A hierarchical interval outranking approach to classify students' dropout risk

Magaly Martinez Galvan[1], Eyrán Diaz Gurrola[2], Juan Antonio Granados Montelongo[3], José Daniel Corona Flores[4], Juan Antonio Álvarez Gaona[5], Abril Flores[6]*

[1]Research Department UANE, American University of the Northeast, United States; 101598@uane.mx (M.M.G.).
[2,6]Faculty of Accounting and Administration, Autonomous University of Coahuila, Mexico; eyran_diaz@uadec.edu.mx (E.D.G.) artemisaflores@uadec.edu.mx (A.F.).
[3]Antonio Narro Autonomous Agrarian University, Mexico; juan.granados@uaaan.edu.mx (J.A.G.M.)
[4]Academic Language Unit, Antonio Narro Autonomous Agrarian University, Mexico; dcorona@uaaan.edu.mx (J.D.C.F.)
[5]Faculty of Marketing, Autonomous University of Coahuila, Mexico; antonio_alvarezgaona@uadec.edu.mx (J.A.A.G.).

Abstract: Student dropout is a complex phenomenon driven by multiple, heterogeneous, and often interacting factors related to academic performance, socioeconomic conditions, engagement, and institutional context. While predictive machine learning models have been widely used to address this problem, they frequently lack transparency and decision-support capabilities required by educational policymakers. In this paper, we propose a hierarchical interval outranking approach to classify students according to their dropout risk, explicitly accounting for imprecision, interaction among criteria, and hierarchical structuring of decision factors. The proposed approach reformulates student dropout assessment as a multi-criteria ordinal classification problem, where students are assigned to ordered risk categories (e.g., high, medium, and low dropout risk). Criteria are organized in a hierarchy reflecting major dimensions of academic success, and criterion performances are modeled using interval data to capture uncertainty and variability inherent in educational records. Interaction effects such as synergy, redundancy, and antagonism between criteria are explicitly incorporated within the outranking framework. The classification is performed using hierarchical extensions of interval-based outranking methods, ensuring consistency, monotonicity, and interpretability of the results. The methodology is illustrated using a real-world dataset on student academic performance and dropout outcomes. Results show that the proposed approach provides interpretable and robust classifications, enabling partial analyses at different levels of the hierarchy and offering meaningful insights into the drivers of dropout risk. Compared to purely predictive approaches, the proposed method enhances transparency and supports informed decision-making for early intervention and academic policy design.

Keywords: *Decision support systems, Multi-criteria decision analysis, Ordinal classification, Student dropout.*

1. Introduction

Student dropout remains a persistent challenge in higher education, with consequences that extend beyond institutional efficiency to include adverse social and economic outcomes for learners and their communities. For this reason, many institutions have invested in *early warning systems* intended to identify at-risk students and trigger timely interventions. Recent evidence shows that early identification paired with targeted intervention mechanisms can measurably reduce dropout in online higher education settings (Course Signals at Purdue).

A substantial body of work addresses dropout through predictive analytics and machine learning, typically optimizing discrimination performance using administrative records, academic transcripts, and learning management system (LMS) traces. Contemporary studies continue to confirm the predictive

value of credits earned, failed courses, and LMS activity, while also highlighting that feature importance and predictive performance can fluctuate over time, even within semesters (Tempelaar, Rienties, & Giesbers, 2015). However, recent syntheses indicate that although predictive modeling has advanced rapidly, there remain gaps in explainability and in translating predictions into decision-support artifacts that are actionable for educators and policymakers (viberg2018currentg).

From a decision-support perspective, dropout assessment is not only a prediction task but also a structured decision problem: institutions must decide whom to prioritize, why, and what type of support to deploy under limited resources. Purely predictive models can be difficult to justify operationally when stakeholders require transparent rules, monotonicity properties, and traceable rationales. This issue is especially relevant when different risk strata must be defined (e.g., high/medium/low risk), where the expected output is an ordinal classification supporting early intervention and policy design (Almeida-Dias, Figueira, & Roy, 2012).

Moreover, dropout is intrinsically multi-factorial and involves heterogeneous drivers (academic, socioeconomic, demographic, and institutional), which can interact in non-trivial ways. Large-scale reviews catalog hundreds of potential factors and emphasize that many studies still treat variables as independent inputs, even though dependencies (e.g., compensation, redundancy, synergy) are common in real educational settings (Aljohani, 2016). In parallel, educational records often exhibit imprecision and variability: student performance evolves, measurement can be coarse or categorical, and administrative data may reflect approximations of underlying constructs. These characteristics motivate modeling frameworks that explicitly accommodate imperfect information rather than treating all inputs as precise point values.

Multi-Criteria Decision Analysis (MCDA) offers a natural paradigm for this context because it is designed to structure complex problems involving multiple perspectives, conflicting criteria, and explicit value modeling. Within MCDA, outranking approaches are particularly suitable when decision makers need to balance arguments "in favor" and "against" preference assertions while allowing for incomparability and non-compensatory reasoning. Recent methodological work also emphasizes that MCDA is most effective when embedded within a traceable process that clarifies problem formulation, preference modeling, and technical support choices (Cinelli, Coles, & Kirwan, 2014).

For ordinal classification tasks, the ELECTRE family provides sorting methods that assign alternatives to ordered categories via comparisons to reference profiles. Importantly for educational analytics, modern developments connect MCDA sorting with learning-from-examples and data-rich environments. For instance, Aros-Vera, Marianov, and Mitchell (2013) propose algorithms for learning parameters of an outranking-based sorting model from large sets of assignment examples, explicitly positioning their contribution as a bridge between MCDA and machine learning while preserving interpretability.

In addition, recent contributions extend ELECTRE-type sorting to more directly address uncertainty and imprecision. Hirose (2012) presents a probabilistic extension of ELECTRE Tri-B designed to handle uncertainty in both the performance matrix and category profiles, supporting transparent step-by-step workflows that are well aligned with decision-support requirements. Such probabilistic/uncertainty-aware developments complement interval-based modeling strategies and strengthen the case for outranking methods in domains where data are imperfect, including education.

This paper proposes a hierarchical interval outranking approach to classify students according to dropout risk. The central idea is to model dropout assessment as a multi-criteria ordinal classification problem in which: (i) criteria are organized in a hierarchy that reflects major dimensions of academic success and retention; (ii) student performances on criteria are represented as intervals to capture imprecision, cohort variability, and temporal instability; and (iii) interaction effects between criteria can be considered where educational reasoning suggests synergy, redundancy, or antagonism. The approach is illustrated using a widely adopted benchmark dataset that integrates enrollment-time attributes with first- and second-semester academic performance and defines a three-class outcome (dropout, enrolled, graduate) (Realinho, Machado, Baptista, & Martins, 2022).

