transparency and strong regulatory environment, supporting Botswana's capital formation. The NGX is a key component of West African capital markets, with performance often linked to macroeconomic factors such as oil prices, exchange rate fluctuations, and political developments. The EGX, one of the continent's oldest exchanges, functions as a strategic financial hub bridging the Middle East and Africa, underscoring Egypt's economic and geopolitical importance.

The data span from 30/01/2012 to 08/08/2024, comprising nearly 3050 observations. All data were downloaded from the website www.investing.com. The index prices were then converted into logarithmic returns $r_t = ln\left(\frac{P_t}{P_{t-1}}\right) = ln(P_t) - ln(P_{t-1})$, where P_t denotes the index daily price and ln corresponds to the natural logarithm.

3.2. Methodology

3.2.1. Multifractal Detrended Cross-Correlation Analysis

In this section, we introduce the Multifractal Detrended Cross-Correlation Analysis (MF-DCCA), originally developed by Zhou [1]. This method integrates key aspects of two earlier techniques: Detrended Cross-Correlation Analysis (DCCA), proposed by Podobnik and Stanley [23] and Multifractal Detrended Fluctuation Analysis (MF-DFA), formulated by Kantelhardt, et al. [24]. MF-DCCA is specifically designed to analyze multifractal characteristics in the cross-correlations between two time series. The procedure involves six main steps, which are outlined below.

Let us consider two time series, $x(k)$ and $y(k)$, , defined over the interval $1 \leq k \leq N$, where N represents the total number of observations. It is assumed that both series are of compact support, implying that the instances where $x(k) = 0$ and $y(k) = 0$ occur only for an insignificant portion of the domain.

Step 1: The first step involves constructing the profiles $= \left(X(i)\right)_{1 \leq i \leq N}$ and $Y = \left(Y(i)\right)_{1 \leq i \leq N}$, which are derived from the original time series $x = \left(x(k)\right)_{1 \leq k \leq N}$ and $y = \left(y(k)\right)_{1 \leq k \leq N}$, respectively:

ISSN: 2576-8484
Vol. 9, No. 8: 586-606, 2025
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$$X(i) = \sum_{k=1}^{N}(x(k) - \bar{x}) \qquad Y(i) = \sum_{k=1}^{N}(y(k) - \bar{y}) \tag{1}$$

Here, $\bar{x}$ and $\bar{y}$ represent the average values of the time series $\left(x(k)\right)_{1\le k\le N}$ and $\left(y(k)\right)_{1\le k\le N}$, respectively.

Step 2: For a selected time scale s satisfying $10 \le s \le \mathrm{N}/3$, the profiles X and Y are divided into $N_s = Int(N/s)$ non-overlapping segments, each of length s, where $Int(.)$ denotes the integer part of a number. Because N is not always an exact multiple of sss, a small portion at the end of each profile may be excluded. To ensure that no data is lost, the segmentation process is repeated from the opposite end of the series. This approach yields a total of $2N_s$ segments. The segmentation is performed in two ways: for $1 \le v \le N_s$, we extract the segments $X\big((v-1)s+1\big)\cdots X\big((v-1)s+s\big)$ and $Y\big((v-1)s+1\big)\cdots Y\big((v-1)s+s\big)$; for $Ns+1 \le v \le 2N_s$, we use the sequences $\big((N-v-N_s)s+1\big)\cdots X\big((N-v-N_s)s+s\big)$ and similarly for Y.

Step 3: For each segment, we apply the Ordinary Least Squares (OLS) technique to model the data locally using a polynomial trend. Specifically, we represent the fitted polynomial of degree m for the profile X in the v-th segment as: $p_{X,v}^{m}(i) = \alpha_0^v + \alpha_1^v . i + \cdots + \alpha_m^v . i^m$, and similarly for the profile Y as: $p_{Y,v}^{m}(i) = \beta_0^v + \beta_1^v . i + \cdots + \beta_m^v . i^m$. Selecting an appropriate *m* is essential to balance model accuracy and prevent overfitting.

Step 4: Once the fitting polynomials $p_{X,v}^{m}(i)$ and $p_{Y,v}^{m}(i)$ have been obtained, the next step is to compute the detrended covariance $f_{X,Y}^{2}(v,s)$ for each segment v (where $11 \le v \le 2N_s$) and for all time scales s.

$$\begin{cases} f_{XY}^{2}(v,s) = \dfrac{1}{s}\displaystyle\sum_{i=1}^{s}\left|X\big((v-1)s+i\big) - p_{X,v}^{m}(i)\right|.\left|Y\big((v-1)s+i\big) - p_{Y,v}^{m}(i)\right| \text{ If } 1 \le v \le N_s \\ f_{XY}^{2}(v,s) = \dfrac{1}{s}\displaystyle\sum_{i=1}^{s}\left|X\big((N-v-N_s)s+i\big) - p_{X,v}^{m}(i)\right|.\left|Y\big((N-v-N_s)s+i\big) - p_{Y,v}^{m}(i)\right| \text{ If } N_s \le \end{cases} \tag{2}$$

Step 5: The fluctuation function $F_q^{XY}(s)$ of order q is then derived by taking the average of the detrended covariances across all segments for a given scale s:

$$\begin{cases} F_q^{XY}(s) = \left[\frac{1}{2N_s}\sum_{v=1}^{2N_s}\left(f_{XY}^{2}(v,s)\right)^{\frac{q}{2}}\right]^{\frac{1}{q}} & \text{For } q \ne 0 \\ F_0^{XY}(s) = exp\left[\frac{1}{4N_s}\sum_{\mathrm{v}=1}^{2N_s} ln\left(f_{XY}^{2}(v,s)\right)\right] & \text{For } q = 0 \end{cases} \tag{3}$$

The main objective of the MF-DCCA method is to analyze how the fluctuation functions $F_q^{XY}(s)$ vary with the time scale s for different values of q. To achieve this, Steps 2 through 4 are repeated across a range of time scales s.

Step 6 : To investigate the multi-scale characteristics of the fluctuation functions $F_q^{XY}(s)$, we examine the slope of the log-log plots of $F_q^{XY}(s)$ versus the time scale s for various values of q. If the time series X and Y exhibit long-range cross-correlations following a power-law, indicative of fractal behavior, then for sufficiently large s, the function $F_q^{XY}(s)$ is expected to follow a power-law relationship:

$$F_q^{XY}(s) \sim s^{H_{XY}(q)} \tag{4}$$

Here, $H_{XY}(q)$ refers to the generalized Hurst exponent, which characterizes the power-law cross-correlation between the two time series X and Y.

Edelweiss Applied Science and Technology
ISSN: 2576-8484
Vol. 9, No. 8: 586-606, 2025
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An estimate of $H_{XY}(q)$ can be obtained by performing a linear regression of $Log\left(F_q^{XY}(s)\right)$ against $Log(s)$:

$$Log\left(F_q^{XY}(s)\right) \approx H_{XY}(q).Log(s) \quad (5)$$

If $H_{XY}(q)$ varies with q, it indicates that the cross-correlation between the two-time series exhibits multifractal behavior; if it remains constant, the relationship is monofractal. To determine the values of $H_{XY}(q)$ across different q levels, a semi-log regression is conducted between the fluctuation function $F_q^{XY}(s)$ and the scale s. Specifically, when $q = 2$, $H_{XY}(2)$ corresponds to the classical Hurst exponent. A value of $H_{XY}(2) = 0.5$ suggests no cross-correlation between the series. If $H_{XY}(2) > 0.5$, the series are positively correlated over long ranges (persistent behavior), whereas $H_{XY}(2) < 0.5$ implies long-range anti-persistent cross-correlations. Furthermore, positive values of q highlight the scaling behavior of segments with large fluctuations, while negative q values emphasize regions with smaller, wavelet-like variations.