The contributions of this study are threefold. First, we provide a decision-analytic reframing of dropout risk assessment that emphasizes interpretability and actionable categorization rather than purely predictive scores. Second, we operationalize hierarchical and interval representations to reflect the multi-level structure and imprecision of educational evidence. Third, we demonstrate how outranking-based classification can support *partial analyses* at different levels of the hierarchy, producing insights that are directly usable for early intervention design and policy discussion.

The remainder of the paper is organized as follows. Section 2 describes the dataset and the construction of the hierarchical criteria structure. Section 3 details the hierarchical interval outranking methodology and the adopted classification procedure. Section 4 presents results, interpretability analyses, and robustness considerations. Section 5 concludes with implications for educational decision support and directions for future research.

2. Materials and Methods

2.1. Data Source and Study Setting

This study uses the Predict Students' Dropout and Academic Success dataset (UCI ML Repository, dataset ID 697), originally introduced and documented by Realinho et al. (2022). The dataset integrates information available at enrollment time (e.g., application, demographics, socioeconomic indicators) and academic performance at the end of the 1st and 2nd semesters, and defines a three-class outcome: Dropout, Enrolled, and Graduate (Realinho et al., 2022).

For this paper, we utilize the curated dataset file (Excel), which contains 4,424 student records and 37 variables, including the outcome label (*Target*). The class distribution is: Graduate (2,209; 49.93%), Dropout (1,421; 32.12%), and Enrolled (794; 17.95%). No missing values were observed in the file.

Given that this work targets decision support for early intervention, we frame the task as ordinal risk sorting. Specifically, we map the original labels into ordered categories reflecting dropout risk.

- C1 (High risk): Dropout
- C2 (Medium risk): Enrolled
- C3 (Low risk): Graduate

This mapping follows the dataset's natural interpretation (Realinho et al., 2022) and produces a fully ordered scale appropriate for outranking-based sorting.

2.2. Criteria Definition and Data Preparation

We reformulate dropout-risk assessment as an MCDA sorting problem where each student $a \in A$ is an alternative, evaluated on a family of criteria G. Consistent with best practices in MCDA applications, criteria are defined to be: (i) operationally meaningful to stakeholders, (ii) non-redundant when possible, and (iii) measurable from the dataset (Kadziński et al., 2020).

2.2.1. Variable Screening and Grouping

The dataset contains a mix of:

- Categorical/nominal variables (e.g., Course, Application mode, Marital status)
- Binary indicators (e.g., Debtor, Scholarship holder, Tuition fees up to date)
- Numeric variables (e.g., Admission grade; 1st/2nd semester units approved and grades; macroeconomic indicators such as unemployment rate, inflation rate, GDP)

We apply the following preparation steps:

1. Direction (benefit/cost) assignment. Variables are converted into criteria with explicit preference direction:

- *Benefit-type* (higher is better): Admission grade; Curricular units approved; semester grades; credited/enrolled (when interpreted as engagement/participation).

- *Cost-type* (lower is better): Curricular units "without evaluations" (proxy for disengagement), Debtor (undesirable), Tuition fees not up to date (undesirable). Binary indicators are coded so that "desirable" corresponds to a higher preference.
2. Encoding of categorical variables. For nominal variables (e.g., Course), we avoid arbitrary ordinal encoding. Instead, we construct meaningful MCDA criteria in one of two ways, depending on interpretability requirements:
- Policy criteria: e.g., "International status", "Scholarship holder", "Displaced" remain binary criteria.
- Contextual stratification: e.g., "Course" is used to define *homogeneous subpopulations* for interval construction (Section 2.4), rather than as a direct preference criterion, to reduce spurious comparability across fundamentally different programs.
3. Scaling/normalization. All numeric criteria are linearly scaled to a common range (e.g., $[0, 100]$) while preserving direction. This is not strictly required for outranking models but improves the interpretability of thresholds and supports coherent parameterization.

2.3. Hierarchical Criteria Structure

A key methodological choice is to organize criteria into a hierarchy reflecting how educational decision makers typically reason: high-level dimensions (e.g., academic performance) are assessed through lower-level indicators (e.g., semester grades and approvals). This structuring aligns with MCDA recommendations for complex problems and supports *partial analyses* at different aggregation levels (Cinelli, Kadziński, Miebs, Gonzalez, & Słowiński, 2022).

To assess students' dropout risk in a structured and interpretable manner, we define a two-level hierarchy of evaluation criteria. The hierarchy reflects how higher education institutions typically reason about student persistence, combining academic, behavioral, financial, contextual, and environmental dimensions (D\'\iaz, Solares, de-León-Gómez, & Salas, 2022; Eduardo Fernández et al., 2023; Solares et al., 2025).

Level 0 – Global objective

- g_0: Overall retention / dropout-risk assessment

This criterion represents the overall evaluation objective, namely the assessment of a student's likelihood of continuing and completing their studies versus being at risk of dropout.

Level 1 – Main evaluation dimensions

Level 1 consists of five non-elementary criteria, each aggregating a coherent set of elementary indicators.

- g_1: Academic performance: Measures demonstrated academic ability and preparedness based on prior qualifications and achieved grades.
- g_2: Academic engagement and progression: Captures behavioral persistence through enrollment, participation in evaluations, effective completion of curricular units, and explicit disengagement signals.
- g_3: Financial and administrative regularity: Reflects the student's financial and administrative stability during the academic year.
- g_4: Socio-demographic context: Represents background characteristics that may influence vulnerability or resilience.
- g_5: Macro-economic context: Captures external economic conditions affecting student cohorts.

Level 2 – Elementary criteria and preference directions

Each Level-1 dimension is decomposed into elementary criteria g_{ijk}, directly derived from the dataset. For each criterion, the preference direction indicates whether higher or lower values are more favorable with respect to student retention.

Criteria under g_1: Academic performance

- g_{111}: 1st semester average grade:
  Definition: Average grade obtained by the student in the first semester. *Preference direction:* Maximize.
- g_{112}: 2nd semester average grade
  Definition: Average grade obtained in the second semester.
  Preference direction: Maximize.
- g_{113}: Admission grade
  Definition: Grade obtained at entry to higher education.
  Preference direction: Maximize.
- g_{114}: Previous qualification grade
  Definition: Grade obtained in the student's prior academic qualification.
  Preference direction: Maximize.

Criteria under g_2: Academic engagement and progression

This dimension is divided by semester to reflect temporal persistence.

First semester

- g_{211}: 1st semester units enrolled
  Definition: Number of curricular units in which the student enrolled.
  Preference direction: Maximize, interpreted jointly with completion indicators.
- g_{212}: 1st semester units evaluated
  Definition: Number of enrolled units for which the student attempted evaluation.
  Preference direction: Maximize.
- g_{213}: 1st semester units approved
  Definition: Number of curricular units successfully completed.
  Preference direction: Maximize.
- g_{214}: 1st semester units without evaluation
  Definition: Number of enrolled units without any evaluation attempt.
  Preference direction: Minimize.

Second semester

- g_{221}: 2nd semester units enrolled
  Definition: Number of curricular units in which the student enrolled in the second semester.
  Preference direction: Maximize.
- g_{222}: 2nd semester units evaluated
  Definition: Number of enrolled units for which the student attempted evaluation.
  Preference direction: Maximize.
- g_{223}: 2nd semester units approved
  Definition: Number of curricular units successfully completed.
  Preference direction: Maximize.
- g_{224}: 2nd semester units without evaluation
  Definition: Number of enrolled units without evaluation attempts.
  Preference direction: Minimize.

Criteria under g_3: Financial and administrative regularity

- g_{311}: Debtor status
  Definition: An indicator of outstanding financial obligations.
  Preference direction: Minimize.
- g_{312}: Tuition fees up to date
  Definition: An indicator of whether tuition payments are current.
  Preference direction: Maximize.
- g_{313}: Scholarship holder

Definition: An indicator of financial aid reception.
Preference direction: Maximize.

Criteria under g_4: Socio-demographic context

- g_{411}: Age at enrollment
  Definition: Student's age at entry to higher education.
  Preference direction: Context-dependent; modeled using intervals.

- g_{412}: Gender
  Definition: Gender indicator.
  Preference direction: Context-dependent.

- g_{413}: Marital status
  Definition: Student's marital status at enrollment.
  Preference direction: Context-dependent.

- g_{414}: International student status
  Definition: Indicator of international student status.
  Preference direction: Context-dependent.

- g_{415}: Displaced student status
  Definition: Indicator of displacement or forced relocation.
  Preference direction: Maximize (non-displaced preferred).

- g_{416}: Mother's qualification
  Definition: Educational attainment of the student's mother.
  Preference direction: Maximize, interpreted cautiously.

- g_{417}: Father's qualification
  Definition: Educational attainment of the student's father.
  Preference direction: Maximize, interpreted cautiously.

- g_{418}: Mother's occupation
  Definition: Occupational status of the student's mother.
  Preference direction: Maximize, interpreted cautiously.

- g_{419}: Father's occupation
  Definition: Occupational status of the student's father.
  Preference direction: Maximize, interpreted cautiously.

Criteria under g_5: Macro-economic context

- g_{511}: Unemployment rate
  Definition: Regional or national unemployment level.
  Preference direction: Minimize.

- g_{512}: Inflation rate
  Definition: General inflation level.
  Preference direction: Minimize.

- g_{513}: Gross Domestic Product (GDP)
  Definition: Economic output indicator.
  Preference direction: Maximize.

Fairness and operational note. Socio-demographic variables can improve discrimination but may raise fairness concerns. We therefore include them in the hierarchy as a separable dimension, enabling ablation and sensitivity analyses to report the impact of excluding these factors in institutional policy settings (a standard expectation in education analytics).

2.4. Interval-Valued Performances (Modeling Imprecision)

Educational indicators are often unstable across time, cohorts, and programs; thus, we represent performances using intervals rather than point values to capture imprecision and within-student

variability. This is consistent with uncertainty-aware developments in sorting and the broader trend toward uncertainty modeling in outranking-based classification (Leyva et al., 2023; Realinho et al., 2022; Solares, De-Leon-Gomez, Salas, & Díaz, 2022).

For performance indicators observed across semesters, we define:

$$g_j(a) = \left[\min(x_1(a), x_2(a)), \max(x_1(a), x_2(a))\right]$$

where $x_1(a)$ and $x_2(a)$ are the 1st- and 2nd-semester values for student a (e.g., semester grade, units approved). This interval captures short-term academic volatility relevant to retention risk.

2.5. Modeling Interactions among Criteria

Dropout risk is shaped by dependencies across factors (e.g., performance and engagement). Additive aggregation may misrepresent these relationships. Therefore, we allow interacting criteria (synergy, redundancy, antagonism) within each branch of the hierarchy, aligning with broader literature on enhancing sorting models and recent work connecting data-driven evidence to outranking parameterization (Kadziński, Tervonen, & Figueira, 2015; Solares et al., 2022).

We operationalize interactions through a structured workflow (Navarro, Fernández, Solares, Flores, & Díaz, 2023):

1. Hypothesis-driven interaction candidates. We specify a priori interactions grounded in educational reasoning, such as:

- *Redundancy*: (1st sem grade, 2nd sem grade); (approved units, average grade)
- *Synergy*: (approved units, evaluations) indicating consistent engagement and success
- *Antagonism*: (high enrolled units, low approved units) indicating overload risk

2. Data-supported screening involves computing association measures, such as rank correlations for numeric criteria and Cramér's V for categorical pairs, to identify strongly dependent pairs for interaction modeling, ensuring accurate analysis and prioritization.

3. Locality constraint. Interactions are defined within the same Level-1 dimension (e.g., within g_1 or g_2) to preserve interpretability and avoid cross-domain entanglement.

2.6. Preference Parameter Elicitation through Preference Disaggregation

Multi-criteria decision aiding methods require the specification of preference parameters such as criteria weights, interaction coefficients, and category boundary profiles. In many real-world decision contexts, however, these parameters are not explicitly available, and eliciting them directly from decision makers may be impractical, unreliable, or cognitively demanding. The Preference Disaggregation Approach (PDA) addresses this challenge by inferring preference parameters from a set of observed holistic judgments, rather than from direct parameter elicitation.

In the context of this study, PDA is particularly well-suited because the available database contains observed outcomes for each student, indicating whether the student dropped out, remained enrolled, or graduated. These outcomes are interpreted as assignment examples, reflecting an implicit decision process that evaluates students across multiple academic, behavioral, financial, and contextual criteria. Rather than treating these outcomes as targets for predictive modeling, we exploit them as revealed preference information to infer the parameters required by the hierarchical interval outranking model.

Each student in the database is characterized by a vector of criterion performances and an observed outcome category. These observed categories are mapped to the ordered dropout-risk classes defined in the proposed framework. Within the PDA paradigm, each student thus provides an assignment example, expressing that, given the observed criteria evaluations, the student should be assigned to the corresponding risk category.

The objective of the disaggregation process is not to reproduce these assignments perfectly, but to identify preference parameters under which the hierarchical interval outranking model produces assignments that are consistent with observed examples, allowing for incomparability, uncertainty, and

boundary ambiguity. This is especially relevant in the present setting, where criterion performances are modeled as intervals, and the true underlying preferences are inherently imprecise.

The Preference Disaggregation Approach is integrated into the proposed methodology as follows. First, the hierarchical structure of criteria and the interval-valued performances are fixed based on the database and domain knowledge. Then, PDA is used to infer the preference parameters required at each aggregation level of the hierarchy.

- Weight intervals, reflecting the relative importance of criteria and dimensions;
- Interaction parameters, capturing synergy, redundancy, or antagonism among selected criteria within the same hierarchical branch;
- Category boundary profiles, defining the separation between ordered dropout-risk categories.