It is widely recognized that the generalized Hurst exponent $H_{XY}(q)$, as derived from the MF-DCCA approach, is closely linked to the multifractal scaling exponent $\tau_{XY}(q)$, also referred to as the Rényi exponent:

$$\tau_{XY}(q) = q.H_{XY}(q) - 1 \quad (6)$$

When the Rényi exponent $\tau_{XY}(q)$ exhibits a nonlinear dependence on q, it indicates that the cross-correlation between the two-time series has multifractal characteristics. In contrast, a linear relationship between $\tau_{XY}(q)$ and q suggests monofractal behavior.

Another effective approach to describe the multifractality of cross-correlations is by analyzing the Hölder spectrum, also known as the singularity spectrum $f_{XY}(\alpha_{XY})$, which is defined in terms of the Hölder exponent α_{XY}. This spectrum is mathematically connected to the Rényi exponent $\tau_{XY}(q)$ via the Legendre transform:

$$\begin{cases} \alpha_{XY} = \tau'_{XY}(q) \\ f_{XY}(\alpha_{XY}) = q.\alpha_{XY} - \tau_{XY}(q) \end{cases} \quad (7)$$

where $\tau'_{XY}(q)$ represents the first derivative of the function $\tau_{XY}(q)$.

The exponent α_{XY} describes the strength of the singularities. When the cross-correlation between the two series exhibits multifractal behavior, the singularity spectrum $f_{XY}(\alpha_{XY})$ typically forms a concave, bell-shaped curve.

The extent of multifractality between the series can be quantified by the range ΔH_{XY}, which is the difference between the minimum and maximum values, or equivalently, by the width of the spectrum $\Delta\alpha_{XY}$ defined as follows:

$$\begin{cases} \Delta H_{XY} = H_{XY-Max} - H_{XY-Min} = H_{XY}(q_{min}) - H_{XY}(q_{max}) \\ \Delta\alpha_{XY} = \alpha_{XY-max} - \alpha_{XY-min} \end{cases} \quad (8)$$

Greater values of ΔH_{XY} and $\Delta\alpha_{XY}$ signify a higher level of multifractality.

3.2.2. Cross-Correlation Significance Test

As an initial step, it is helpful to qualitatively assess whether cross-correlations exist between the series. For this purpose, Podobnik, et al. [25] introduced the Q_{CC} statistical test. Consider two time series $(X_t)_{1\leq t\leq N}$ and $(Y_t)_{1\leq t\leq N}$, each of length N. The authors defined the cross-correlation function C_i, for $1 \leq i \leq N-1$, as follows:

$$C_i = \frac{\sum_{k=i+1}^{N} X_k.Y_{k-1}}{\sqrt{\sum_{k=1}^{N} X_k^2 \cdot \sum_{k=1}^{N} Y_k^2}} \quad (9)$$

The cross-correlation statistic Q_{CC} is defined for $1 \leq s \leq N-1$ as follows:

ISSN: 2576-8484
Vol. 9, No. 8: 586-606, 2025
DOI: 10.55214/2576-8484.v9i8.9367

$$Q_{CC}(s) = N^2 . \sum_{i=1}^{s} \frac{C_i^2}{N - s} \quad (10)$$

Podobnik, et al. [25] showed that the statistic $Q_{CC}(s)$ approximately follows a chi-square distribution with s degrees of freedom. This test can be applied to evaluate the null hypothesis that none of the first sss cross-correlation coefficients differ significantly from zero. The authors suggested plotting $Q_{CC}(s)$ against the corresponding chi-square critical values $\chi^2(s)$ over a wide range of degrees of freedom s. If $Q_{CC}(s)$ consistently exceeds the critical values at a 95% confidence level across many s, it indicates the presence of significant and potentially long-range cross-correlations. Nevertheless, since this test statistic is based on correlation coefficients, it primarily measures linear cross-correlations. As noted by Podobnik, et al. [25] this test should be employed only as a qualitative tool to detect the existence of cross-correlations.

3.2.3. Origins of Multifractal Cross-Correlations

It is widely recognized that multifractality in the cross-correlations of bivariate time series mainly arises from two sources: long-term temporal cross-correlations and heavy-tailed probability distributions. To assess the relative impact of these factors on the overall multifractal behavior, two data transformations are commonly employed on the original return series: random permutation (shuffling) and phase randomization (surrogate data generation).

Random permutation rearranges the order of the return series, preserving the distribution of values and moments while effectively removing any long-range temporal dependencies. Consequently, the shuffled data retain the original distribution but lose all temporal correlations or memory effects.

Phase randomization, on the other hand, disrupts long-term correlations by randomly altering the phases of the time series in the frequency domain, while maintaining the original amplitude spectrum and overall fluctuation structure. This process isolates the effect of long-range correlations on multifractality.

Several methods exist for phase randomization, all relying on Fourier transform techniques, including:

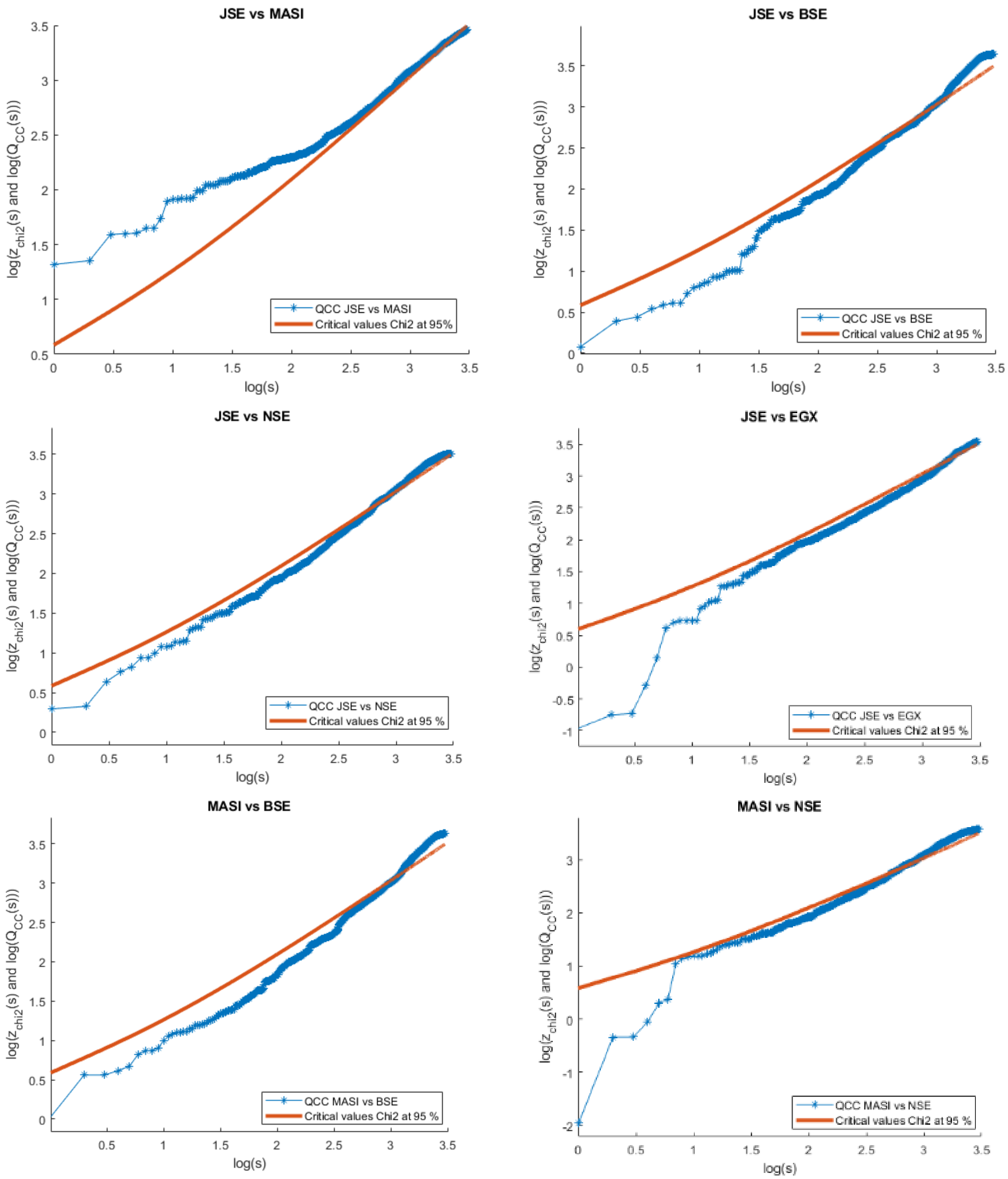
- The Inverse Fast Fourier Transform (IFFT) method (Proakis and Manolakis [26]).
- The Iterative Amplitude Adjusted Fourier Transform (iAAFT) algorithm (Schreiber and Schmitz [27]).
- The Statically Transformed Autoregressive Process (STAP) approach (Kugiumtzis [28]).