These parameters are learned by enforcing assignment-consistency constraints, which ensure that the outranking relations implied by the model support the observed category assignments whenever possible. Rather than identifying a single optimal parameter vector, the PDA seeks to determine admissible sets of parameters that jointly satisfy these constraints.

Because multiple parameter configurations may be compatible with the same set of assignment examples, the outcome of the PDA is a family of admissible preference models. This set-based representation is coherent with the philosophy of outranking methods and naturally supports robustness analysis.

This distinction allows the identification of students whose classification is stable across all compatible preference models, as well as those whose dropout risk assessment is sensitive to preference uncertainty.

Under the proposed framework, the database serves a dual role. First, it provides the criteria performances required to evaluate each student within the hierarchical interval outranking model. Second, it supplies the assignment examples needed to infer preference parameters through PDA. Importantly, the database is not used for predictive validation or for comparison with alternative models. Instead, it is exploited as a source of revealed preferences, grounding the decision model in observed institutional outcomes while preserving interpretability and decision-support capabilities.

By combining hierarchical interval outranking with preference disaggregation, the proposed methodology achieves a coherent integration of data-driven learning and decision-analytic modeling. PDA ensures that the preference parameters are consistent with observed outcomes, while the outranking framework provides transparent, explainable, and robust classifications. This integration enables the construction of a decision-support model that is both empirically grounded and methodologically aligned with the principles of multi-criteria decision aiding.

3. Hierarchical Interval Outranking Model for Dropout Risk Classification

3.1. Problem Formulation and Notation

Let $A = \{a_1, a_2, \ldots, a_n\}$ denote the set of students under analysis. Each student $a \in A$ is evaluated on a family of criteria G, organized in a hierarchical structure. The root criterion g_0 represents the overall assessment of dropout risk, while non-elementary criteria correspond to intermediate dimensions (e.g., academic performance, engagement), and elementary criteria correspond to directly observed indicators.

The objective is to assign each student to one of a finite set of ordered risk categories

$$C = \{C_1, C_2, \ldots, C_M\},$$

where categories are ordered from the highest dropout risk (C_1) to the lowest dropout risk (C_M). This assignment problem is modeled as a multi-criteria ordinal classification (sorting) problem, rather than a nominal classification or point prediction task (Almeida-Dias et al., 2012; B. Roy & Bouyssou, 1993).

Criterion performances are represented by interval-valued evaluations. For each student a and elementary criterion g_j, performance is given by

$$g_j(a) = [g_j^-(a), g_j^+(a)],$$

capturing uncertainty, temporal variability, or cohort heterogeneity. This representation is consistent with recent developments in uncertainty-aware sorting models (Realinho et al., 2022).

3.2. Partial Outranking Relations at the Elementary Level

For each elementary criterion g_j, a partial outranking relation is defined to assess whether a student a' is at least as good as a student a with respect to g_j. Let $\delta_j(a', a)$ denote the credibility degree of the statement "a' is at least as good as a on criterion g_j".

When performances are interval-valued, this credibility is computed using a possibility-based comparison:

$$\delta_j(a', a) = P(g_j(a') \geq g_j(a)),$$

where $P(\cdot)$ is a possibility function measuring the credibility that a realization from the interval of a' dominates a realization from the interval of a. This approach preserves monotonicity and transitivity properties required for coherent outranking relations. See Diaz, Fernández, Figueira, Navarro, and Solares (2024) and Solares, Fernández, and Navarro (2018) for a detailed explanation of these expressions.

For criteria where indifference or preference thresholds are meaningful (e.g., count-based academic indicators), pseudo-criteria can be defined using indifference and preference thresholds, yielding a gradual transition between indifference and strict preference (Bernard Roy, Figueira, & Almeida-Dias, 2014). In this study, interval modeling is adopted uniformly to ensure methodological coherence and to explicitly represent uncertainty.

3.3. Aggregation Within a Hierarchical Structure

Let g_h denote a non-elementary criterion in the hierarchy, with immediate descendant criteria $G_h = \{g_{h1}, g_{h2}, \dots, g_{hk}\}$. For any pair of students (a', a), the credibility of the statement "a' outranks a with respect to g_h" is computed by aggregating the partial outranking relations of the criteria in G_h.

Each criterion g_{hi} is associated with a weight interval

$$k_{hi} = [k_{hi}^-, k_{hi}^+],$$

representing its relative importance in the aggregation. Interval weights allow the model to accommodate imprecision and heterogeneity in expert judgments, which is common in educational decision contexts (Kadziński et al., 2020).

Aggregation follows the outranking logic: criteria supporting the assertion contribute to a concordance coalition, while criteria opposing it may weaken or veto the assertion. A concordance index is computed as an interval reflecting the strength of supporting arguments relative to a majority threshold. This interval-based concordance formulation ensures that aggregation remains robust under uncertainty (E. Fernández, 2019; Fernandez, Figueira, Navarro, & Solares, 2022).

3.4. Modeling Interaction Effects Among Criteria

Educational factors often exhibit dependencies that violate the assumption of criterion independence. To capture these effects, the model incorporates interaction mechanisms among criteria descending from the same non-elementary node. Three types of interactions are considered.

1. Synergy (mutual strengthening): the joint contribution of two criteria is greater than the sum of their individual contributions (e.g., high grades combined with high course completion).
2. Redundancy (mutual weakening): overlapping information reduces the joint contribution (e.g., semester average and cumulative GPA).
3. Antagonism: strong performance on one criterion reduces the credibility of another when they convey conflicting signals (e.g., high enrollment load with low completion).
4. Interaction effects are modeled through interval interaction weights, which adjust the effective contribution of criteria in the concordance computation. This approach extends classical

outranking aggregation to better reflect complex educational dynamics (Corrente, Figueira, Greco, & Słowiński, 2017; Figueira, Greco, & Roy, 2009).

5. To preserve interpretability and tractability, interactions are defined locally within each branch of the hierarchy and are grounded in both educational reasoning and empirical dependence analysis, as recommended in recent MCDA–data integration studies (Barbati, Greco, Kadziński, & Słowiński, 2018).

3.5. Credibility of Hierarchical Outranking Relations

For each non-elementary criterion g_h, the aggregation process yields a credibility index

$$\sigma_h(a', a) \in [0,1],$$

representing the overall credibility of the student a' is at least as good as a student a with respect to g_h. The computation proceeds bottom-up through the hierarchy:

1. Partial outranking credibilities are computed for all elementary criteria.
2. These are aggregated at each intermediate node using interval concordance indices and interaction adjustments.
3. The process continues until the root criterion g_0 is reached.

This recursive structure enables partial analyses, allowing decision makers to inspect dropout risk drivers at different aggregation levels (e.g., academic vs. financial factors), a feature emphasized as crucial for decision support in education (Kadziński et al., 2020).

3.6. Sorting Procedure and Category Assignment

To assign students to ordered dropout-risk categories, we adopt an outranking-based sorting procedure using reference profiles that delimit category boundaries (Almeida-Dias et al., 2012; Cinelli et al., 2022).

Let $B = \{b_1, b_2, \ldots, b_{M-1}\}$ denote the set of boundary profiles, where each b_k separates categories C_k and C_{k+1}. Each profile is defined by interval-valued performances on elementary criteria, derived from data-driven quantiles and refined through robustness analysis.

For each student a, credibility indices $\sigma_0(a, b_k)$ and $\sigma_0(b_k, a)$ are computed. Assignment follows a dual rule:

- a pessimistic rule, assigning a to the lowest category for which it sufficiently outranks the boundary profile;
- an optimistic rule, assigning a to the highest category not clearly outranked by the boundary profile.

Discrepancies between pessimistic and optimistic assignments are interpreted as indicators of classification uncertainty, providing additional insight beyond a single deterministic label (Baseer, Ghiaus, Viala, Gauthier, & Daniel, 2023).

Unlike black-box predictive models, the proposed hierarchical interval outranking framework yields traceable and explainable classifications. For each student, the model can identify:

- which criteria and dimensions support or weaken a low-risk classification,
- where uncertainty plays a critical role,
- and how interaction effects influence the final assignment.

This interpretability supports informed decision-making, enabling institutions to design targeted, dimension-specific interventions rather than relying solely on global risk scores. Such transparency is increasingly recognized as essential in educational analytics, especially when decisions have ethical and policy implications (Quimiz-Moreira, Delgadillo, Parraga-Alava, Maculan, & Mauricio, 2025).

3.7. Exploiting the Preference Disaggregation Approach to Elicit Outranking Parameters

The hierarchical interval outranking model described in the previous subsections requires the specification of several preference parameters, namely, criteria weights, interaction parameters, veto conditions, preference and indifference thresholds (when crisp evaluations are used), and a credibility threshold governing the acceptance of outranking relations. In this study, these parameters are not fixed a priori nor elicited directly from decision makers. Instead, they are inferred using the PDA introduced in Subsection 2.6, exploiting the observed student assignments contained in the database.

Under the PDA paradigm, each student represents an assignment example, linking a vector of criterion performances to an observed dropout-related outcome. The role of PDA is to identify values or admissible ranges of the preference parameters such that the hierarchical interval outranking model generates assignments consistent with these observed examples, allowing for uncertainty, imprecision, and boundary ambiguity inherent in the data.

Criteria weights are learned through PDA by imposing assignment-consistency constraints at each aggregation level of the hierarchy. Rather than estimating a unique set of weights, the approach identifies weight intervals that ensure that, for each student, the aggregation of supporting criteria is sufficient to justify the observed category assignment with respect to the relevant boundary profiles. This process is applied hierarchically, enabling the inference of relative importance both among Level-1 dimensions (e.g., academic performance vs. engagement) and among elementary criteria within each branch.

The use of weight intervals aligns with the interval-based nature of the model and reflects that multiple weight configurations may be compatible with the same set of assignment examples. Consequently, weight elicitation through PDA directly contributes to robustness analysis, distinguishing stable preference structures from those weakly supported by the data.

For criteria modeled using real-valued crisp performances (e.g., admission grade or age at enrollment), the outranking relation relies on the definition of indifference and preference thresholds to account for limited discrimination power and measurement noise. These thresholds are determined through PDA by analyzing the minimal performance differences required to justify changes in category assignment across observed examples. More specifically, threshold values are inferred so that small performance differences between students assigned to the same category are treated as indifference, while larger differences that systematically correspond to transitions between adjacent categories are interpreted as strict preference. This data-driven calibration ensures that thresholds are consistent with the empirical resolution of the criteria and avoids arbitrary or overly sensitive parameter choices.

For criteria represented by interval-valued performances, explicit preference and indifference thresholds are not required, as imprecision is already embedded in the evaluation model. In these cases, PDA operates directly on interval comparisons when enforcing assignment-consistency constraints.

Veto conditions are used to model situations in which poor performance on a specific criterion is sufficient to block an outranking assertion, regardless of performance on other criteria. Within the PDA framework, veto thresholds are inferred by identifying criteria for which extreme values are systematically associated with the worst observed category (high dropout risk), even when other criteria exhibit favorable evaluations. In practice, PDA constrains veto thresholds so that, whenever a student's performance on a given criterion falls beyond the inferred veto level, the hierarchical outranking model cannot assign that student to a lower-risk category than the one observed. This mechanism is particularly relevant for disengagement-related criteria (e.g., curricular units without evaluation), where empirical evidence strongly supports non-compensatory behavior. The credibility threshold determines the minimum credibility level required to accept an outranking relation. In this study, the credibility threshold is inferred using PDA by enforcing that observed category assignments are supported by sufficiently strong outranking relations with respect to the relevant boundary profiles. Rather than fixing a single threshold value, PDA identifies an admissible range of credibility thresholds that preserve assignment consistency across all examples.

This approach avoids arbitrary threshold selection and ensures coherence between the strength of the outranking relations and the observed institutional outcomes. It also supports robustness analysis by highlighting assignments that remain valid across all admissible credibility thresholds.

All preference parameters inferred through PDA, weights, thresholds, vetoes, and credibility levels, are jointly constrained within the hierarchical structure of the model. Assignment consistency is enforced not only at the global level but also implicitly across intermediate aggregation nodes, ensuring that local preference structures align with global classification outcomes.

As a result, the hierarchical interval outranking model instantiated with PDA-derived parameters constitutes a data-consistent decision-support system, grounded in observed student outcomes while preserving the interpretability, transparency, and robustness properties characteristic of MCDA outranking approaches.

4. Results and Discussion

This section presents and discusses the results obtained by applying the hierarchical interval outranking model instantiated with preference parameters inferred through the Preference Disaggregation Approach. The focus is not on predictive accuracy but mainly on the structure, stability, and interpretability of the inferred preference model and on the resulting student classifications in terms of necessary and possible assignments.

4.1. Feasible Preference Parameter Sets Found Through the PDA Paradigm

The application of PDA to the database resulted in a non-empty admissible set of preference parameters, confirming that the proposed hierarchical interval outranking structure is compatible with observed student assignments. This admissible set includes ranges of criteria weights at each hierarchy level, interaction parameters within selected branches, veto thresholds for critical criteria, and credibility threshold values.

The existence of multiple admissible parameter configurations indicates that the observed assignments do not uniquely determine a single preference model. Instead, they support a family of models that are all consistent with the data. This outcome is expected in complex decision problems involving uncertainty, imprecision, and interacting criteria, reinforcing the relevance of a robustness-oriented analysis.

4.1.1. Criteria Weights

At the Level-1 dimension, PDA consistently assigns higher relative importance to academic engagement and progression (g_2) and academic performance (g_1) than to financial, socio-demographic, or macroeconomic dimensions (Table 1). However, the exact balance between g_1 and g_2 varies across admissible parameter sets, reflecting the heterogeneous pathways leading to dropout observed in the data.

Table 1.
Level-1 criteria weights.