In this study, we implemented two shuffling methods using MATLAB functions "randperm" and "randi." For phase randomization, the Inverse Fast Fourier Transform (IFFT) technique was applied.

4. Results and Discussion

4.1. Cross-Correlation Significance Test Results

In this section, we qualitatively assess the existence of cross-correlations among the five African indices using the Q_{CC} statistic. For each pair of indices, we plotted the base-10 logarithm of the test statistic Q_{CC} against the base-10 logarithm of the critical chi-square values $\chi^2_{0,95}(s)$ at the 95% confidence level, covering a wide range of degrees of freedom s from 1 to 3000. The corresponding results are illustrated in the figure below.



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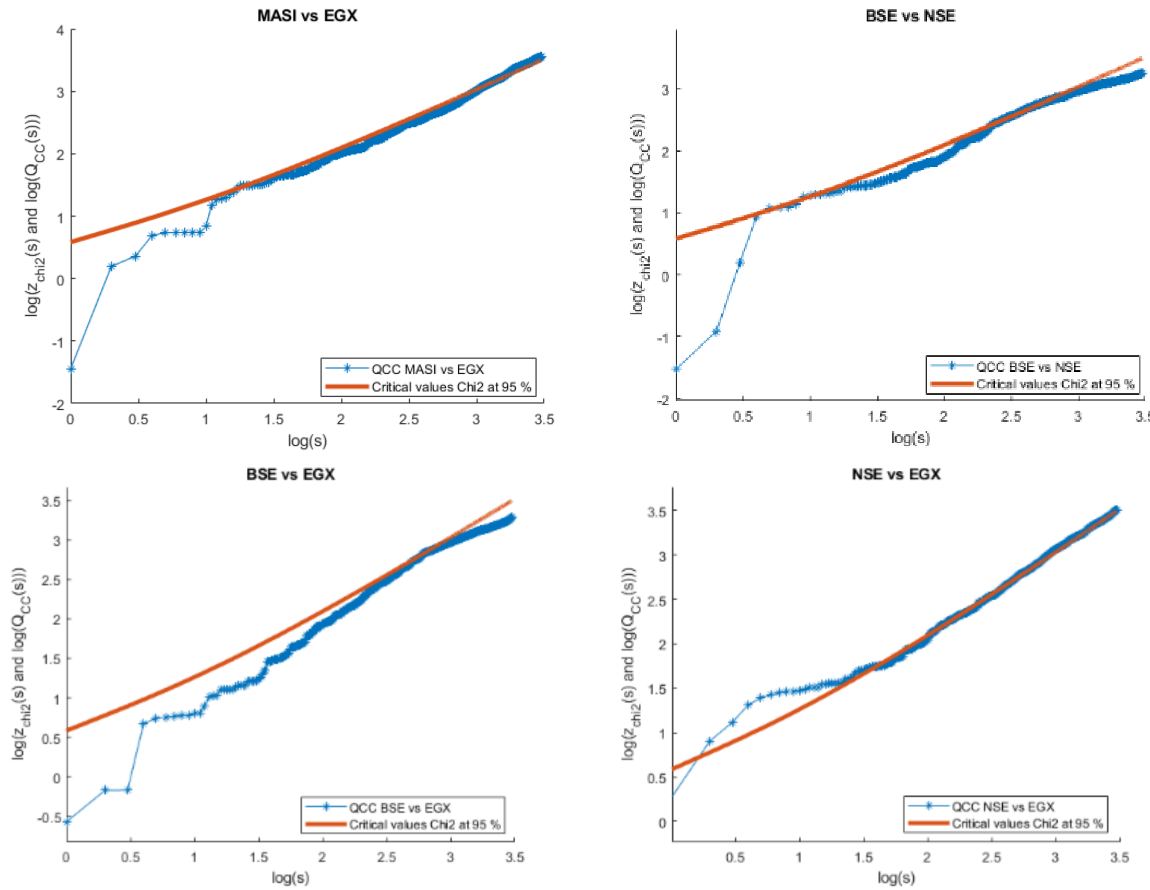


Figure 1.
$Log(Q_{CC}(s))$ and $Log(\chi^2_{0,95}(s))$ vs. $Log(s)$ for all pairs of indices.

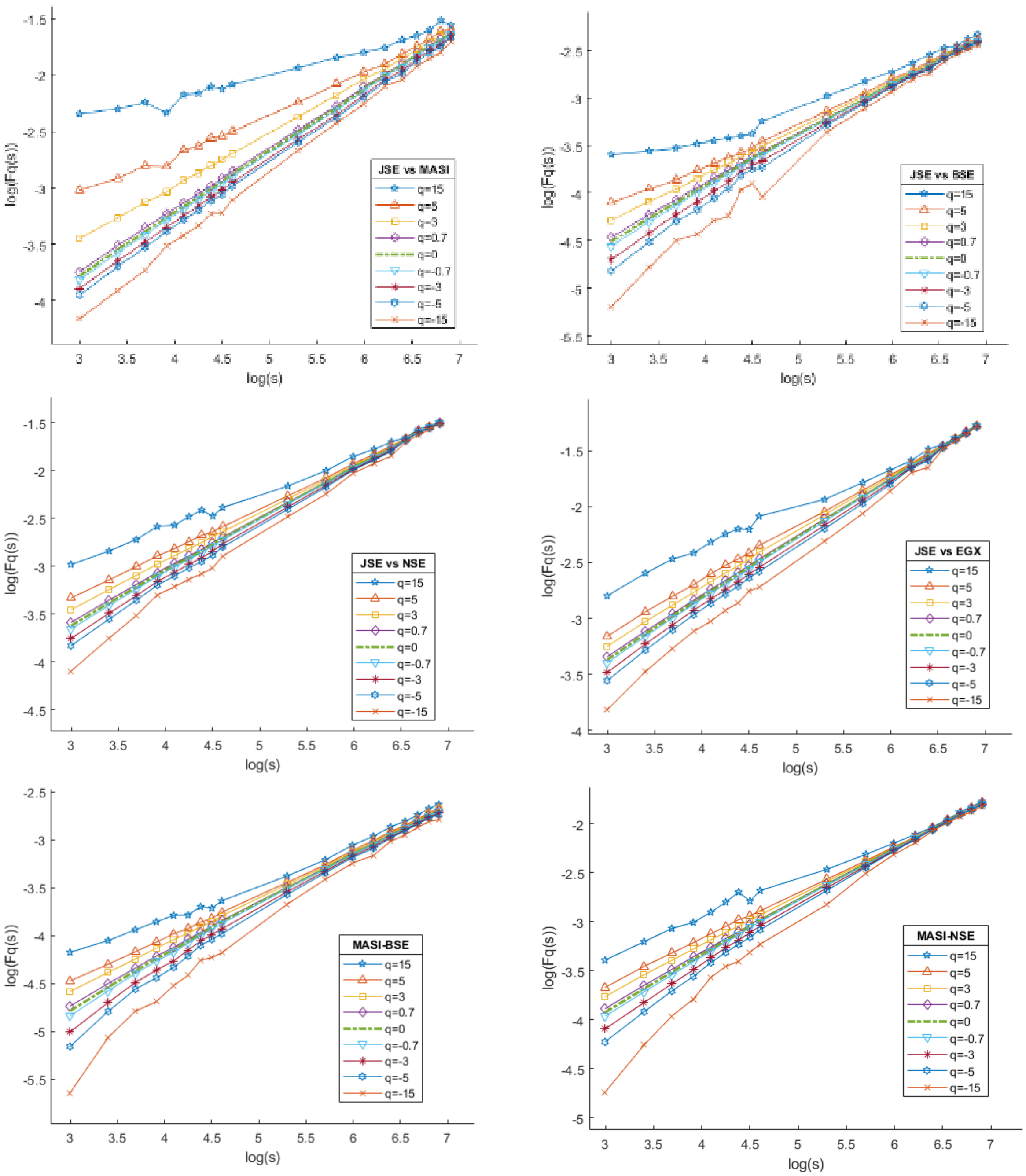
It can be observed that, for nearly all index pairs, the $Q_{CC}(s)$ statistic consistently exceeds the corresponding critical values $\chi^2_{0,95}(s)$, suggesting the presence of statistically significant cross-correlations. However, since this test primarily captures linear dependencies and provides only a qualitative assessment, the findings should be further validated using the MF-DCCA method.