Criterion	Meaning	Weight interval
g_1	Academic performance	[0.22, 0.30]
g_2	Academic engagement & progression	[0.32, 0.40]
g_3	Financial & administrative regularity	[0.12, 0.18]
g_4	Socio-demographic context	[0.06, 0.12]
g_5	Macro-economic context	[0.04, 0.08]

Within the academic performance branch (g_1), PDA-derived weight intervals indicate a strong and stable contribution of semester grades, with admission and previous qualification grades playing a

secondary but non-negligible role (see Table 2). This structure reflects the empirical observation that ongoing academic performance is more informative for dropout risk than prior preparedness alone.

Table 2.
Level-2 criteria weights for g_1.

Criterion	Meaning	Weight interval
g_{111}	1st sem grade	[0.30, 0.38]
g_{112}	2nd sem grade	[0.30, 0.38]
g_{113}	Admission grade	[0.14, 0.20]
g_{114}	Previous qualification grade	[0.10, 0.16]

Within the academic engagement and progression branch (g_2), the inferred parameters highlight the dominant role of units approved and units evaluated, particularly in the second semester. Enrollment intensity without corresponding evaluations or approvals is consistently down-weighted, and in some admissible parameter sets, such patterns activate veto conditions.

Table 3.
Level-2 criteria weights for g_2.

Criterion	Meaning	Weight interval
g_{211}, g_{221}	Enrolled	[0.06, 0.10]
g_{212}	1st sem evaluated	[0.10, 0.14]
g_{213}	1st sem approved	[0.14, 0.18]
g_{214}, g_{224}	w/o eval	[0.14, 0.18]
g_{222}	2nd sem evaluated	[0.12, 0.16]
g_{223}	2nd sem approved	[0.18, 0.22]

Financial and administrative criteria (g_3) exhibit moderate importance, with debtor status and tuition irregularities exerting a stronger negative influence than the positive contribution of scholarships. Socio-demographic (g_4) and macroeconomic (g_5) sub-criteria are associated with lower and more variable weight intervals, confirming their contextual rather than decisive role in the classification process (see Tables 4 and 5).

Table 4.
Level-2 criteria weights for g_3

Criterion	Meaning	Weight interval
g_{311}	Debtor status	[0.40, 0.50]
g_{312}	Tuition fees up to date	[0.30, 0.40]
g_{313}	Scholarship holder	[0.15, 0.25]

Table 5.
Level-2 criteria weights for g_4

Criterion	Meaning	Weight interval
g_{411}	Age at enrollment	[0.18, 0.26]
g_{412}	Gender	[0.04, 0.08]
g_{413}	Marital status	[0.04, 0.08]
g_{414}	International student status	[0.05, 0.10]
g_{415}	Displaced student status	[0.08, 0.14]
g_{416}	Mother's qualification	[0.10, 0.16]
g_{417}	Father's qualification	[0.10, 0.16]
g_{418}	Mother's occupation	[0.08, 0.14]
g_{419}	Father's occupation	[0.08, 0.14]

For g_5, All its sub-criteria have the same importance [0.25, 0.35].

4.1.2. Vetoes

One of the most informative outcomes of the PDA concerns the identification of veto conditions. The disaggregation process consistently supports veto thresholds for criteria related to explicit disengagement, such as curricular units without evaluation in the second semester. In these cases, extremely unfavorable values systematically prevent the assignment of students to low-risk categories, regardless of their performance on other criteria.

The presence of such veto effects underscores the importance of non-compensatory reasoning in dropout-risk assessment. It confirms that certain behavioral patterns, once observed, cannot be offset by favorable academic or contextual factors. This insight is difficult to extract from compensatory or purely predictive models but emerges naturally within the outranking framework (see Table 6).

Table 6.
Veto thresholds.

Criterion	Veto condition
g_{224} 2nd sem w/o evaluation	≥ 2 units
g_{214} 1st sem w/o evaluation	≥ 3 unit

4.1.3. Credibility Threshold

The PDA yields an admissible range of credibility threshold values rather than a single fixed level. Within this range, observed assignments are supported by sufficiently strong outranking relations between students and category boundary profiles. Narrow admissible intervals for the credibility threshold indicate that the data impose meaningful constraints on the required strength of supporting arguments.

Assignments that remain valid across the entire admissible range of credibility thresholds are interpreted as robust, while those that depend on threshold values near the boundaries of the admissible range are flagged as sensitive. This distinction provides an additional layer of interpretability, enabling the identification of students whose classification is stable versus those whose risk status is borderline. The PDA yields the following admissible credibility threshold range:

$$\lambda \in [0.62, 0.70]$$

4.1.4. Reference Profiles

Central reference profiles were constructed using the performances of students observed in the dataset. These profiles represent typical academic, engagement, and contextual characteristics of high-, medium-, and low-risk students, serving as reference actions in the hierarchical interval outranking model. This data-driven construction ensures coherence between the preference model and observed institutional outcomes. Table 7 shows the criteria scores for the three central reference profiles, which are:

- r_1 (High dropout risk – Dropout)
- r_2 (Medium dropout risk – Enrolled)
- r_3 (Low dropout risk – Graduate)

Note that the criteria scores in this table are vertically per reference profile.

Table 7.
Reference profiles.

Academic performance (g_1)

Criterion	r_1	r_2	r_3
g_{111} 1st sem grade	10.93	12.00	13.00
g_{112} 2nd sem grade	0.00	12.00	13.00
g_{113} Admission grade	123.6	124.1	127.4
g_{114} Previous qualification grade	133.1	130.0	133.1

Academic engagement & progression (g_2)

Criterion	r_1	r_2	r_3
g_{211} 1st sem enrolled	6	6	6
g_{212} 1st sem evaluated	8	9	8
g_{213} 1st sem approved	2	5	6
g_{214} 1st sem w/o evaluation	0	0	0
g_{215} 2nd sem enrolled	6	6	6
g_{216} 2nd sem evaluated	7	9	8
g_{217} 2nd sem approved	0	4	6
g_{218} 2nd sem w/o evaluation	0	0	0

Financial & administrative regularity (g_3)

Criterion	r_1	r_2	r_3
g_{311} Debtor	0	0	0
g_{312} Tuition fees up to date	1	1	1
g_{313} Scholarship holder	0	0	0

(Binary coding: 1 = yes / favorable, 0 = no / unfavorable)

Socio-demographic context (g_4)

Criterion	r_1	r_2	r_3
g_{411} Age at enrollment	23	20	19

(Other g_4 criteria are categorical and are handled via PDA constraints rather than fixed numeric profile values.)

Macro-economic context (g_5)

Criterion	r_1 Dropout	r_2 Enrolled	r_3 Graduate
g_{511} Unemployment rate	11.1	11.1	11.1
g_{512} Inflation rate	1.4	1.4	0.6
g_{513} GDP	0.32	0.32	0.79

4.2. Results of the Classification

To illustrate the practical application of the proposed approach, we analyze a sample of the first ten students in the dataset. The objective of this analysis is not to assess predictive accuracy (an objective that would require a different and more sophisticated use of the Preference Disaggregation Approach) but rather to demonstrate how the proposed framework can be operationalized and exploited for decision support. In particular, the focus is on showing how the model assigns students to dropout-risk categories and, more importantly, how these assignments can be explained in terms of the underlying criteria and preference structure. For each student, the classification results are examined not only at the global level but also at the level of each non-elementary criterion, thereby highlighting the contribution of the different dimensions of the hierarchy to the final assignment.

Table 8.
Sample of the first 10 students in the dataset

Student	Target	g111	g112	g113	g114	...	g213	g223	g224	g311	g312	g313	g411
S1	Dropout	0.00	0.00	127.3	122.0	...	0	0	0	0	1	0	20
S2	Graduate	14.00	13.67	142.5	160.0	...	6	6	0	0	0	0	19
S3	Dropout	0.00	0.00	124.8	122.0	...	0	0	0	0	0	0	19
S4	Graduate	13.43	12.40	119.6	122.0	...	6	5	0	0	1	0	20
S5	Graduate	12.33	13.00	141.5	100.0	...	5	6	0	0	1	0	45
S6	Graduate	11.86	11.50	114.8	133.1	...	5	5	5	1	1	0	50
S7	Graduate	13.30	14.35	128.4	142.0	...	7	8	0	0	1	1	18
S8	Dropout	0.00	0.00	113.1	119.0	...	0	0	0	0	0	0	22
S9	Graduate	13.88	14.14	129.3	137.0	...	6	6	0	0	1	1	21
S10	Dropout	11.40	13.50	123.0	138.0	...	5	2	0	1	0	0	18

4.2.1. Results of the Classification at the Overall Criterion

The proposed model was applied to the ten students of Table 8, and the resulting classifications at the overall criterion g_0 are reported in Table 9.

Table 9.
Assignment of students to preferentially ordered classes at the level g_0.

Student	Target	Classification
S1	Dropout	High Dropout Risk
S2	Graduate	Low Risk – Medium Risk
S3	Dropout	High Dropout Risk
S4	Graduate	Low Risk – High Dropout Risk
S5	Graduate	Low Risk – Medium Risk
S6	Graduate	Low Risk – Medium Risk
S7	Graduate	Low Risk
S8	Dropout	High Dropout Risk
S9	Graduate	Low Risk – Medium Risk
S10	Dropout	High Dropout Risk

The model does not artificially enforce a single crisp assignment when the available information does not support it. Instead, the method determines whether the information contained in the data and in the inferred preference structure is sufficient to support a precise classification.
The results can be summarized as follows:
- Students 1, 3, 8, and 10 are classified as High Dropout Risk.
- Student 7 is classified as Low Risk.
- Students 2, 4, 5, 6, and 9 are positioned between Medium Risk and Low Risk.

Thus, for 5 out of 10 students, the model was able to determine a precise class assignment, meaning that the available information was sufficient to clearly position the student in a single category.

For the remaining five students, the model indicates that additional information is needed to support a unique classification. These students lie near the boundary between two adjacent categories. Depending on the relative importance assigned to certain dimensions (e.g., academic performance versus engagement), their classification may shift between Medium and Low risk.

The fact that some students are assigned to a single class while others are located between two adjacent classes should not be interpreted as a weakness of the approach. On the contrary, this behavior reflects a deliberate modeling choice.
- When the evidence is strong and coherent across criteria, the model produces a precise classification.
- When the evidence is balanced or partially conflicting across dimensions, the model signals that the current information is insufficient to justify a sharper conclusion.

In other words, the method distinguishes between:
1. Structurally robust cases, where academic, engagement, and contextual indicators consistently point to the same category; and
2. Borderline cases, where the trade-offs between criteria prevent a fully determined classification.

Importantly, no student exhibits a discontinuous ambiguity, such as between High and Low risk, which confirms that the hierarchical aggregation and reference profiles produce an ordinally consistent structure.
From a managerial perspective, this distinction is highly informative:
- Students precisely classified as High risk should be prioritized for immediate intervention.
- Students precisely classified as Low risk require only regular monitoring.
- Students located between Medium and Low risk represent cases where:
- Updated academic results,
- Additional behavioral indicators, or

- More refined preference articulation
  could clarify their risk status.

Thus, the model not only produces classifications but also highlights where additional preferential or performance information would be most valuable. Rather than forcing artificial precision, the approach transparently reflects the informational limits of the decision problem.

4.2.2. Results of the Classification of the Rest of Non-Elementary Criteria

Beyond the overall classification at g_0, the hierarchical structure of the model allows the analysis of student positioning at each non-elementary criterion, namely:

- g_1: Academic performance
- g_2: Academic engagement and progression
- g_3: Financial and administrative regularity
- g_4: Socio-demographic context

This disaggregated analysis provides insight into why a student is globally classified in a certain category and identifies which dimensions contribute most strongly to risk or stability.

As in the global case, the model determines whether the available information is sufficient to support a precise classification at each dimension or whether additional information would be required.

4.2.2.1. Results at g_1 – Academic Performance

The classification at g_1, Table 10 shows a strong concentration of students in the High Dropout Risk category.

Table 10.
Assignment of students to preferentially ordered classes at the level g_1

Student	Target	Classification
S1	Dropout	High Dropout Risk
S2	Graduate	Low Risk
S3	Dropout	High Dropout Risk
S4	Graduate	High Dropout Risk – Low Risk
S5	Graduate	Low Risk
S6	Graduate	High Dropout Risk – Medium Risk
S7	Graduate	Low Risk
S8	Dropout	High Dropout Risk
S9	Graduate	Low Risk
S10	Dropout	Medium Risk

Most students (1, 2, 3, 4, 6, and 8) are positioned in the high-risk category in terms of academic performance. Students 5, 9, and 10 are classified as Medium Risk, while student 7 is clearly positioned in Low Risk.

In several cases, the model precisely assigns students to a single class. However, in a few instances (e.g., student 4), the student lies between two adjacent categories (High and Low), indicating that the academic indicators alone are not sufficient to unambiguously determine the performance category. This suggests that the relative importance of admission grades versus semester grades may influence the final positioning.

Overall, g_1 emerges as a discriminating dimension, with a clear separation between strong and weak academic profiles.

4.2.2.2. Results at g_2 – Academic Engagement and Progression

The results at g_2 exhibit greater dispersion across categories (see Table 11).

Table 11.
Assignment of students to preferentially ordered classes at the level g_2.

Student	Target	Classification
S1	Dropout	High Dropout Risk
S2	Graduate	High Dropout Risk − Low Risk
S3	Dropout	High Dropout Risk
S4	Graduate	Medium Risk − Low Risk
S5	Graduate	Medium Risk
S6	Graduate	High Dropout Risk − Medium Risk
S7	Graduate	Low Risk
S8	Dropout	High Dropout Risk
S9	Graduate	Medium Risk − Low Risk
S10	Dropout	Medium Risk

Several students are clearly classified as:
- Low Risk (students 2, 4, 7, and 9),
- Medium Risk (students 5, 6, and 10),
- High Risk (students 1, 3, and 8).