4.2. MF-DCCA Analysis

In this part of the study, the MF-DCCA method is employed to investigate the multifractal cross-correlation properties of the bivariate series of logarithmic returns.

4.2.1. Multi-Scale Analysis of Cross-Correlation Fluctuation Functions

We examined how the cross-correlation fluctuation functions $F_q^{XY}(s)$ behave across multiple time scales sss, specifically within the range [20:10:100, 200:100:1000], for various values of q taken from the intervals [-45:5:-5,-3.1:0.1:-0.1,0.1:0.1:3.1,5:5:45]. The figure below presents log-log plots of $Log\left(F_q^{XY}(s)\right)$ versus $Log(s)$ for nine selected q values: $-15, -5, -3, -0.7, 0, 0.7, 3, 5, 15$, across 9 different index pairs :



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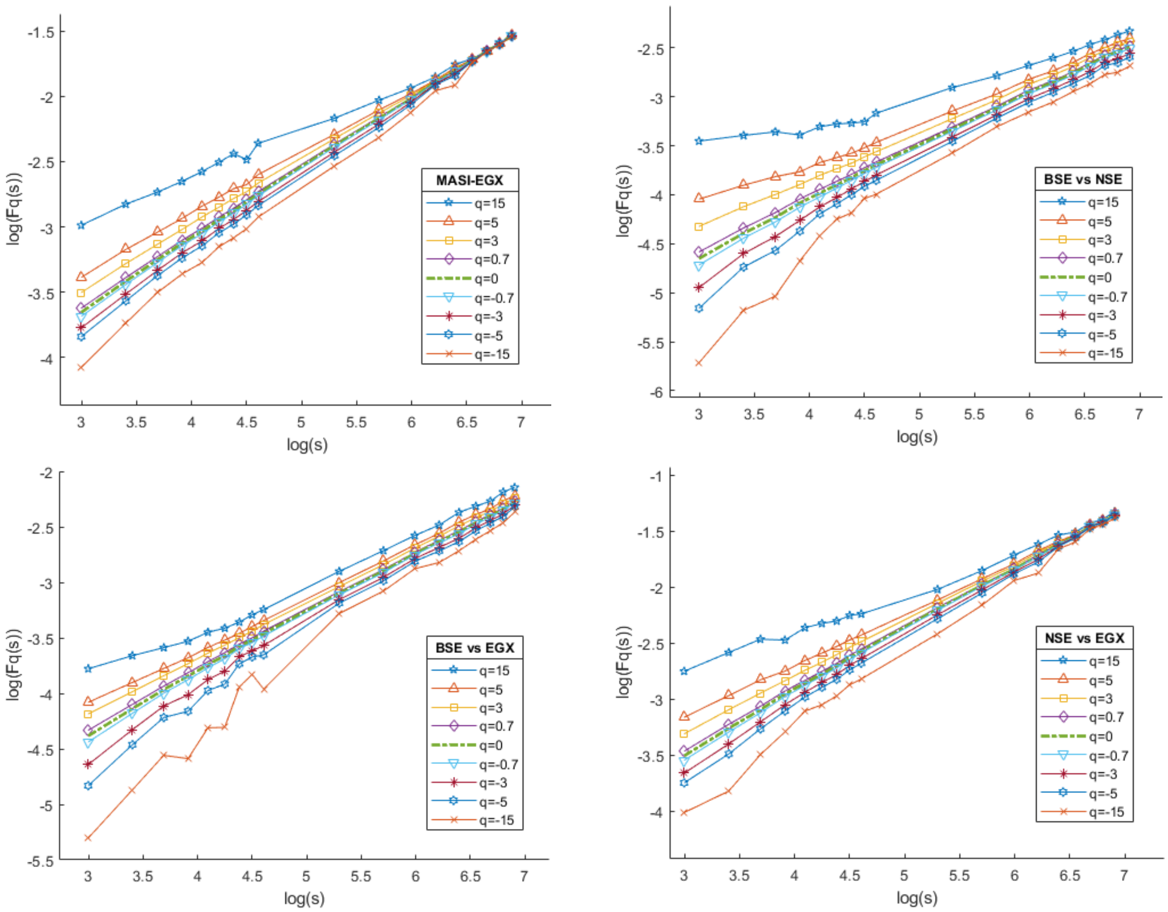


Figure 2.
$Log\left(F_q^{XY}(s)\right)$ vs. $Log(s)$ for $q \in \{-15, -5, -3, -0.7, 0, 0.7, 3, 5, 15\}$.

As illustrated in Figure 2, the functions $F_q^{XY}(s)$ exhibit a nonlinear increase with respect to both the time scale *s* and the moment q, demonstrating a consistent power-law behavior across all index pairs. This pattern indicates the existence of long-range cross-correlations, highlighting the persistent interconnectedness of African stock markets. It suggests that market disturbances may propagate and have prolonged effects across the region. These results carry important implications: foreign investors should account for regional interdependencies when making investment decisions; portfolio managers may benefit from integrating multifractal techniques into their risk assessment and hedging frameworks; and policymakers should remain attentive to cross-market dynamics, promoting coordinated actions to enhance market stability and reduce the likelihood of systemic contagion.

4.2.2. Multifractal Properties and Persistence in Cross-Correlations

The following figure presents the generalized Hurst exponent $H_{XY}(q)$, the Rényi scaling exponent $\tau_{XY}(q)$, and the singularity spectrum $f_{XY}(\alpha)$ for each of the ten index pairs.

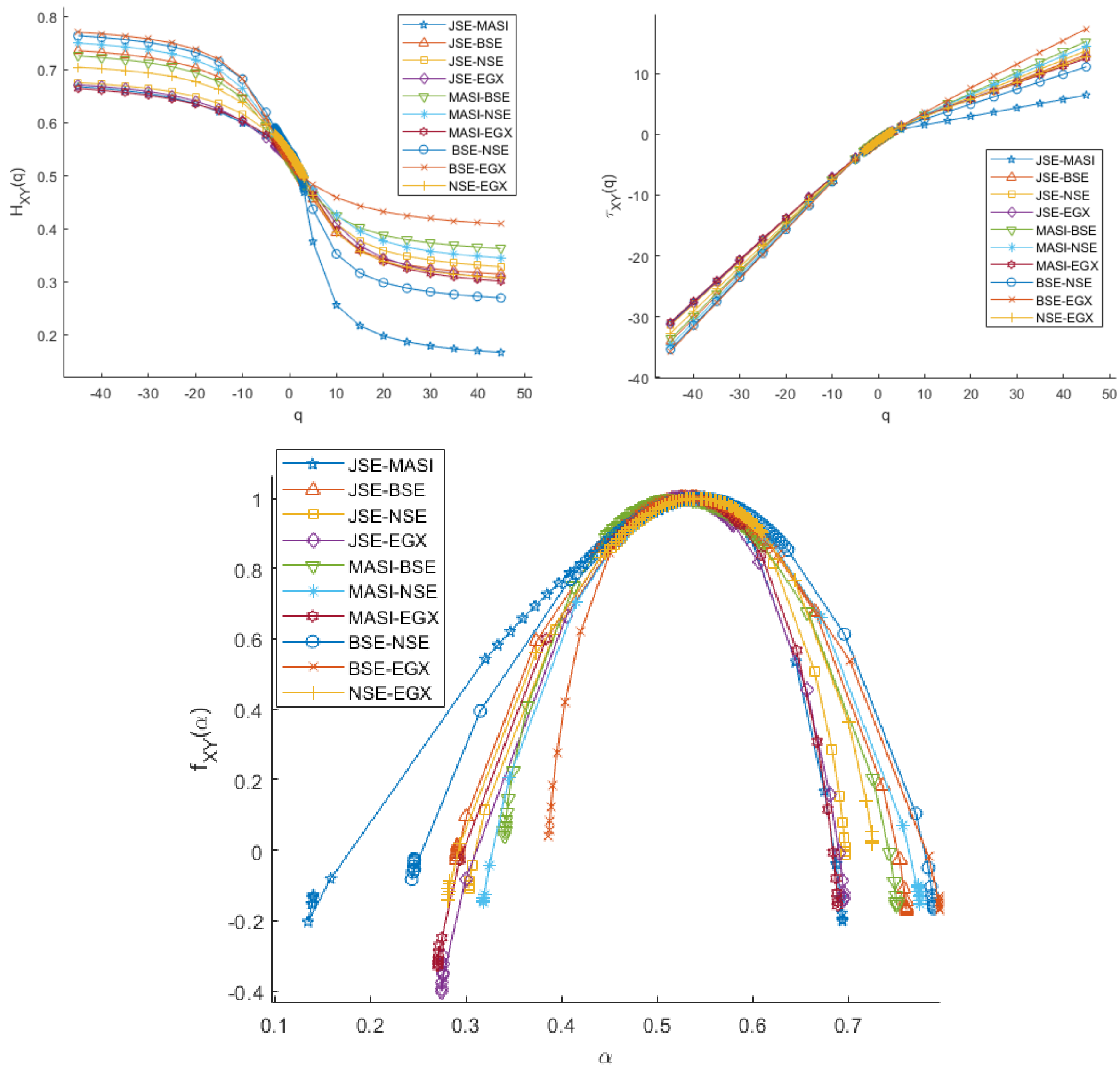


Figure 3.
Plots of $H_{XY}(q)$, $\tau_{XY}(q)$ and $f_{XY}(\alpha)$ for all pairs of indices.

The figure illustrates that across all index pairs, the generalized Hurst exponent $H_{XY}(q)$ decreases in a nonlinear manner as q increases from -45 to 45, while the Rényi exponent $\tau_{XY}(q)$ exhibits a nonlinear upward trend over the same range. Additionally, the singularity spectra $f_{XY}(\alpha)$ display inverted, concave bell-shaped curves. These observations collectively confirm the presence of multifractal cross-correlation structures among the ten index pairs. This multifractality indicates inefficiencies in the interactions between the markets, meaning that the cross-market relationships are complex and exhibit persistent, non-random dependencies over multiple time scales. Such inefficiencies suggest that shocks or information in one market can have lasting and nonlinear effects on others, leading to prolonged interdependence and challenging the assumption of fully efficient and independent markets.

The level of persistence in the cross-correlations can be evaluated by examining the values of $H_{XY}(2)$. The table below presents the corresponding $H_{XY}(2)$ values for each index pair.

Table 1.
Values of $H_{XY}(2)$ for the 10 pairs of indices.

Pairs of indices	$H_{XY}(2)$
JSE vs MASI	0.511
JSE vs BSE	0.504
JSE vs NSE	0.514
JSE vs EGX	0.509
MASI vs BSE	0.497
MASI vs NSE	0.513
MASI vs EGX	0.510
BSE vs NSE	0.508
BSE vs EGX	0.506
NSE vs EGX	0.514

It is observed that the generalized Hurst exponents $H_{XY}(2)$ are approximately 0.51 for the majority of index pairs, with the exception of the MASI-BSE pair, which has a value of $H_{XY}(2) = 0.497$. This means that for 9 out of the 10 index pairs, $H_{XY}(2) > 0.5$, indicating the presence of long-range persistent cross-correlations among those pairs.

The observation that the generalized Hurst exponents $H_{XY}(2)$ are around 0.51 for most index pairs, with the exception of the MASI-BSE pair at 0.497, indicates that nine out of ten pairs exhibit long-range persistent cross-correlations. This persistence reflects a significant degree of market inefficiency, as it implies that past interactions between these markets influence their future behavior over extended periods. Such inefficiencies suggest that information or shocks are not fully or instantaneously absorbed, allowing predictable patterns or dependencies to persist. Consequently, these persistent cross-correlations reveal opportunities for market participants to potentially exploit these predictable dynamics, while also highlighting the need for careful risk management given the prolonged interdependencies between the African stock markets.

4.2.3. Measuring the Strength of Multifractality

The intensity of multifractality in the cross-correlations can be quantified using ΔH_{XY} and $\Delta \alpha_{XY}$, as outlined in Equation (8). The table below displays the multifractality levels for the ten index pairs based on these two indicators.

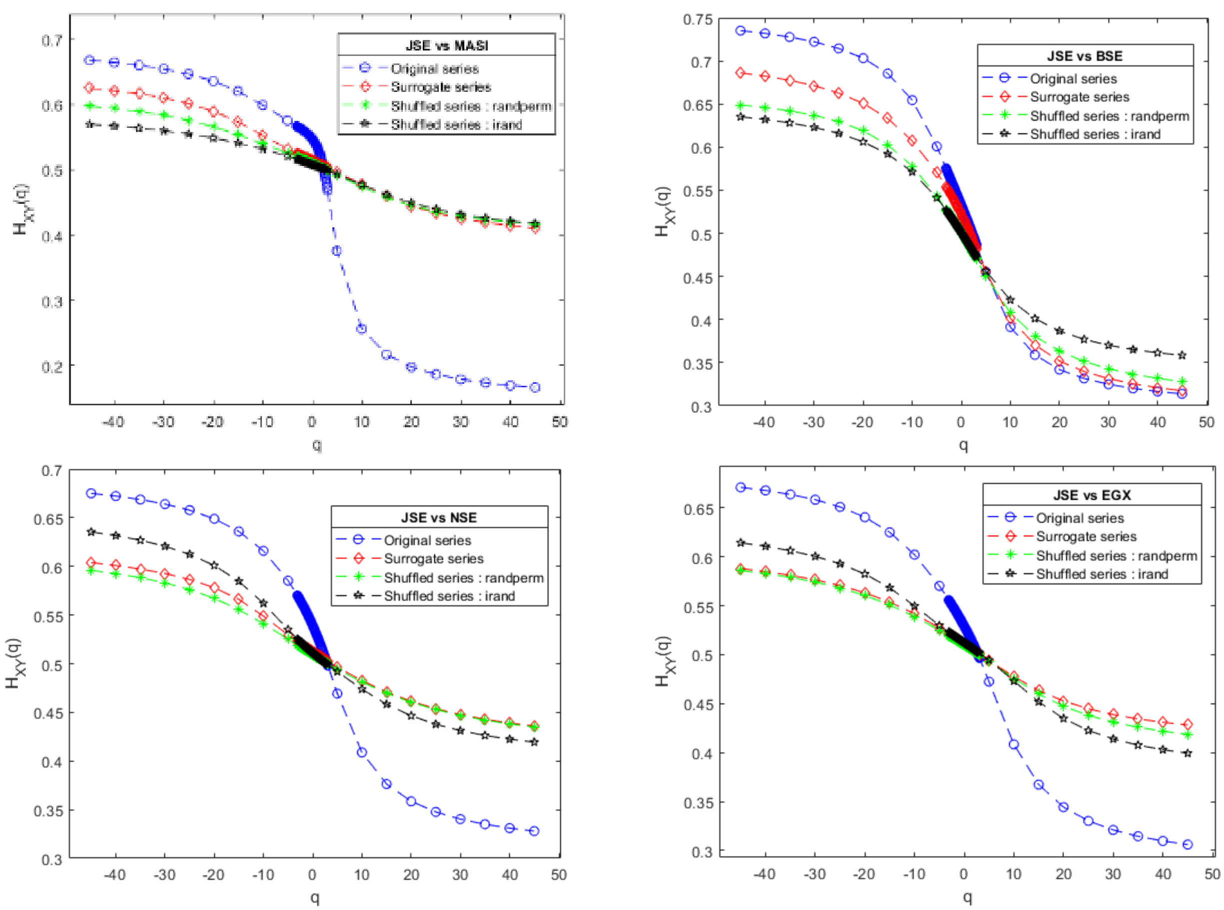
Table 2.
Degrees of multifractality of the ten pairs cross-correlations based on ΔH_{XY} and $\Delta \alpha_{XY}$.