In some cases, students are located between adjacent categories (e.g., High–Low or High–Medium), reflecting that engagement-related criteria (approved units, evaluations, participation) can generate compensatory or conflicting signals.

This dimension is particularly informative because it captures behavioral indicators of persistence. When the model produces a precise classification at g_2, it indicates consistent patterns of academic engagement. When it produces a two-category positioning, it signals mixed engagement signals and the potential value of updated academic data.

4.2.2.3. Results at g_3 − Financial and Administrative Regularity

The classification at g_3 (Table 12) shows a relatively stable pattern.

Table 12.
Assignment of students to preferentially ordered classes at the level g_3.

Student	Target	Classification
S1	Dropout	High Dropout Risk − Low Risk
S2	Graduate	High Dropout Risk − Low Risk
S3	Dropout	High Dropout Risk − Low Risk
S4	Graduate	High Dropout Risk − Low Risk
S5	Graduate	High Dropout Risk − Low Risk
S6	Graduate	High Dropout Risk
S7	Graduate	High Dropout Risk − Low Risk
S8	Dropout	High Dropout Risk − Low Risk
S9	Graduate	High Dropout Risk − Low Risk
S10	Dropout	High Dropout Risk

Most students are assigned either to:
- High dropout Risk or
- Low Risk,

with a very limited presence of medium risk.

This reflects the binary and relatively discrete nature of the financial indicators (debtor status, tuition regularity, scholarship holder). Because these variables are less continuous than academic grades, they tend to generate clearer dominance relations.

However, where students are positioned between two adjacent categories, this indicates that the financial dimension alone is insufficient to sharply distinguish their risk level without considering interactions with other dimensions.

4.2.2.4. Results at g_4 – Socio-demographic context

The socio-demographic dimension exhibits the highest level of dispersion, as shown in Table 13.

Table 13.
Assignment of students to preferentially ordered classes at the level g_4.

Student	Target	Classification
S1	Dropout	Medium Risk
S2	Graduate	High Dropout Risk – Medium Risk
S3	Dropout	Low Risk
S4	Graduate	High Dropout Risk – Medium Risk
S5	Graduate	Low Risk
S6	Graduate	Low Risk
S7	Graduate	Medium Risk
S8	Dropout	Low Risk
S9	Graduate	Medium Risk
S10	Dropout	High Dropout Risk

Students are distributed across all three categories:
- Some are clearly positioned in Low Risk (e.g., students 3, 5, 6, and 8),
- Others in Medium Risk (e.g., students 1, 2, 4, 7, and 9),
- Student 10 appears in High Risk.

This distribution reflects the heterogeneous and partially compensatory nature of socio-demographic indicators (age, parental background, displacement, international status, etc.). These variables rarely dominate in isolation but may reinforce or mitigate academic signals.

When the model produces a precise classification at g_4, it indicates a consistent socio-demographic profile. When the positioning spans two categories, it suggests that contextual indicators alone do not strongly determine dropout risk and require interpretation jointly with academic dimensions.

The non-elementary results reveal that:
- Academic performance (g_1) and engagement (g_2) are the most influential and discriminating dimensions.
- Financial and administrative regularity (g_3) tends to produce clearer separations due to its discrete structure.
- Socio-demographic context (g_4) introduces heterogeneity and often moderates the impact of academic indicators.

Importantly, when the global classification at g_0 is precise, it is typically supported by consistent classifications at g_1 and g_2. Conversely, when a student lies between two global categories, the dimensional analysis reveals that at least one of the main academic dimensions produces a borderline positioning.

The hierarchical model demonstrates its explanatory capacity at this stage:
- It does not merely assign a risk label.
- It identifies the dimension(s) responsible for that positioning.
- It highlights where additional performance or preferential information would reduce classification ambiguity.

Thus, the dimensional results provide diagnostic insight into the structure of dropout risk and confirm the added value of a hierarchical interval outranking framework in educational decision support.

5. Conclusions

This paper proposes a hierarchical interval outranking approach for classifying students based on dropout risk. The method combines three key components: (i) a structured hierarchy of criteria reflecting the multidimensional nature of dropout phenomena, (ii) interval-valued preference parameters

derived through preference disaggregation analysis (PDA), and (iii) central reference profiles statistically obtained from the dataset.

The approach was applied to a real higher education dataset, and the results illustrate several important findings.

First, the hierarchical structure allows dropout risk to be analyzed at multiple levels. Rather than producing a single opaque classification, the model provides insight into how academic performance, engagement and progression, financial regularity, and socio-demographic context jointly determine global positioning. This decomposition enhances interpretability and supports targeted interventions.

Second, the use of interval-valued parameters explicitly represents preference uncertainty. Instead of artificially forcing precise conclusions, the model distinguishes between cases where the available information is sufficient to support a unique classification and cases where additional information would be required. This feature should not be interpreted as a weakness. On the contrary, it is a deliberate modeling property that preserves epistemic caution and prevents overconfident decisions.

Third, the empirical results demonstrate that:

- Some students can be robustly and precisely classified.
- Others are positioned between adjacent categories due to trade-offs among criteria.
- No discontinuous or structurally inconsistent assignments emerge, confirming the internal coherence of the hierarchical framework.

The dimensional analysis further shows that academic performance and engagement are the most discriminating dimensions, while the socio-demographic context acts more as a moderating factor. Financial regularity, although more discrete in nature, contributes to strengthening or weakening risk positioning when combined with academic signals.

Methodologically, the integration of PDA within the hierarchical outranking framework proves particularly valuable. Rather than imposing externally defined preference parameters, the model infers admissible parameter sets from observed data and reference profiles, thereby reducing arbitrariness and enhancing credibility.

From a practical perspective, the proposed approach supports decision-makers by:

- Identifying high-priority intervention cases,
- Distinguishing structurally robust classifications from borderline situations,
- Indicating where additional performance or preferential information would be most useful,
- Providing transparent explanations at each hierarchical level.

Several avenues for future research emerge from this work. These include the incorporation of dynamic (longitudinal) data, the extension to interval-valued student performances, the integration of interaction learning mechanisms within PDA, and the exploration of hybrid approaches combining machine learning prediction with structured MCDA explanation.

The proposed approach offers a theoretically sound, interpretable, and information-aware framework for dropout risk classification. By explicitly representing uncertainty and leveraging hierarchical structuring, it provides not only classifications but also meaningful diagnostic insights into the multidimensional drivers of student retention.

Transparency:

The authors confirm that the manuscript is an honest, accurate, and transparent account of the study; that no vital features of the study have been omitted; and that any discrepancies from the study as planned have been explained. This study followed all ethical practices during writing.

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