Pairs of indices	ΔH_{XY}	$\Delta \alpha_{XY}$
JSE vs MASI	0.502	0.559
JSE vs BSE	0.421	0.470
JSE vs NSE	0.347	0.395
JSE vs EGX	0.365	0.422
MASI vs BSE	0.363	0.410
MASI vs NSE	0.405	0.457
MASI vs EGX	0.364	0.418
BSE vs NSE	0.494	0.545
BSE vs EGX	0.362	0.409
NSE vs EGX	0.397	0.445

All measured multifractality levels exceed 0.3, confirming the presence of multifractal characteristics in the cross-correlations among the ten pairs of indices. The JSE-MASI pair shows the strongest multifractality, followed in descending order by BSE-NSE, JSE-BSE, MASI-NSE, NSE-EGX, JSE-EGX, MASI-EGX, MASI-BSE, BSE-EGX, and JSE-NSE pairs. The notably high multifractality in the JSE-MASI pair carries important implications: investors should be aware of the persistent and potentially volatile patterns between these markets, adjusting their strategies accordingly. Portfolio managers need to consider differences in multifractality when managing diversification and risk, especially given the close interdependence of the JSE-MASI pair. Policymakers should understand that market shocks or regulatory actions in one market may have prolonged impacts on others, highlighting the need for coordinated policies addressing both immediate and long-term effects. From a risk management perspective, the complex, nonlinear dependencies observed in the JSE-MASI pair underline the importance of employing sophisticated models capable of capturing these dynamics to enhance risk prediction and hedging approaches.

4.2.4. Origins of Multifractal Cross-Correlations

As mentioned earlier, multifractality in cross-correlations arises mainly from two factors: long-term temporal cross-correlations and heavy-tailed distributions. To assess the individual impact of each factor on the overall multifractality, we applied two transformations to the original logarithmic return series: shuffling and phase randomization (surrogate). In this analysis, two shuffling methods, “randperm” and “randi”, were utilized, while phase randomization was performed using the Inverse Fast Fourier Transform (IFFT) technique. Figures 4 and 5 below present a comparison of the generalized Hurst exponent $H_{XY}(q)$ and the singularity spectra $f_{XY}(\alpha)$ curves for the original index return pairs alongside those of the shuffled and surrogate series.



Edelweiss Applied Science and Technology
ISSN: 2576-8484
Vol. 9, No. 8: 586-606, 2025
DOI: 10.55214/2576-8484.v9i8.9367

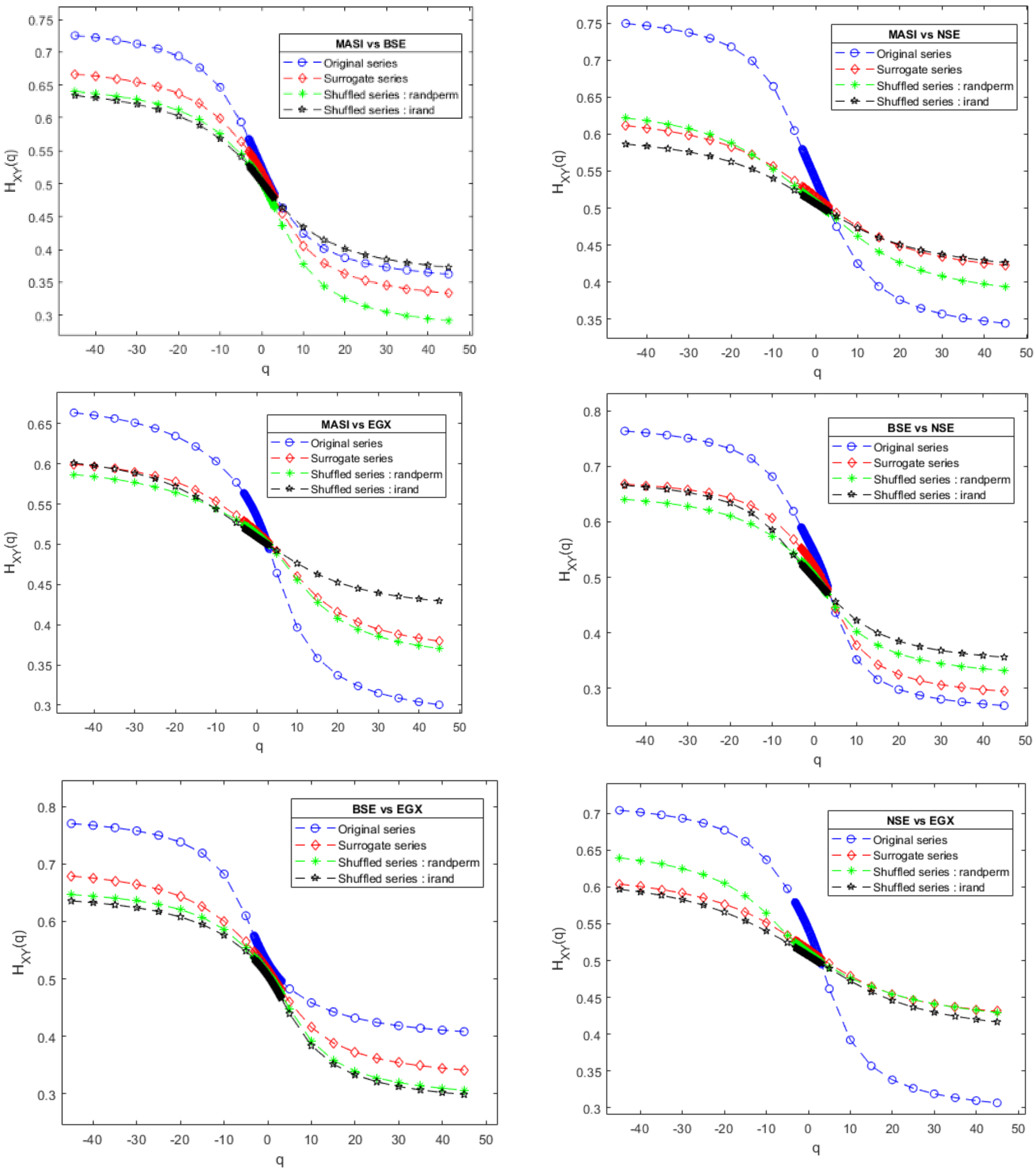


Figure 4.
Generalized Hurst exponent $H_{XY}(q)$ vs. q for all the indices' pairs series of original, surrogate and shuffled.

Edelweiss Applied Science and Technology
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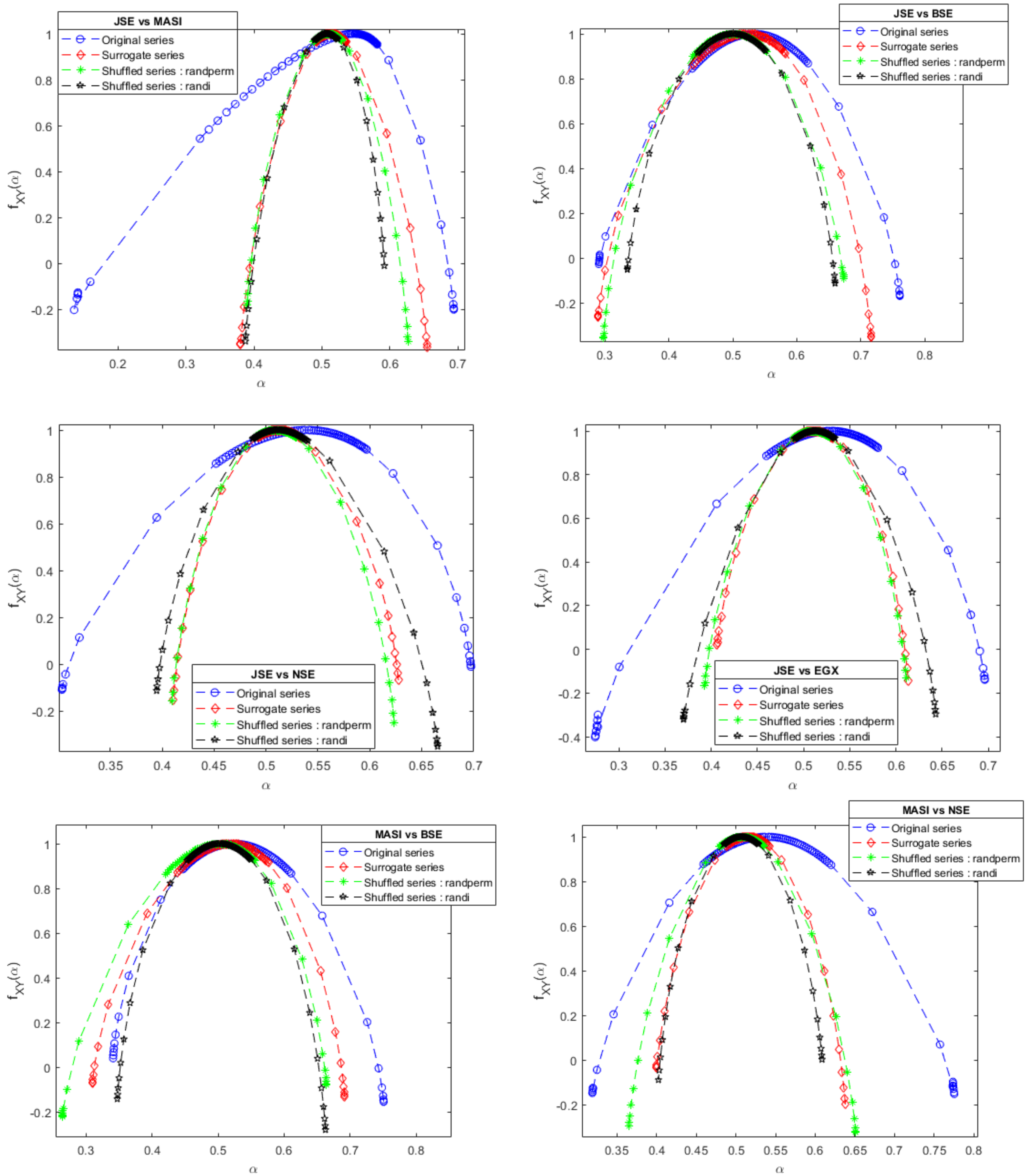

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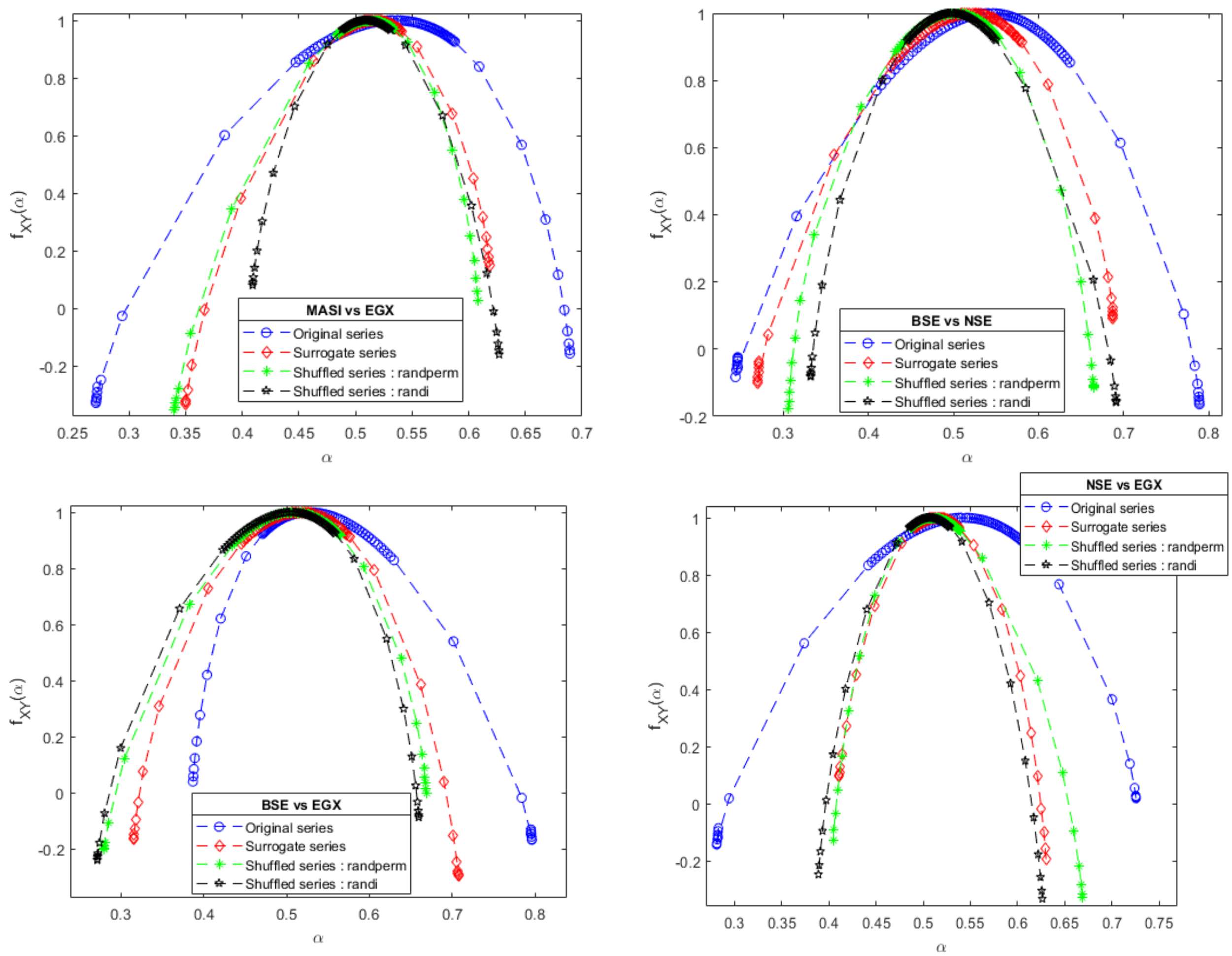

Figure 5.
Singularity spectra $f_{XY}(\alpha)$ vs. α for all the indices' pairs series of original, surrogate and shuffled.

Figures 4 and 5 demonstrate that both shuffling and surrogate transformations lead to a noticeable decrease in the multifractality levels of the original series. To quantify this reduction, we computed the values of ΔH_{XY} and $\Delta \alpha_{XY}$ for all ten pairs of indices. The MF-DCCA analysis was repeated 100 times in MATLAB for each pair. While the original series yielded consistent results, the surrogate and shuffled series produced varying outcomes across runs due to the random nature of the permutation algorithms used. Despite this variability, the original series consistently showed higher values of ΔH_{XY} and $\Delta \alpha_{XY}$ compared to both the surrogate and shuffled series in every simulation. The table below displays the results from a representative simulation out of the 100 runs.

Table 3.
Degrees of multifractality of original, surrogate and shuffled series based ΔH_{XY} and $\Delta\alpha_{XY}$.

	Original		**Surrogate**		**Shuffled-randperm**		**Shuffled-randi**	
Pairs	ΔH_{XY}	$\Delta\alpha_{XY}$	ΔH_{XY}	$\Delta\alpha_{XY}$	ΔH_{XY}	$\Delta\alpha_{XY}$	ΔH_{XY}	$\Delta\alpha_{XY}$
JSE vs MASI	0.502	0,559	0.215	0,314	0.180	0,237	0.153	0,204
JSE vs BSE	0.421	0,470	0.369	0,427	0.321	0,375	0.277	0,325
JSE vs NSE	0.347	0,395	0.168	0,318	0.160	0,216	0.216	0,270
JSE vs EGX	0.365	0,422	0.158	0,206	0.167	0,218	0.215	0,273
MASI vs BSE	0.363	0,410	0.333	0,382	0.349	0,400	0.261	0,316
MASI vs NSE	0.405	0,457	0.188	0,238	0.228	0,286	0.161	0,207
MASI vs EGX	0.363	0,418	0.219	0,268	0.217	0,269	0.171	0,218
BSE vs NSE	0.394	0,545	0.371	0,418	0.308	0,359	0.310	0,359
BSE vs EGX	0.362	0,409	0.338	0,392	0.341	0,389	0.337	0,389
NSE vs EGX	0.397	0,445	0.174	0,221	0.209	0,260	0.180	0,237

The outcomes from the 100 simulations reveal that for all ten index return pairs, the original series consistently exhibit higher values of ΔH_{XY} and $\Delta\alpha_{XY}$ compared to both the surrogate and shuffled series, as shown in the previous table. This demonstrates that the multifractality in cross-correlations diminishes following either surrogate or shuffled transformations. Therefore, it can be concluded that both long-term temporal cross-correlations and heavy-tailed distributions play significant roles in shaping the multifractal nature of the cross-correlations among the ten index return pairs.

In conclusion, the analysis of generalized Hurst exponents and singularity spectra reveals that both long-term cross-correlations and heavy-tailed distributions play key roles in the multifractal characteristics of returns across the ten index pairs, highlighting persistent inefficiencies between the markets where price movements in one market continue to influence others over long periods, indicating delayed information transmission and imperfect market integration. This insight carries important consequences for different market participants. The observed inefficiencies between the markets imply that price adjustments are not instantaneous, leading to opportunities for arbitrage but also increasing vulnerability to prolonged shocks and delayed responses. Investors should be aware that market trends often exhibit persistence, but the probability of extreme events such as sharp crashes or rallies is elevated, necessitating stronger risk management approaches. Portfolio managers need to incorporate nonlinear dependencies among assets and prepare for potential extreme co-movements, adjusting their risk models to perform effectively during both normal and turbulent market conditions. Policymakers must acknowledge that their actions can have lasting and unpredictable impacts, potentially increasing systemic vulnerabilities due to the imperfect integration of these markets, and thus should develop regulations that consider extreme scenarios and market interconnections. From a risk management perspective, conventional models fall short in capturing these nonlinear dynamics, tail risks, and inefficiencies, underscoring the importance of adopting advanced techniques, such as multifractal analysis and tail-risk-focused measures, to more accurately evaluate and mitigate extreme market risks.

4.3. Comparison with Previous Research

Research examining cross-correlations or co-movements among African stock markets—particularly from a multifractal perspective—remains limited. This study's results reveal notable co-movements and strong interdependencies between African stock markets. These findings are in line with those of Owusu-Junior, et al. [29] who investigated correlations and information flow during the COVID-19 pandemic, though their analysis emphasized short-term dynamics. In contrast, our application of the MF-DCCA method captures long-term, persistent cross-correlations. Similarly, our conclusions partly coincide with Ferrouhi [30] who employed Granger causality and Johansen cointegration to detect both short- and long-term co-movements; however, our study advances this by identifying the multifractal and multi-scale characteristics inherent in these relationships. Tweneboah,

ISSN: 2576-8484
Vol. 9, No. 8: 586-606, 2025
DOI: 10.55214/2576-8484.v9i8.9367

et al. [31] also reported long-term dependencies using wavelet analysis, yet our approach offers a more detailed exploration of the underlying complexity beyond what wavelet methods typically reveal. Furthermore, Yaya, et al. [32] used QVAR dynamic connectedness to highlight the shifting roles of individual markets during different phases, pinpointing key markets driving shock transmission. While their work focuses on dynamic market leadership, our research centers on the persistent multifractal structure of cross-market linkages over time.

In summary, although previous studies provide valuable insights, this study contributes a more nuanced and comprehensive understanding of the multifractal cross-correlations, intricate interdependencies, and notable inefficiencies characterizing African stock markets across extended time horizons.

5. Conclusion

This study provides a detailed investigation of cross-correlations among the five largest African stock markets - Johannesburg Stock Exchange, Casablanca Stock Exchange, Botswana Stock Exchange, Nigerian Exchange, and Egyptian Exchange - employing the Multifractal Detrended Cross-Correlation Analysis (MF-DCCA) method. The analysis of nearly 3050 daily observations, spanning from January 30, 2012, to August 8, 2024, has produced significant findings.

The preliminary application of Cross-Correlation Significance Test revealed that cross-correlations among almost all pairs of indices are statistically significant. Utilizing the core components of the MF-DCCA method, such as Generalized Hurst exponents, Rényi exponents, and Hölder Singularity Spectrum, further confirmed that the pairs of indices display long-range persistent cross-correlations and multifractal behavior. These results indicate that the markets are deeply interconnected, with multifractal dynamics influencing their interactions. However, the presence of long-range persistent cross-correlations alongside multifractal features also suggests inefficiencies between the markets, implying that price movements are not fully random or efficient. This inefficiency can create opportunities for arbitrage but also signals potential risks due to prolonged dependencies and delayed information transmission across markets.Moreover, the investigation into the sources of multifractality through surrogate and shuffling transformations revealed that both long-term cross-correlations and heavy-tailed distributions play significant roles in the multifractal nature of the cross-correlations observed.

The findings of this study provide practical implications for various market players. Investors should incorporate multifractal and cross-correlation analyses into their decision-making processes to enhance risk management and capitalize on dynamic market relationships. For policymakers, the findings highlight the need to consider the broader impact of regulatory changes. Policies affecting one market can have substantial ripple effects on others due to their strong interconnections. Therefore, regulators should develop coordinated strategies that address these interdependencies to promote overall market stability and prevent systemic risks. Financial institutions are advised to integrate multifractal analysis into their risk management frameworks. By understanding the long-term stability and multifractal behavior of cross-correlations, they can better anticipate and mitigate risks associated with market fluctuations.

Transparency:

The authors confirm that the manuscript is an honest, accurate, and transparent account of the study; that no vital features of the study have been omitted; and that any discrepancies from the study as planned have been explained. This study followed all ethical practices during writing.

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